GARCH-Class Analysis of Bitcoin—A Comparison with Gold

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Abstract. Bitcoin establishes itself as an investment asset and is often named the New Gold. This study, however, shows that the two assets are different in univariate and multivariate aspects. First, we construct GARCH, APARCH and APARCH-in-Mean models to analyze and compare conditional variance properties of Bitcoin and Gold, and find Bitcoin does not have the significant inverse leverage effect as Gold. Then we apply the BEKK-GARCH model to estimate time-varying conditional correlations between Bitcoin and Gold with other major market indexes. The results show that Bitcoin can not hedge the market risk, especially when a crash occurs. So we conclude that Bitcoin and Gold feature fundamentally different properties as assets and linkages to equity markets.

Keywords: Bitcoin · APARCH-M · Leverage Effect · BEKK · Dynamic Correlation

1 Introduction

In recent years, Bitcoin has been favoured progressively because of its numerous excellent characteristics, such as high security and decentralization. At the same time, Bitcoin can serve as an investment vehicle, which has attracted a lot of venture capitalists. However, with the great uncertainty and lack of regulation in the cryptocurrency market, its risks cannot be ignored. Here, instead of exploring the Bitcoin market itself [7], we focus on whether Bitcoin has the same fine properties as other precious metal—Gold.

Our research is mainly divided into two parts. The first part will focus on the univariate volatility of Bitcoin return. According to the study of Blau (2017) [3], until 2014, the volatility of Bitcoin return is not related to speculative trading. This is contrary to the findings of Cheah and Fry (2015) [5] and Cheung, Roca, and Su (2015) [1]. In our study, we hope to establish a univariate volatility model, which can analyze the volatility structure of Bitcoin.

The leverage effect is defined as a phenomenon where past negative returns increase volatility more than positive returns of the same magnitude do [8]. Regarding cryptocurrencies, Dyhrberg (2016) [2] reports an insignificant leverage effect of Bitcoin. However, Catania and Grassi (2017) [6] found the inverse leverage effect was significant. This is the main contention where our study will put concentration on.

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The second part of our research is to explore the hedged nature of Bitcoin and compare it with Gold. We apply the definition of hedge by Baur and Lucey (2010) [4]: if an asset is uncorrelated or negatively related to another asset, we consider this asset to have a hedge function. In this part, we used the BEKK-GARCH model to regress the dynamic correlation between Bitcoin and other assets. Furthermore, we studied the difference in the performance of Bitcoin and Gold in a short window period when the market was in turmoil. Thus, we can compare Bitcoin's performance with Gold's during market distress and judge whether Bitcoin can serve as a safe haven from a deeper insight.

2 Data and Preliminary Analysis

In our study, six sets of time series data were used: Bitcoin prices, prices of Gold and Silver in USD per oz, crude oil prices for the West Texas Intermediate (WTI), the S&P 500 index and the Morgan Stanley Capital International (MSCI) World Index. The period is from 2014-01-01 to 2020-09-25, and the daily closing price is adopted (denoted by P_t). The Bitcoin data came from *coindesk.com*, and the prices of other assets were obtained from *Datastream.*

Since Bitcoin and the MSCI World Index are traded continuously, and other assets are traded on working days, we only use working-days data to match these six sets of time series. We calculated the natural logarithm of the prices and took differences between consecutive twos to obtain the daily log returns in percentage: $r_t = 100 \times \log(P_t/P_{t-1})$ and ended up with 1691 observations for each asset.

