Decision-Making Analysis of Selected Subjects for Comprehensive Reform of College Entrance Examination Based on SEU Model

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Abstract. Under the background of the comprehensive reform of the college entrance examination, there are many main factors that affect the decision-making of the elective subjects, such as the relevant documents required by the elective subjects, the scoring method of the elective subjects, the number of elective subjects in the province, etc., integrating senior high school career planning Decision-making. For high school students in provinces with comprehensive reform of the college entrance examination, the SEU model can be used to analyze and make decisions. Based on the comprehensive reform mode of the college entrance examination and the classification of the subject scope, this paper will use the SEU model as the theoretical basis to model and analyze the decision-making of the selected subjects.

Keywords: comprehensive reform of the college entrance examination · selected subjects · SEU model

1 Introduction

Under the background of the comprehensive reform of the college entrance examination, there are many main factors that affect the decision-making of the elective subjects, such as the relevant documents required by the elective subjects, the scoring method of the elective subjects, the number of elective subjects in the province, etc., integrating senior high school career planning Decision-making. For high school students in provinces with comprehensive reform of the college entrance examination, the SEU model can be used to analyze and make decisions.

At present, the comprehensive reform of the college entrance examination is divided into two modes: “3 + 3” and “3 + 1 + 2”, of which the “3 + 1 + 2” mode is a further refinement of the “3 + 3” mode. details as shown in Fig. 1.

Based on the Fig. 1, it can be seen that when candidates make decisions on subjects for examination, they mainly choose from physics, history, politics, geography, chemistry, biology, and technology (only in Zhejiang). According to the requirements in the “Guidelines for Subject Selection for Undergraduate Admissions in General Colleges
and Universities (General Edition)” issued by the Ministry of Education in 2021, the
decision-making of subjects for examination is divided into the following two subject
ranges.

**Fig. 1.** Comparison of two comprehensive reform modes of college entrance examination.

<table>
<thead>
<tr>
<th>subject area</th>
<th>elective subjects</th>
<th>Selection method</th>
</tr>
</thead>
<tbody>
<tr>
<td>subject area one</td>
<td>physics, history</td>
<td>Choose one or choose all</td>
</tr>
<tr>
<td>subject area two</td>
<td>politics, geography, chemistry, Biology, Technology (Zhejiang only)</td>
<td>Choose two out of four or one out of four</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Choose two out of five or one out of five (Zhejiang only)</td>
</tr>
</tbody>
</table>
Table 2. Decision table for elective subjects

<table>
<thead>
<tr>
<th>elective subjects</th>
<th>e_1</th>
<th>e_2</th>
<th>...</th>
<th>e_n</th>
</tr>
</thead>
<tbody>
<tr>
<td>k_1</td>
<td>r_{11}</td>
<td>r_{12}</td>
<td>...</td>
<td>r_{1n}</td>
</tr>
<tr>
<td>k_2</td>
<td>r_{21}</td>
<td>r_{22}</td>
<td>...</td>
<td>r_{2n}</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td></td>
<td>...</td>
</tr>
<tr>
<td>k_m</td>
<td>r_{m1}</td>
<td>r_{m2}</td>
<td>...</td>
<td>r_{mn}</td>
</tr>
</tbody>
</table>

Based on the comprehensive reform mode of the college entrance examination and the classification of the subject scope, this paper will use the SEU model as the theoretical basis to model and analyze the decision-making of the selected subjects.

2 Description of the Decision-Making Problem of the Selected Subjects

According to the comprehensive reform mode of the college entrance examination and the requirements for the scope of selected subjects, this paper prefers to describe the problem by establishing a decision table for selected subjects [3]. First, we define the set of elective subjects, \( K = \{k_1, k_2, \ldots, k_m\} \), that is to say, the decision makers of the elective subjects may choose multiple elective subjects according to their own preferences or interests, which is, Internal factors. The next step is the impact of decision-making on the total score of candidates in the college entrance examination, which is, external factors, external environmental factors such as the number of subjects selected for each subject, the difficulty of the test questions and other external environmental factors when making decisions [5]. Simply put, it is the environment, so we define the environment set, \( E = \{e_1, e_2, \ldots, e_n\} \), which indicates the impact of the decision on the subject selection on the final college entrance examination score. Finally, the result set \( r_{ij} \) of elective subjects In the result set \( R \) of elective subjects, the results of elective subjects obtained by decision makers are determined by the selected subjects \( k_i \) and the environment \( e_j \) in which they are located, as shown below:

\[
\text{Elective subject } \oplus \text{ Total score } \rightarrow \text{ Result, } k_i \oplus e_j \rightarrow r_{ij}
\]

According to the above description of the decision-making problem of elective subjects, we can establish the decision-making table of elective subjects (as shown in Table 1) (Table 2).

