The Application of Modern Portfolio Theory in US Stock Market with the Use of Python Programming

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Abstract. The purpose of our study is to investigate the application of Modern portfolio Theory in the stock markets and design an investment strategy for risk-averse investors. The principle of the Modern portfolio theory is mean-variance analysis, that is we use mean and variance to measure the expected return and risk of portfolios respectively, and select an optimal portfolio based on it. The core idea is diversification of investments, which means to divide the investment into a combination of different financial assets or shares of different companies, such as stocks and securities. We investigate the adjusted closing stock price of five companies: Facebook, Amazon, Apple, Netflix and Google, from 2011 to 2021. Our results show that the minimum risk (annualized volatility at 23%) can be obtained when the expected annualized return is at level of 28%. A higher expected annualized return (at the level of 33%) might be achieved if the investors are willing to take more risk (annualized volatility at 25%).

Keywords: Modern Portfolio Theory · Investment · Finance · Stock Market · Python Programming

1 Introduction

Portfolio is a combination of financial assets including bonds, stocks, options etc. It focuses on minimizing the risk meanwhile maximizing the return. The concept of portfolio was introduced by Markowitz [3] (1952), which is the start of the methodology of financial mathematical analysis and developed by Sharp [6] (1964). The key idea of the portfolio theory is the mean-variance analysis.

MPTs have been applied in various areas. For example, it has successfully applied in US EB-5 [1] investment program, which helps investors worldwide achieve permanent residency or green cards in the US. EB-5 investors face 2 big risks: potentially not be able to get green cards and lose investment. The Atlantic American Opportunities Fund was such EB-5 portfolio fund that diversified the risk by having a number of different assets classes and from different geographic locations across US. The Atlanta American opportunities fund successfully conducts EB-5 with a portfolio model and has been approved. The fund disperses investors' funds into different projects and requires

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investors to select a specific project to meet the requirements of job creation. In other words, in order to meet the requirements of job creation, the success or failure of I-526 and I-829 applications still depends on the completion of a project, but the return and risk of investment funds are dispersed in the investment pool.

The combination of funds established by multiple projects can help investors spread risks. Project bridging funds can help such funds minimize construction risks and start the project in time before EB-5 funds are available. According to Markowitz's portfolio theory, the smaller the relevance of each item in the portfolio, the lower the volatility of the whole portfolio. In theory, the volatility of a portfolio composed entirely of technology company stocks or energy company stocks is greater than that of a portfolio containing stocks in different industries. Theoretically, this EB-5 portfolio fund can reduce investment volatility through the amount of funds in the portfolio, and may further reduce variables by adding different asset classes in different geographical locations in the United States to the portfolio. The lower the homogeneity of the project, the fewer variables of the portfolio according to the portfolio theory.

The investment portfolio has several extensions but also faces various challenges, since it is based on various assumptions. Based on MPTs, capital asset pricing model (CAPM) was proposed, which describes the linear relationship between the return and risk of a single stock or a portfolio [5]. However, MPTs relies on 2 assumptions. First, investors are rational and risk-averse, who seek to maximize returns on their investments. Second, the market is efficient, that is, the asset price has reflected all the information about the company. Fama [2] (1970) put forward the efficient market hypothesis theory which was the foundation of MPTs and has a wide influence on the market. He divided the stock market into three categories: weak, semi-strong and strong effective markets. Currently, it is generally believed that the market is not completely effective. Another popular theory is arbitrage pricing theory (APT), assuming an arbitrage pricing model in market equilibrium. It believes that the return on assets is affected by many factors, so it can be determined by the factor model [7]. The APT is an alternative to CAPM, it involves fewer assumptions but might be harder to implement. In addition, Rom and Ferguson (1993) [4] proposed the post-modern portfolio theory, which uses the downside risk to capture the negative returns.

The purpose of this paper is to introduce the modern portfolio theory and apply it in real stock markets, hence design strategies and methods for risk-averse investors.

2 Methodology

2.1 Assumptions

MPT is based on some assumptions. We give a brief description of some simplified assumptions. For readers who are interested in the complete list of assumption, we refer to Markowitz (1952).

- Investors are risk-averse, that is, investors would prefer high return when the risks are high. They choose higher return at a given level of risk or choose lower risk at a given level of return.