Table 1 summarizes the descriptive statistics and preliminary time series tests for these assets. Std. Dev. stands for standard deviation, Min. and Max. are minimum and maximum values. For hypothesis testing, we denote the 10%, 5% and 1% levels of

Table 1. DESCRIPTIVE STATISTICS AND PRELIMINARY ANALYSIS

	Bitcoin	Gold	Silver	WTI	S&P 500	MSCI World
Mean	0.16	0.02	0.01	0.00	0.03	0.02
Std. Dev.	4.62	0.99	1.71	3.40	1.13	0.95
Min.	−31.59	−10.60	−12.35	−28.14	−12.77	−10.44
Max.	24.99	10.47	7.57	42.58	8.97	8.41
Skewness	−0.24	0.11	−0.57	1.36	−1.06	−1.52
Kurtosis	4.95	17.53	7.18	37.65	22.26	24.83
Jarque-Bera	1745.35 ***	21716.63 ***	3733.06 ***	100672.68 ***	35341.09 ***	44218.83 ***
Ljung-Box(20)	20.08	38.35 ***	40.95 ***	163.28 ***	443.78 ***	284.55 ***
ADF	−10.77 ***	−12.029 ***	−11.16 ***	−10.84 ***	−11.23 ***	−11.43 ***

significance by *, ** and ***. Jarque-Bera test checks the distribution's normality, and the significance level of *** can verify that these six groups of log return data all do not follow the normal distribution. Ljung-Box(20) is the simultaneous test for autocorrelation conducted at the 20^{th} lag. Except for Bitcoin, the test results for the remaining assets strongly reject the null hypothesis of no autocorrelation from first to twentieth lags. Lastly, ADF is the augmented Dickey-Fuller test, and the test results reject the null hypothesis of a unit root, which means all series are stationary.

3 Methodology

3.1 Univariate Volatility Modelling

In this section, we focus on Bitcoin volatility modelling itself. We start with general features like volatility clustering and leverage effect of financial data, construct models that can describe these features, and further find out the unique properties of Bitcoin.

First, we check the distribution of the Bitcoin log return series. The horizontal axis of Fig. 1 represents the asset's log return in percentage, and the vertical axis is the density. We use a red dash line to depict the Bitcoin log returns and a solid blue line to represent normal-distributed density. The difference between sample density and normal density shows that a t-distribution is suitable. The extremely small p-value ($p < 2.2 \times 10^{-16}$) in the Jarque-Bera test convinced our choice. Figure 2 plots the daily log return series of Bitcoin, indicating the series has no unit root, and the ADF test confirmed that it is stationary. Besides, the insignificant coefficient in Ljung-Box(20) test and the within-the-bound Auto Correlation Function (ACF) coefficients in Fig. 3 all indicate that there is no autocorrelation in the Bitcoin log return series.

In order to characterize the volatility structure of Bitcoin log returns, we employ the Generalized Auto-Regressive Conditional Heteroskedasticity (GARCH) family models and start with the classical GARCH(1,1) model:

$$r_t = \mu + \varepsilon_t \tag{1}$$

$$\varepsilon_t = \sigma_t \eta_t,\ \eta_t \sim i.i.d.\ \mathrm{t}_\upsilon(0, 1) \tag{2}$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{3}$$