3 Adaptation of SEU Model Axioms in Elective Subjects

3.1 Subjective Probability

The starting point of subjective probability in elective subjects is to assume that given any two environmental factors, A and B, there is no need to be mutually exclusive,
decision makers have an inherent judgment on relative probabilities, and therefore it can be said that candidates believe in one of the following three situations:

A is more likely than B;
A and B have the same probability;
A is less likely than B.

In this regard, decision makers do not need to judge which of the two environmental factors is more likely to occur, but only need to rank the views of likelihood [6]. This article uses the following notation for description:

\[ A \succ= \ell B \] - The decision maker believes that A is at least as likely to occur as B;
\[ A \succ \ell B \] - The decision maker believes that A is significantly more likely to occur than B;
\[ A \sim \ell B \] - The decision maker believes that A and B are equally likely to occur;

At the same time, this paper uses subscripts \( \ell \) to distinguish decision makers’ judgments and preferences for relative probability. At the same time, based on the above situation, we can deduce the nature of probability

\[ A \succ= \ell B \iff P(A) \geq P(B) \]

At the same time, candidates can continue to draw the following conclusions when making decisions on subjects for examination:

1. For all environmental factors A, B and C, \( A \succ= \ell B, B \succ= \ell C \), means \( A \succ= \ell C \), that if A is at least as likely to occur as B and B is at least as likely to occur as C, then the decision maker will consider A to occur Likely at least as likely as C.

2. For \( \forall A, B, \) and C there are \( A \cap C = \emptyset = B \cap C, A \succ= \ell B \iff A \cup B \succ= \ell B \cup C \).
That is, assuming that A and C do not occur together, and B and C do not occur together, if the decision maker believes that the probability of A occurring is at least as great as that of B, then the decision maker will also think that A or C and B or C is equally likely to occur.

### 3.2 Preference for Subject Combination

When making elective subject decisions, we assume that the decision maker must choose among many combinations of elective subjects, and the outcome of the decision is determined only by some simple and clear mechanisms, so the probability of any outcome is well established and beyond doubt, We call these a combination of elective subjects.

Assuming that there are only a limited number of results that can improve the candidate’s college entrance examination ranking, let \( R = \{r_1, r_2, \ldots, r_i\} \) be the possible result set. A typical combination of elective subjects is expressed as follows:

\[ L = <p_1, r_1; p_2, r_2; \ldots; p_i, r_i> \]

Here \( p_j \geq 0 \), is the probability of obtaining \( r_j(j = 1, 2, \ldots, i) \), and \( \sum p_j = 1 \).

In the process of considering decision makers’ preferences for members in the above environmental factor A, this paper makes several reasonable assumptions related to the consistency of decision makers’ preferences [2]. First, it is assumed that decision makers
are rational. If you want to show these assumptions, you need to show that the utility function $u(.)$ of the decision makers in the R set satisfies:

For any $r_i$ in R set, $r_i$ has $r_i \geq r_j \iff u(r_i) \geq u(r_j)$
and

$<p_1, r_1; p_2, r_2; \ldots; p_t, r_t> \geq <p'_1, r_1; p'_2, r_2; \ldots; p'_t, r_t>$

$\iff \sum_j p_j u(r_i) \geq \sum_j p'_j u(r_i)$

For any combination of elective subjects in set A, the first condition shows that $u(.)$ is an ordinal value function in the resulting R set, and the second condition shows the expected utility property that $u(.)$ has in the combination of elective subjects.

### 3.3 Subjective Expected Utility

We assume that the decision-maker’s arrangement of the elective subjects will form the following situation:

1. Subjective probability distribution, $p(.)$, represents the decision maker’s perception of the unknown environment.
2. The utility function, $u(.)$, represents the preference of the decision maker.

When applying the SEU model to the decision-making of elective subjects, we only need to associate each elective subject $k_i$ in Table 1 with the preference of the combination of elective subjects, that is, we get

$<p(e_1), r_{i1}; p(e_2), r_{i2}; \ldots; p(e_n), r_{in}>$

Based on this, when using the SEU model to make decisions on elective subjects, the decision makers arrange the combination of elective subjects in the table through the close combination of ideas and preferences, thereby forming the expected utility:

$$Eu[k_i] = \sum_{j=1}^{n} u(r_{ij})p(e_i)$$

### 4 SEU Modeling Analysis

In this paper, the SEU model is adapted to the decision-making of elective subjects. Next, this article will combine the SEU model and decision tree to model and analyze the decision-making of elective subjects.