- Our portfolio performance is only determined by these expected return and risk, where the expected return is measured by the mean of returns and risk is measured by the variance of the expected returns.
- Market is competitive and efficient. Investors have access to all information and have expectation for the same investment.
- No transaction costs and no taxes.
- Stocks can be divided into infinitely many shares. The number of stocks purchased is not necessary to be an integer.

2.2 Model

We consider a portfolio and denote it by p. Assume there are n assets in this portfolio. The return of each individual asset by R_i and the expected return is $\mathrm{E}[R_i]$. Let w_i be the weight of money invested in each stock i, where $i = 1, \ldots, n$. The expected return of the portfolio, $E\left[R_p\right]$, is the weighted average return of individual stocks, which is given by

$$E\left[R_p\right] = \sum\nolimits_{i=1}^{n} w_i E[R_i] \tag{1}$$

where

$$\sum\nolimits_{i=1}^{n} w_i = 1, w_i \geq 0. \tag{2}$$

Basically, risk can be classified into two categories: systematic risk and unsystematic risk. By MPT, the unsystematic risk can be smoothed out by diversification, and only unsystematic risk left. Variance or standard deviation of the portfolio are applied to measure the risk. We use σ_p^2 represent the variance of the portfolio and σ_p the standard deviation of the portfolio, which are given by

$$\sigma_p^2 = \sum\nolimits_{i=1}^{n} \sum\nolimits_{j=1}^{n} w_i w_j \sigma_{ij} \tag{3}$$

And

$$\sigma_p = \sqrt{\sigma_p^2}. \tag{4}$$

Here σ_{ij} is the covariance between stocks i and j ($i = 1, \ldots, n; j = 1, \ldots n; i \neq j$), w_i and w_j stand for the weights of stocks i and j representatively. It shows that the risk of the portfolio depends on not only the risk of each stock but also the correlation of different stocks.

There are two strategies to select stocks in a portfolio. The first one is to maximize the expected return for a given level of risk, that is,

$$\max E\left[R_p\right] = \max \sum\nolimits_{i=1}^{n} w_i E[R_i] \tag{5}$$

subject to

$$\sigma_p^2 = \sum\nolimits_{i=1}^{n} \sum\nolimits_{j=1}^{n} w_i w_j \sigma_{ij} \tag{6}$$

And

$$\sum\nolimits_{i=1}^{n} w_i = 1, w_i \geq 0. \tag{7}$$

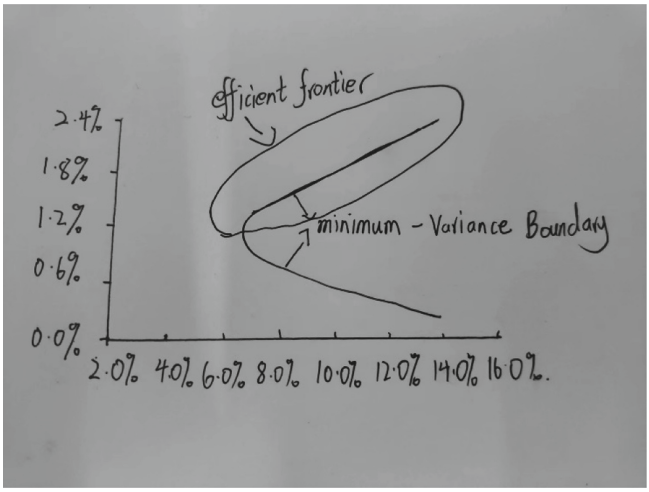

Fig. 1. Efficient frontier. (Photo credit: Original)

2.3 Efficient Frontier

The volatility of portfolios depends on the weights of individual assets in the portfolio and covariance between each other. By different weights, we can get the feasible region of all portfolios. For risk averse investors, they prefer higher returns and lower risk. Thus, we can find a frontier which gives us the lowest risk for a given level of expected return, which we call minimum-variance boundary of the feasible region.

The efficient frontier is a set of portfolios that obtain the maximum return at a given a risk or the lowest risk for a given level of expected return. It is the top of the minimum-variance boundary, as shown in Fig. 1. Here the $y-$ axis represents the expected return of the portfolio, and the $x-$ axis is the standard deviation of the expected return, which shows the risk the portfolio. The portfolios that lie on the curve are considered as optimal. They are the portfolios that gives highest level of return at any level of risks. Those on the right to it are considered as sub-optimal because risks are higher for the determined return. While those below the curve are also sub-optimal because return are lower for the given risk. It is impossible for any existence of portfolios at its left.