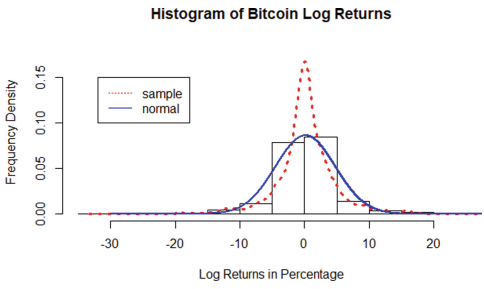

Fig. 1. Histogram of Bitcoin log returns

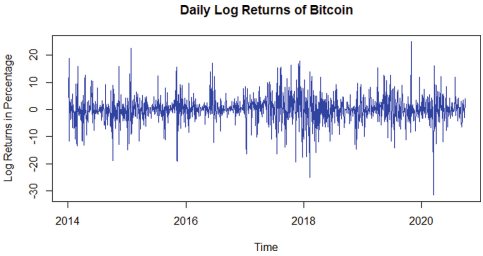

Fig. 2. Time series plot of Bitcoin log returns

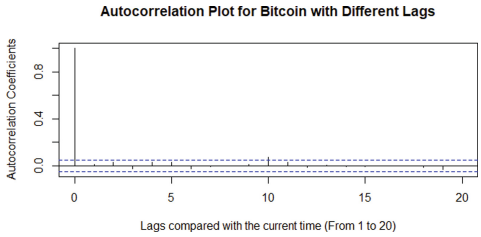

Fig. 3. Autocorrelation plot of Bitcoin log returns

Equation (1) models the expected return, in which r_t is the log returns we obtained above, μ stands for the conditional mean of r_t and ε_t for the residuals. Equations (2) and (3) are constructed to fit the conditional variance. We denote the conditional volatility by σ_t. Standardized residual η_t is an independent and identically distributed (*i.i.d.*) random variable, following a t-distribution with the degree of freedom υ. Finally, ω is the intercept for σ_t^{δ} term, while α and β are the ARCH(1) and GARCH(1) coefficients, respectively.

However, in the financial time series, large negative residuals in returns tend to increase future volatility more than positive ones, widely acknowledged as the "leverage effect". The standard GARCH model given above can not describe such effect due to the conditional variance σ_t^2 being the function of ε_t^2—whether the past values of ε_t are positive or negative is not taken into account.

To overcome this deficiency, we exploit the APARCH model raised by Ding, Granger and Engle in 1993 [11]. APARCH is an acronym standing for the Asymmetric Power Auto-Regressive Conditional Heteroskedasticity, and APARCH(1,1) is constructed as follow:

$$g_{\gamma}(\varepsilon_{t-1}) = \alpha(|\varepsilon_{t-1}| - \gamma\varepsilon_{t-1})^{\delta} \tag{4}$$

$$\sigma_t^{\delta} = \omega + g_{\gamma}(\varepsilon_{t-1}) + \beta\sigma_{t-1}^{\delta} \tag{5}$$

We use Eqs. (4) and (5) to replace the variance model expressed in Eqs. (3). Compared with traditional GARCH models, we newly introduce the power parameter δ and the leverage parameter γ. The parameter δ determines the appropriate power to model the standard deviation σ_t, more flexible than the standard GARCH models. The leverage parameter γ allows the effect of ε_{t-1} upon σ_t passing through the function $g_{\gamma}(\varepsilon_{t-1})$:

when $\gamma > 0$, $g_\gamma(\varepsilon_{t-1}) > g_\gamma(-\varepsilon_{t-1})$ for any negative ε_{t-1}, thus the negative residuals ε_{t-1} would increase the volatility more considerable. So we say that γ captures the leverage effect of the log returns. A significant negative value of γ is the evidence of "inverse leverage effect", which is a leverage effect in the opposite direction to what is mentioned above—past positive residuals ε_{t-1} would have a more substantial impact on current volatility than past negative residuals of the same magnitude.

Further ameliorating the fitting, we adopt the APARCH-in-Mean (APARCH-M) model, proposed by Engle, Lilien and Robin in 1987 [9], to better account for the unique properties of financial assets.

APARCH(1,1)-M model takes the form:

$$r_t = \mu + \lambda\sigma_t^\delta + \varepsilon_t \tag{6}$$

$$\varepsilon_t = \sigma_t\xi_t, \xi_t \sim i.i.d.\ \mathrm{t}_\upsilon(0,1) \tag{7}$$

$$\sigma_t^\delta = \omega + \alpha(|\varepsilon_{t-1}| - \gamma\varepsilon_{t-1})^\delta + \beta\sigma_{t-1}^\delta \tag{8}$$

The additional parameter λ in Eq. (6) allows us to model the risk premium. According to modern portfolio theory, the increased volatility leads to increased risk, and investors would require higher expected returns for additional risk as a reward. The presence of λ successfully captures this feature and enables the model to better fit financial time series.

We first apply the APARCH(1,1)-M model to the Bitcoin series for fit testing. The estimated results of the model are as follows:

$$r_t = 0.103 + 0.003\sigma_t^{0.841} + \varepsilon_t \tag{9}$$

$$\varepsilon_t = \sigma_t\xi_t, \xi_t \sim i.i.d.\ \mathrm{t}_{2.54}(0,1) \tag{10}$$

$$\sigma_t^{0.841} = 0.109 + 0.231(|\varepsilon_{t-1}| + 0.011\varepsilon_{t-1})^{0.841} + 0.849\sigma_{t-1}^{0.841} \tag{11}$$

To test the goodness of fit, we plotted the ACF of the squared standard residuals ξ_t. Figure 4 shows that the autocorrelation coefficients are all within the test bound of 0.05, which indicates no significant autocorrelation in the squared standardized residuals. So we have sufficient evidence to say that APARCH(1,1)-M is a suitable fitting model, and we will use it to fit all six sequences in the following part.

