The Dynamics of Prices Under Nominal Interest Rate Policy

Stability of Interest Rate Instruments and Price Effects Under Four Typical Policies

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Abstract. Nominal interest rate policy is one of the most fundamental financial regulation tools in the world nowadays and it plays a rather important role in the economic stability and development of society. What exactly are the effects of interest rate hikes, rate cuts and various central bank interest rate strategies on the real economy? Is their impact policy stable? Under what conditions is it stable? We construct a model containing four typical nominal interest rate strategies under the ideal assumption of an arbitrage free economy to study the differential equations of nominal interest rate and price dynamics. The relationship between the two is shown by mathematical and computer methods such as system stability analysis, changing the initial value to study the dynamic change of the model with numerical simulation and MATLAB graphing. The model reveals that the interest rate system is often unstable and the initial value of interest rate policy is extremely sensitive to prices, and interest rate cuts do not necessarily lead to deflation or inflation. Moreover, the actual price impact of interest rate policy is not only related to the target rate but also to the path of interest rate adjustment. This suggests that, in practice, it is essential to adjust interest rate policy flexibly according to socioeconomic conditions.

Keywords: Interest Rate Policy · Inflation · Negative Interest Rate · Financial Stability

1 Introduction

In recent years, the new crown epidemic has swept through various countries around the world, and various industries in many countries have been affected, among which,

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the economies of various countries have been hit to different degrees. At the same time, monetary easing, lower interest rates and other financial policies have been widely used, which also include negative interest rate policies [2].

However, also for financial stimulus, direct issuance of money (the central bank directly increases M1) and interest rate cuts (increases M2 mainly by facilitating money turnover) have different effects on the economy. In general, direct issuance of money leads to inflation, but the relationship between interest rate cuts and the occurrence of inflation is uncertain [9], due to the fact that interest rate cuts generally have the effect of boosting production, which is expected to increase the demand for money. When the cost of acquiring capital is low, the confidence of manufacturers to expand production is enhanced and the cost and risk of borrowing funds is reduced, thus the size of the economy is often expanded. The increase in the aggregate supply of goods causes the economy's demand for the quantity of money, M2, to grow as well. In this case, the surplus of production may also trigger a fall in prices if the demand in the consumer market fails to expand effectively [5]. Therefore, the combined effect of the interest rate cut policy is stronger than the direct issuance of money M1, and the potential inflationary pressure may be less. Interest rate cuts are therefore less inflationary risky than direct issuance of consumption vouchers. Of course, inflation may also be triggered if employment continues to languish and the economy fails to receive a stimulus from the interest rate cut and the mere amount of money is released.

After two years of financial easing strategies [6], the United States enters a cycle of interest rate hikes starting around 2022 to curb the previously higher inflation rate. We would like to theoretically analyze the effectiveness and stability of this interest rate policy by establishing differential equations or systems of differential equations describing the dynamics of prices and nominal interest rates. In other words, is the effect of interest rate policy related to the initial value of prices and interest rates? Under what circumstances is the conventional policy effective?

Article Structure and Innovation:

In the Sect. 1, introduction, literature review and formulation of the problem are presented, and the parameter variables are defined.

In the Sect. 2, the interest rate equation model is derived from both consumption and investment, and the model structure is treated and expanded in the vicinity of the steady state to initially explore the stability of the interest rate system, the effectiveness of monetary policy and other issues.

In the Sect. 3, the dynamic equations and policy functions of price and nominal interest rates are studied under several typical nominal interest rate strategies and practical analysis is given.

In the Sect. 4, discussion and generalization, the strengths and weaknesses of the model are evaluated and directions for further improvement are given.

Main Innovation Points:

- Deriving the dynamic equation of nominal interest rates and prices from Fisher equation and price of capital.
- Abstractly summarizing nominal interest rate strategies into four types, and research the quite different policy effects that result from the different directions of price deviations from the steady state in this implementation process (Table 1).

Table 1. Main variables and functions.

	Symbol	Variable Name	Symbol
Nominal interest rate	i	Initial nominal interest rate	i_0
Nominal interest rate adjustment target	i_{aim}	Nominal interest rate adjustment limit	i_∞
Real interest rate	r	Initial real interest rate	r_0
Price level	p	time	t
Adjustment cycle	T	Coefficient of Logistics Function	h

2 Construction and Exploration of Interest Rate Equation Model

Assuming that p is the price level, for convenience, we model it in an ideal economy. Since there is no arbitrage in the ideal economy, we do not distinguish between the prices of consumer goods and capital goods. We derive the equations for the real and nominal interest rates in terms of both commodities and investment goods.

The real interest rate for money capital as a commodity is the physical rental value per unit of capital per unit of time, and the nominal interest rate is its monetary value-price, so that it should satisfy:

$$i = rp \tag{1}$$

Of course, this is for the more ideal market, if the commodity market and the capital market are separate and there are difficulties in liquidity, the prices of the two may not be the same.