The efficient frontier can provide investors with direct investment suggestions with the aim to construct an optimal portfolio with maximum investment return. It can help investors to analyse the potential risk and return, and then choose the best investments.

2.4 Sharpe Ratio

The Sharpe ratio shows the risk-adjusted performance of a portfolio. It measures the trade-off between risk and return of a portfolio, which is given by the portfolio's risk premium or divided by its volatility. This formula is defined as

$$Sharpe\ Ratio = \frac{E(R_p) - r_f}{\sigma_p} \tag{8}$$

where $E(R_p)$ represents the expected return of the portfolio, r_f is the risk-free rate. We use $E(R_p) - r_f$ to measure the risk premium or the excess of the portfolio. The σ_p represents the standard deviation of the risk. Sharpe ratio is a useful tool that it can help investors understand the return to risk. Investors would prefer portfolios with higher Sharpe ratio, because it gives higher return or lower risk.

3 Data Analysis

With the aim to apply the mean-variance analysis in the U.S. stock market, we consider the stocks of five successful companies but from different industries. They are Facebook, Amazon, Apple, Netflix and Google. We use the adjusted closing price of those shares over 10 years, from 2011 to 2021, obtained from Yahoo Finance[i]. In our case, 25000 portfolios are produced by randomly generated weights. The risk-free rate of the portfolios chosen to be 0.1%, which is rate of the 52-week treasury Bill provided by the U.S. Department of TheTreasury[ii].

Figure 2 shows the adjusted price of shares of five companies over 10 years. All five companies' price have shown an increasing trend. The closing prices of Amazon and Google increase sharply and much higher than the other three. Meanwhile Apple's prices do not vary a lot, it shows a stable and smooth increase over the ten years, so do Facebook and Netflix'.

Figure 3 shows the daily returns of the five companies. We can see that Netflix, Amazon and Facebook are volatile since Netflix has both highest positive and negative spikes and Amazon and Facebook also have significant spikes. Compared with these three companies, Google and Apple are more stable which means they are less risky stocks. Figure 4 shows the cumulative daily return, the sum of total returns on consecutive days. Before 2017, the daily cumulative return of the five companies at a similar level. After 2017, the daily cumulative return of Netflix increases most rapidly with a relatively large extent of fluctuation, followed by Amazon Their daily cumulative return is less volatile compare with that of Netflix. Apple's increased smoothly and in the year of

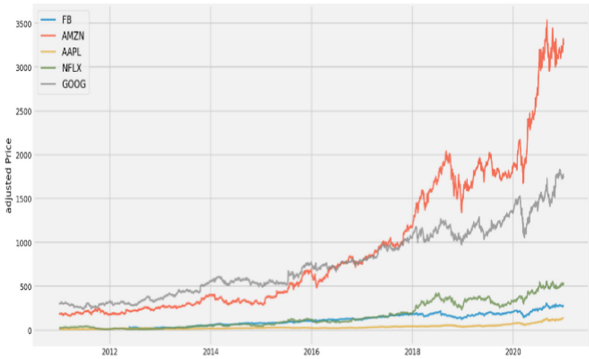

Fig. 2. Adjusted closing prices of Facebook, Amazon, Apple, Netflix and Google from 2011 to 2021. (Photo credit: Original)