3.2 Dynamic Correlation Modelling

From a perspective of market linkages, we modelled conditional heteroskedasticity to compare Bitcoin and Gold concerning the markets. We applied the BEKK-GARCH model proposed by Engle and Kroner in 1995 [10]. The construction of the model is as follows:

First, we let $\boldsymbol{r}_t$ be a k-dimensional vector of log returns observations at time t, denoted as:

$$\boldsymbol{r}_t = \boldsymbol{\mu}_t + \boldsymbol{\varepsilon}_t \tag{12}$$

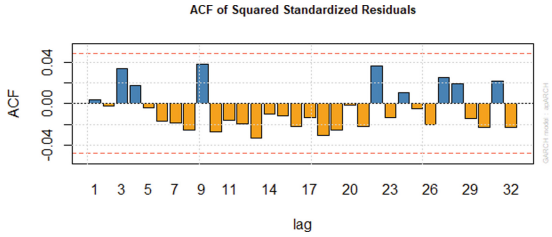

Fig. 4. Autocorrelation of squared standardized residuals

where $\boldsymbol{\mu}_t$ is k-dimensional conditional mean and $\boldsymbol{\varepsilon}_t$ is a k-dimensional vector. Assuming $\boldsymbol{\varepsilon}_t|F_{t-1} \sim N(0, \mathbf{H}_t)$, we model the conditional heteroskedasticity:

$$\boldsymbol{\varepsilon}_t = \mathbf{H}_t^{1/2}\boldsymbol{\zeta}_t \tag{13}$$

Equation (13) expressed how the residual vector $\boldsymbol{\varepsilon}_t$ is decomposed: $\mathbf{H}_t$ is a k × k sized conditional variance matrix, and we take the square root of it. $\boldsymbol{\zeta}_t$ is a k-dimensional vector of independent and identically distributed (*i.i.d.*) standard normal random variables with zero mean and $\boldsymbol{\zeta}_t$ satisfies $\boldsymbol{E}\left[\boldsymbol{\zeta}_\tau,\ \boldsymbol{\zeta}'_\tau\right] = \boldsymbol{I}_k$.

Next, the BEKK-GARCH model could be used to determine the matrix $\mathbf{H}_t$:

$$\mathbf{H}_t = \mathbf{C}\mathbf{C}^\top + \mathbf{A}\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}^\top\mathbf{A}^\top + \mathbf{G}\mathbf{H}_{t-1}\mathbf{G}^\top \tag{14}$$

$$\mathrm{C} = \begin{pmatrix} c_{11} & 0 \\ c_{21} & c_{22} \end{pmatrix} \tag{15}$$

$$\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}^\top = \begin{pmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon_{2,t-1}^2 \end{pmatrix} \tag{16}$$

$$\mathbf{A} = \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix} \tag{17}$$

$$\mathrm{G} = \begin{pmatrix} g_{11} & 0 \\ 0 & g_{22} \end{pmatrix} \tag{18}$$

Equation (14) is the definition of $\mathbf{H}_t$, and the complete expression of $\mathbf{H}_t$ can be obtained by plugging Eqs. (15) to (18) into Eq. (14). $\mathbf{A}$ and $\mathbf{G}$ diagonal matrices, and $\mathbf{C}$ is the lower triangle matrix, all of which are parameter matrices. Based on the symmetric parameterization of the model, if $\mathbf{C}\mathbf{C}^\top$ is positive-definite, then $\mathbf{H}_t$ is positive-definite almost everywhere. The model allows for dynamic dependencies between volatility series.