Money capital as an investment good, based on the principle of equivalence between the value of physical and monetary investments, has the traditional Fisher equation, where the real interest rate, the nominal interest rate and the inflation rate (rate of price change) are related as follows:

$$(1 + i) = (1 + r)(1 + \dot{p}/p) \tag{2}$$

When all variables are not zero, taking the growth rate for both sides of Eq. (1), we get:

$$\dot{i}/i = \dot{r}/r + \dot{p}/p \tag{3}$$

Combining Eq. (2) and eliminating the real interest rate r, we obtain the differential equation describing the price and nominal interest rates:

$$\dot{p} = (p - 1)ip/(p + i) \tag{4}$$

Now exploring the stability of the system for this differential equation, we let $i = f(t)$. Then:

$$\dot{p} = f(t)p(p - 1)/(p + \mathrm{f(t)}) = \mathrm{F(t, p)}$$

Let F(t, p) = 0, and get: p = 1.
Taylor expansion of F(t, p) at p = 1 [1]:

$$\begin{aligned}\dot{p} &= F(t,1) + \partial F(t,1) \big/ \partial p(p-1) + o(p-1) \\ &= F(t,1) + f(t) \big/ (1+f(t))(p-1) + o(p-1)\end{aligned}$$

When $f(t) \big/ (1+f(t)) < 0$, we can get $-1 < f(t) = i < 0$, and Eq. (4) is stable at this point, indicating that the price system is stable under this interest rate policy, and this just shows the feasibility of the negative interest rate policy sometimes adopted in some countries such as Japan [3].

When $f(t) \big/ (1+f(t)) > 0$, we can get $f(t) = i > 0$, at this point Eq. (4) is unstable, indicating that the price system is unstable in this case and prices will deviate.

In the following we study the dynamics of prices in the ideal economy under several specific purely financial (without considering the regulation of real economic variables) nominal interest rate strategies.

3 Establishing Four Nominal Interest Rate Strategies and Further Exploring the Price Dynamics Model Based on Them

3.1 Constant Nominal Interest Rate Functions and Price Dynamics

Under an ideal no-arbitrage economy, holding the nominal interest rate constant is $i = i_0$ [7], then at this point, the price dynamics equation is as follows:

$$\dot{p} = (p-1)i_0 p \big/ (p+i_0) \tag{5}$$

Making dynamic images of prices with MATLAB:

The steady state of price p is 1. When the central bank sets four fixed nominal interest rate policies of "−0.01,0.01,0.03,0.05", as shown in Fig. 1, the effect of the same interest

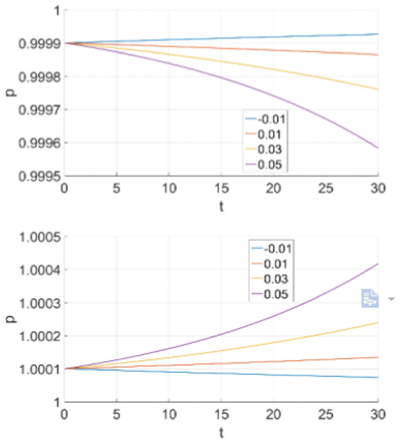

Fig. 1. Price dynamics for initial values less than 1 and greater than 1 under interest rate condition 3.1

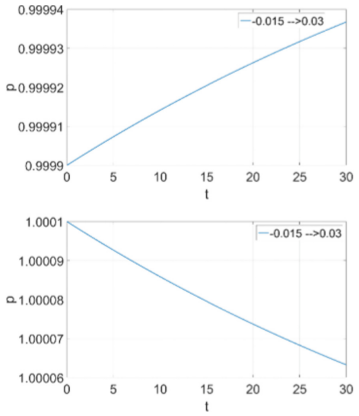

Fig. 2. The rate hike process when the initial price is less than 1 and greater than 1

rate policy in the following 30 periods (30 years) is different under different initial values of prices.

When the fixed nominal interest rate is negative, interest rate policy can have the policy effect of making prices less than 1 increase and making prices greater than 1 decrease, in which case the adjustment of prices is stable and the price level is a convergent damped vibration around the mean. When the nominal interest rate is greater than zero, a fixed nominal interest rate policy can only make prices greater than 1 larger and make prices less than 1 smaller, although the change is very slow.