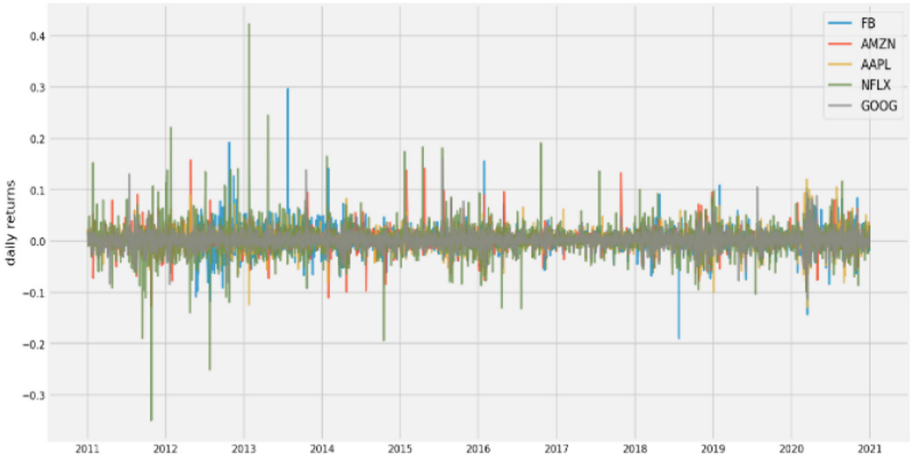

Fig. 3. Daily returns of Facebook, Amazon, Apple, Netflix and Google from 2011 to 2021. (Photo credit: Original)

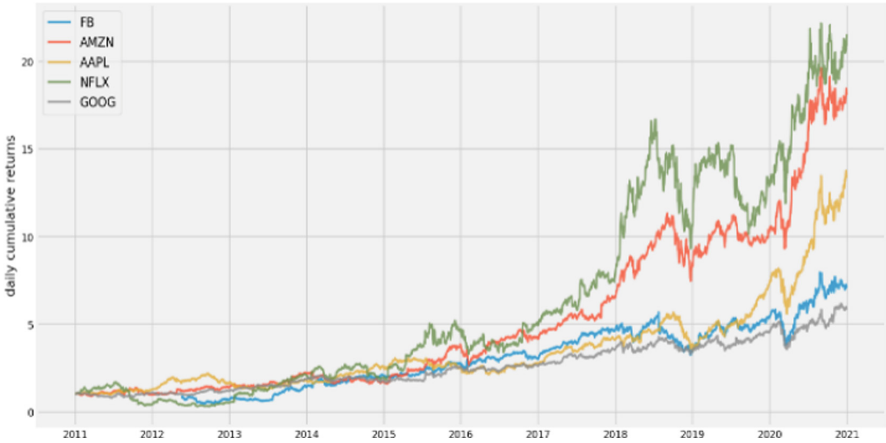

Fig. 4. Cumulative returns of Facebook, Amazon, Apple, Netflix and Google from 2011 to 2021. (Photo credit: Original)

2020, it experienced a large extent of increase. Facebook and Google increase smoothly. In general, we can recognize a sharp decrease in their daily cumulative return.

In Fig. 5, the blue dots represent all feasible portfolios. We randomly generate 25000 weights and obtain 25000 portfolios corresponding to these weights. The red star represents the portfolio with highest Sharpe Ratio, the green star represents the portfolio with lowest volatility. As the dots get closer to the red star, the colour of the dots gets darker, which means that their Sharpe Ratio is higher. The top line of the blue area is the efficient frontier. We consider the investment portfolios on its right as inefficient portfolios.

There are two strategies for investment. The first is to choose the one with higher return. The portfolio represents by the red star is preferable for risk-averse investors. They

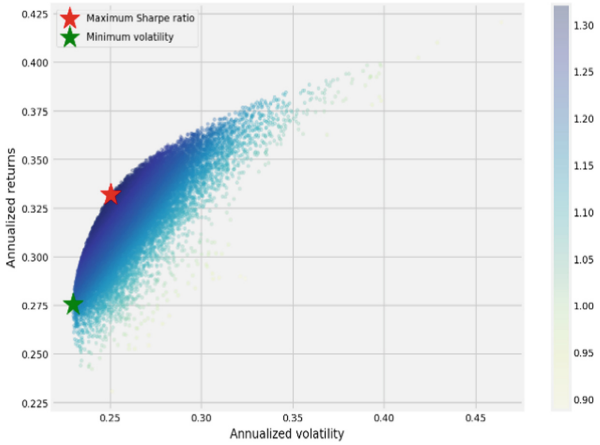

Fig. 5. Portfolio optimization and efficient frontier. (Photo credit: Original)

Table 1. Allocation of portfolios with maximum Sharpe Ratio and minimum volatility.