Since we compare the pairwise correlation of Bitcoin and Gold with other markets and assets, we set $k = 2$ and apply the BEKK-GARCH model.

4 Results

4.1 Estimation Result for Univariate Volatility

According to the layer-by-layer analysis in the Methodology part, we plug the six groups of time series data into the APARCH(1,1)-M model for fitting.

Table 2. ESTIMATION RESULTS OF APARCH(1,1)-M MODEL

	Bitcoin	Gold	Silver	WTI	S&P 500	MSCI World
ω	0.11 **	0.02 ***	0.06 *	0.02 ***	0.04 ***	0.03 ***
α	0.23 ***	0.05 ***	0.04 ***	0.07 ***	0.14 ***	0.11 ***
β	0.85 ***	0.95 ***	0.94 ***	0.94 ***	0.86 ***	0.88 ***
γ	−0.01	−0.47 **	−0.23 ***	0.87 ***	0.99 ***	1.00 ***
δ	0.84 ***	1.05 ***	2.34 ***	0.83 ***	0.80 ***	0.92 ***
υ	2.54 ***	3.68 ***	3.38 ***	5.27 ***	5.56 ***	6.34 ***
LL	−4562.12	−2085.57	−2937.27	−3641.23	−1841.29	−1614.15
BIC	5.50	2.53	3.56	4.39	2.24	1.97

Table 2 lays out the estimation result of the APARCH(1,1)-M model. Statistically significant parameters are indicated with asterisk *, **, *** for 10%, 5%, and 1% significance level.

In addition to the estimated parameters, we also tabulate LL and BIC in the table, which stands for the Log-Likelihood value and Bayesian Information Criterion, respectively.

We will first focus on the leverage effect parameter γ. The Table 2 shows that although Bitcoin's γ coefficient shares the same negative sign with Gold and Silver, it is not significant. In other words, we cannot reject the null hypothesis that γ of Bitcoin's log return series is equal to zero. This distinction strongly points to the first difference between Bitcoin and Gold: Gold has a significant "inverse leverage effect" while Bitcoin does not.

The γ values of the remaining three assets are significantly greater than zero, showing a significant leverage effect—past negative residuals in return increase current volatility more substantially than positive residuals of the same magnitude do, which is a feature shared by much financial time series.

From the estimation results of the power parameter δ, we can figure out the coefficient of Silver is around two and is very significant, indicating that modelling variance appears better than modelling volatility for Silver. The δ estimates of remaining assets, including Bitcoin and Gold, are significantly around one, suggesting that modelling volatility is a better choice.

Lastly, in all cases, the estimated degree-of-freedom parameter ν in the APARCH(1,1)-M model are highly significant.

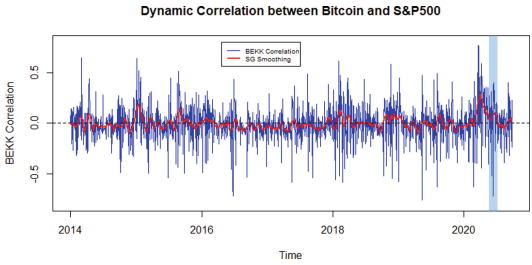

Fig. 5. Dynamic correlations of Bitcoin and S&P 500

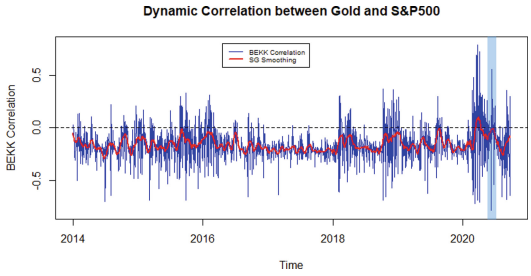

Fig. 6. Dynamic correlations of Gold and S&P 500

4.2 Dynamic Correlations of Bitcoin and Gold to Financial Markets

In this part, we analyzed the dynamic correlation between Bitcoin and S&P 500 and compared it with the dynamic correlation between Gold and S&P 500 based on the BEKK-GARCH framework. In Figs. 5 and 6, we use the navy-blue line to plot the unfiltered BEKK correlations, the red line to show the smoothed correlations by the Savitzky-Golay-smoothing method.