3.2 Logistic Nominal Interest Rate Function and Price Dynamics

When the nominal interest rate slowly increases to a certain value, the trajectory of price dynamics is as follows:

$$i = i_\infty i_0 e^{ht} \Big/ (i_\infty + i_0(e^{ht} - 1)) \tag{6}$$

This nominal interest rate strategy is a gradual process from the initial value of i_0 to i_∞. And it can be either an increase or a decrease in interest rates.

$$\lim_{t\to\infty} i(t) = i_\infty, \quad \lim_{t\to o} i(t) = i_0$$

At this point, the dynamic equation for the price is as follows (Figs. 2 and 3):

$$\dot{p} = \frac{i_\infty i_0 p(p-1)e^{ht}}{p(i_\infty - i_0) + (p + i_\infty)i_0 e^{ht}} \tag{7}$$

In this case, the interest rate policy of raising interest rates makes more sense, and the process of raising interest rates sees the price trajectories that deviate from the steady state price of 1 all return to 1. High prices are suppressed and low prices are restored.

However interest rate cuts may not trigger inflation [8].When the initial value of prices is greater than the steady state price of 1, the interest rate cut does trigger price

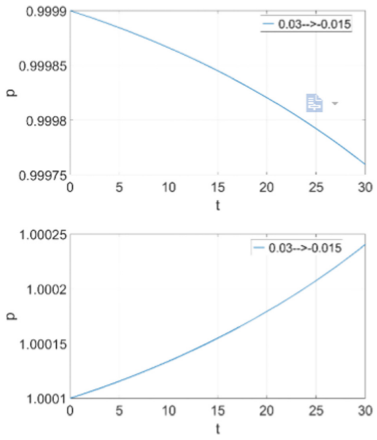

Fig. 3. The rate reduction process when the initial value of price is less than 1 and greater than 1

growth. However, when the initial value of prices is less than 1, the interest rate cut still leads to a decrease in prices and the economy has a deflationary tendency. Therefore, the interest rate cut does not necessarily trigger inflation, but depends on the initial value of prices.

3.3 Straight-Line Nominal Interest Rate Path and Price Dynamics

When the nominal interest rates adjust on a straight-line trajectory, the price dynamics trajectory is as follows:

$$i = \frac{1}{T}((T-t)i_0 + ti_{aim}) \tag{8}$$

At this point, the dynamic equation of the price is (Figs. 4 and 5):

$$\dot{p} = \frac{(p-1)(i_0T + (i_{aim} - i_0)t)p}{T(p+i_0) + (i_{aim} - i_0)t} \tag{9}$$

The dynamic adjustment curves of prices all show a U-shape when the nominal interest rate is increased from -0.015 to 0.03 in a uniform rate increase: an inverted U-shape that increases and then decreases when the initial value of the price is less than 1, and a U-shape that decreases and then increases when the initial price value is greater than 1.

When the nominal interest rate is reduced from 0.03 to -0.015 in a uniform interest rate cut, the dynamic adjustment curve of prices also shows a U-shape: a U-shape that decreases and then increases when the initial value of prices is less than 1, and an inverted U-shape that increases and then decreases when the initial value of the price is greater than 1.

The difference with the path of interest rate adjustment in 3.2: the movement of prices is monotonic under the gradual adjustment of the interest rate policy of the logistic curve. In contrast, under a straight-line interest rate policy adjustment, the price movement instead appears as a U-shaped [10].

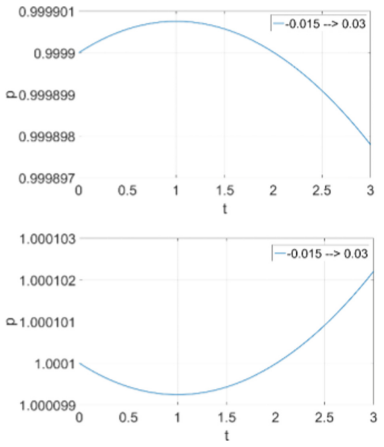

Fig. 4. The interest rate hike process when the initial price is less than 1 and greater than 1

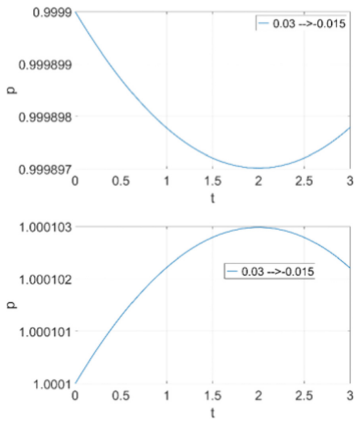

Fig. 5. The interest rate reduction process when the initial value of price is less than 1 and greater than 1

3.4 Cyclical Nominal Interest Rate Path and Price Dynamics

With cyclical nominal interest rates, price dynamics change with a cyclical T:

$$i = i_0(1 + \sin(2\pi T^{-1}t))\Big/2 \tag{10}$$

At this point, the dynamic equation of the price is (Fig. 6):

$$\dot{p} = \frac{i_0(p-1)(1+\sin 2\pi T^{-1}t)p}{2p + i_0(1+\sin 2\pi T^{-1}t)} \tag{11}$$

Under this policy of cyclical nominal interest rates [4], initial Price greater than 1 lead to waves of price increases, triggering potential inflation. In turn, price primaries less than one lead to successive waves of price declines, triggering deflation.