Maximum Sharpe Ratio Portfolio Allocation					
Annualised Return: 0.33					
Annualised Volatility: 0.25					
Companies	Facebook	Amazon	Apple	Netflix	Google
Allocation (weights%)	10.55	35.35	40.16	13.46	0.48
Minimum Volatility Portfolio Allocation					
Annualized Return: 0.28					
Annualized Volatility: 0.23					
Companies	Facebook	Amazon	Apple	Netflix	Google
Allocation (weights%)	7.83	15.09	35.09	3.04	38.95

choose to bear some risk in order to get higher return. As shown in Table 1, if 75.51% of the total investment is invested in Amazon and Apple, and around 24% in Netflix and Facebook, and the rest of the investment in Google. We can obtain the annualized return is 0.33, and the annualized volatility is 0.25. A large portion is in Amazon, because Amazon and Apple have relatively high daily cumulative returns and less volatility, and Apple has lowest adjusted price. Netflix enjoys high cumulative return, but also high adjusted price and high volatility. Since Google generates lowest return, so we only invest a little in it. This combination of portfolios gives relative higher cost and risk, but it has highest Sharpe ratio.

Second strategy is to choose the portfolio with lowest volatility. This portfolio is preferable for conservative investors. They would like to sacrifice some return to seek for low risk. In the portfolio represented by the green star, the annualized return is 0.28,

and annualized volatility is 0.23. We can see that 75.04% of the total investment is invested in Google and Apple. This is because Apple and Google's daily return are less volatile according to Fig. 3. There is 15.09% invested in Amazon, because Amazon has high level of daily cumulative return.

4 Conclusion

In this paper, we study the modern portfolio theory and its application in the U.S. stock market. Modern portfolio theory is a widely used tool for portfolio selection, especially for risk averse investors. The core idea of it is to use mean-variance analysis to measure the performance of portfolios (risk and return), and thus, we can provide investment strategies to investors.

We consider portfolios of five companies: Facebook, Amazon, Apple, Netflix and Google, using the adjusted closing price of those shares from 2011 to 2021. We construct 25000 portfolios by randomly generated weights and generate an efficient frontier, which is a set of portfolios with highest expected return at a certain level of risk. Our results show if 75.51% of the total investment is invested in Amazon and Apple then we can obtain highest risk-adjusted return portfolio, and if 75.04% of the total investment is invested in Google and Apple, then we can obtain minimum risk portfolio.

There are several advantages. Investors can apply the theory to allocate investment, hence risks will be distributed into a wider investment pool. This can be used to spread unsystematic risks. In addition, if investors hold stocks from different companies, their risk can be offset. As the price for some stocks are falling, some stocks' price may increase, hence the total loss may be offset. By using diversification, risks will hence be distributed into more investment portfolios. This method is easy to understand and implement. It analyses the return and risk of different portfolios and generate combinations of portfolio for different preferences. Return and risks are calculated by the mean and variance.

However, there are some limitations. MPT cannot be used to eliminate systematic risks, for example the war, economic recession.

Meanwhile, the theory is based on some assumptions. The theory is not appropriate for risk lover investors, who are willing to accept a very high level of risk so that they can generate a very high level of return. We also assume that the market is competitive and efficient. That is investors have access to all information and have expectation for the same investment. But in real life, we believe that the market is not completely efficient, since the price in the market does not reflect all the changes in the market, hence investors cannot have access to all information in the market. In addition, not all of the investors are rational. Some markets are not competitive, such as the existence of monopoly and oligopoly. Meanwhile, we assume that there are no transaction costs and taxes. However, taxes and transaction costs exist in the real markets. Hence the accuracy of the MPT will be affected. Another assumption is that stocks can be divided into infinitely many shares, that is the number of stocks purchased is not necessary to be an integer. But we can only buy positive integer number of shares in the stock market. All these factors effect the accuracy of the result of implementing modern portfolio theory. Investors have to analyse any changes that exist in the market or have effects on the market.

In this article, I had given a solution of investment in US stock market. For future investigation, I would like to investigate the effect of the efficiency of market on the application of MPT. By this research, we can further improve the MPT so that it would be more accurate and helpful to generate more benefits for investors. Meanwhile, due to the future changes in technology, cultural differences and business activities, there may be great differences in the stock market in different countries. Data may not be able to reflect those changes just in time. I would like to investigate the difference between the market in China and America to make contribution for adjusting and improving Chinese stock market. However, due to lack of ability and effort, I was not able to achieve this. I wish in the future I or somebody can contribute more to the improvement on MPT or Chinese stock market, hence there will be more people benefited.

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