In view of Baur and Lucey (2010) [4], the time of market turmoil or distress is of particular interest in analyzing the hedge function of a specific asset. Here we choose the COVID-19 epidemic, a profound and sudden global blow, to denote a market crash. By definition provided by Wikipedia, this period of market distress is labelled as "the 2020 stock market crash", which began on February 20th, 2020, and ended on April 7th in the same year. So we used the light-blue region to identify the S&P 500 market crash period during the 2020 pandemic.

From Fig. 5, we could observe that the correlation is highly volatile, fluctuating between positive and negative values throughout the years, showing that there is no stable correlation. This phenomenon may be mainly due to Bitcoin itself having very high and erratic returns. At the same time, we can see that during the market crash in 2020, when the S&P 500 index fell, the correlation became positive. This phenomenon reveals that Bitcoin is not a good safe haven, in other words, cannot hedge away the risk when the market is in the downturns.

The correlation between Gold and S&P 500, however, shows an entirely different trend. From Fig. 6, we can see that the correlation is mainly negative throughout the

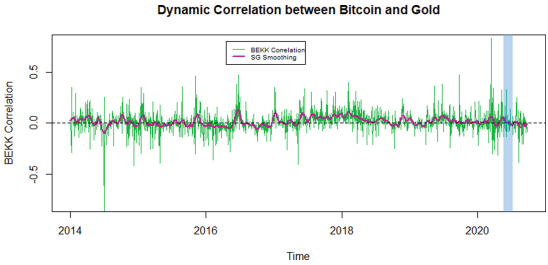

Fig. 7. Dynamic correlations of Bitcoin and Gold

years. According to the definition raised by Baur and Lucey (2010) [4], the negative correlations can serve as evidence of an asset's function as a hedge.

To better study the differences and connections between Bitcoin and Gold, we also analyzed the correlation of these two assets. Here, we follow the method used just now to study dynamic correlations, using a thin green line to represent the BEKK correlation and the thick purple line to depict the smoothed coefficients. Again, we apply the light-blue region to highlight the market downturns. If we assume Bitcoin is very similar to Gold, a positive and stable correlation from the plot would be expected. However, Fig. 7 reflects very unstable correlations between these two assets, with coefficients fluctuating between negative to positive and mainly around 0, regardless of the market is in turmoil or not. This phenomenon shows that there is no notable relationship between Gold and Bitcoin, not to say a significant positive correlation. So this piece of new evidence also supports our conclusion that Bitcoin shares not many similarities with Gold.

5 Conclusion

Based on the data up to September 2020, we used GARCH-family models to study the properties of univariate volatility and finally concluded that Bitcoin does not have the same significant negative leverage effect as Gold. That is to say, if the residual in log return of the previous day is positive, the volatility of Gold will be higher, but Bitcoin does not have this prominent feature.

In the multivariate variance-covariance analysis, we further explored how Bitcoin differs from Gold in its correlation with other assets. Our conclusion shows no correlation between Bitcoin and the major market index. It exhibits a high degree of volatility, which means Bitcoin cannot be used as a haven in a severe market downturn. The performance of Gold is entirely different: Gold has a negative correlation with market volatility, which means Gold can be a powerful tool for hedging market risks.

To further study the functions of these two assets, we extracted the natural market distress window period caused by the COVID-19 pandemic in 2020 and analyzed the performance of Bitcoin and Gold on the market. We found that Bitcoin, as an asset, cannot serve as a hedging tool when the market index plunges sharply. At last, we drew a dynamic correlation diagram between Bitcoin and Gold, the graphic support our judgment that Bitcoin and Gold are not similar.

For now, we can reach our conclusion that Bitcoin, as a cryptocurrency asset, bears not much resemblance with the conventional financial asset Gold, from both univariate volatility and multivariate volatility aspects. Given the high uncertain as well as the high volatile trend of Bitcoin, we have vast potential to conduct further research to explore its other properties, which is highly worthwhile and deserves waiting and seeing.

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