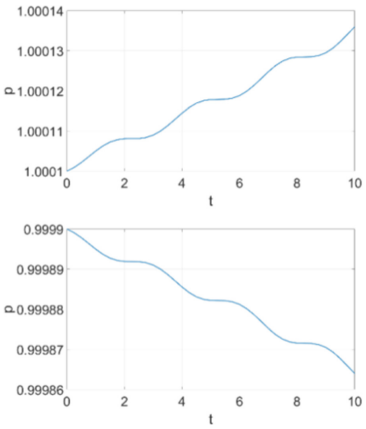

Fig. 6. Price dynamics when the initial value of prices is greater than 1 and less than 1 under cyclical nominal interest rate policy

4 Conclusion and Outlook

4.1 Conclusion

Firstly, interest rate policy is extremely sensitive to the initial value of prices, and raising or lowering interest rates does not necessarily lead to deflation or inflation.

Secondly, the real price impact of interest rate policy is not only related to the target interest rate, but also to the path of interest rate adjustment.

Thirdly, under negative nominal interest rates, the financial system steady state has stability, and perturbations that deviate from prices have a tendency to return to steady state.

4.2 Outlook

In reality, because physical goods have a limited shelf life and liquidity, and the acquisition of money capital requires certain qualifications, commodity prices and money prices are generally different. The economy does not easily form an ideal equilibrium. Then we have to distinguish between the price index on the capital side and the price index on the consumer side, which describes inflation as the price index on the consumer side, and its relationship with the nominal interest rate is perhaps more complex. Exploring its dynamics requires the involvement of more real economy variables, forming a more complex differential dynamical system.

In a later study, we will continue to discuss in depth the dynamic equations of the nominal interest rate, leading to a two-dimensional or even high-dimensional differential dynamical system with stability and path form analysis of its initial values and shocks to further reveal the nature of the financial system on the model.

References

1. Belke, A. & Klose, J. (2013), "Modifying Taylor reaction functions in the presence of the zero-lower-bound - Evidence for the ECB and the Fed", ECONOMIC MODELLING, Vol. 35515–527.
2. Berg, D. B., Shelomentsev, A. G., Khatmullina, O. I., Taubayev, A. & SGEM (2016) NEGATIVE INTEREST RATES POLICY, ECONOMIC CRISIS AND ENVIRONMENTAL MANAGEMENT. ECOLOGY, ECONOMICS, EDUCATION AND LEGISLATION CONFERENCE PROCEEDINGS, SGEM 2016, VOL III. 16th International Multidisciplinary Scientific Geoconference (SGEM 2016).
3. Czudaj, R. L. (2020), "Is the negative interest rate policy effective?", JOURNAL OF ECONOMIC BEHAVIOR & ORGANIZATION, Vol. 17475–86.
4. Duncan, R. (2014), "Institutional quality, the cyclicality of monetary policy and macroeconomic volatility", JOURNAL OF MACROECONOMICS, Vol. 39113–155.
5. Lucas, G. D. (2021), "The (dampened) wage-price spiral: Conflict, endogenous markup and inflation", STRUCTURAL CHANGE AND ECONOMIC DYNAMICS, Vol. 59558–566.
6. Lukas, L. (2019) Dynamic AD-AS macroeconomic model of Mankiw type with generalized expectations. IN HOUDA, M. & REMES, R. (Eds.) 37TH INTERNATIONAL CONFERENCE ON MATHEMATICAL METHODS IN ECONOMICS 2019. 37th International Conference on Mathematical Methods in Economics (MME).
7. MACKINNON, K. & SMITHIN, J. (1993), "AN INTEREST-RATE PEG, INFLATION AND OUTPUT", JOURNAL OF MACROECONOMICS, Vol. 15 No. 4, pp. 769–785.
8. Smirna, T. G. (2015) THE EFFECTS OF NEGATIVE NOMINAL INTEREST RATES ON ECONOMIC INTEGRATION. IN ROMAN, A., ANTON, S. G. & BILAN, I. (Eds.) EURO AND THE EUROPEAN BANKING SYSTEM: EVOLUTIONS AND CHALLENGES. International Conference on Euro and the European Banking System - Evolutions and Challenges.
9. Watt, R. (2009), "How does interest rate policy affect inflation? A simple general equilibrium model with the interest rate as the policy instrument", REVIEW OF ECONOMIC DESIGN, Vol. 13 No. 4, pp. 345–360.
10. Wu, S. Y., Hu, B. L. & Pan, Q. Z. (2019), "The Impact of the US Interest Rate Hike on Emerging Market Economies and the Belt and Road Initiative", CHINA & WORLD ECONOMY, Vol. 27 No. 3, pp. 126–142.