Suppose a candidate is faced with the decision of choosing a subject, then we make a simple classification. Choosing a subject can be divided into three categories: very feasible, generally feasible and very infeasible. A very feasible elective subject will improve candidates’ college entrance examination scores and help candidates enter better colleges and universities; a generally feasible elective subject will not help candidates’ college entrance examination results and the improvement of college level; a very unfeasible
selected subjects will lower the candidates’ college entrance examination scores and lower the college level.

Candidates can seek professional consultants to help analyze the decision-making of elective subjects if they wish, before making decisions about elective subjects. Of course, this requires candidates to spend a certain amount of study time and money, which will also have a certain impact and interference on the candidates’ learning. Usually, professional consultants do not make external predictions about the feasibility of the subjects to be tested, and they often express their opinions on the decision-makers’ selection of the subjects by giving suggestive approval or disapproval.

According to the above problem analysis, we can use the above figure to represent. The first question for decision makers is to decide whether to seek professional help, which is represented by the square node on the left side of the figure. The upper branch corresponds to seeking the help of professional consultants and continues to the right. This branch is separated according to the possible results of the consultation situation at the opportunity point, indicating that the consultation report gives the feasibility assessment of the selected subjects. The assessment may be positive or negative, and in each case the decision maker has to decide whether to take the subject. In Fig. 2, at point A, the decision-maker instructs the consultant to make a favorable conclusion for the subject, at point B, the assessment conclusion is known to be disapproval, and at point C, no conclusion is obtained. Therefore, the decision maker’s judgment on the feasibility of the elective subject will be different at each point.
Suppose, at the start of elective subjects, the decision makers assess the feasibility of elective subjects as follows:

\[ p(\text{very feasible}) = a \cdot \cdot \cdot p(\text{generally feasible}) = b \cdot \cdot \cdot p(\text{very infeasible}) = c, \]

of which, \( a + b + c = 1 \)

The evaluation of these probabilities will enable decision-makers to take the average feasibility of the elective subjects as their subconsciousness when evaluating whether to choose the elective subjects, and then based on the decision-makers’ selection of the elective subjects in previous years and previous years The results obtained by the candidates will be considered comprehensively \([1, 4]\).

The next thing that decision makers need to consider is that, according to the information the decision makers may obtain from the consultants, the decision makers’ perception of the subjects to be tested will change to a certain extent. In this case, the evaluation probability is as follows:

\[ p(\text{very feasible} | \text{approval}) \]
\[ p(\text{generally feasible} | \text{approval}) \]
\[ p(\text{very infeasible} | \text{approval}) \]

In the same way, the decision maker will have a similar conditional probability as above in the case of receiving a disapproving conclusion.

At the beginning of the decision-making of the elective subjects, the initial or priority probability has been evaluated before receiving the conclusion from the consultant, and even when making a decision on whether to seek a consultant for consultation, the decision-maker has already made a certain assessment \([7]\). At the same time, the parents of the decision makers will also evaluate the decision makers’ decision-making on the subjects for examination, but the decision makers generally do not consider adopting the suggestions that the parents have no experience with. Decision makers often expect the possibility that if the subject is very feasible and the conclusion of the consultation is positive, and the probability of this ideal situation should be 100%, but few consultants are absolutely reliable, based on this, This article makes assumptions, assuming that the consultant’s assessment is as follows:

\[ p(\text{approval} | \text{very feasible}) = x \]
\[ p(\text{disapproval} | \text{very feasible}) = y, \text{ while } x + y = 1 \]
\[ p(\text{approval} | \text{generally feasible}) = z \]
\[ p(\text{disapproval} | \text{generally feasible}) = w, \text{ while } z + w = 1 \]

By analogy, we can make assumptions about the probability of further conditions, that is, make assumptions about generally feasible and very infeasible aspects, so as to
obtain a decision table for the estimated conclusions under the feasibility conditions of the selected subjects, as shown in Table 3.

According to the above conditions, the probability required in the analysis can be calculated according to Bayes’ theorem, as follows:

\[
p(\text{very feasible}|\text{approval}) = \frac{p(\text{approval}|\text{very feasible}) \times p(\text{very feasible})}{p(\text{approval})}
\]

here

\[
p(\text{approval}) = p(\text{approval}|\text{very feasible}) \times p(\text{very feasible}) + p(\text{approval}|\text{generally feasible}) \times p(\text{generally feasible}) + p(\text{approval}|\text{very infeasible}) \times p(\text{very infeasible}) = ax + bz + cy
\]

Similarly

\[
p(\text{disapproval}) = p(\text{disapproval}|\text{very feasible}) \times p(\text{very feasible}) + p(\text{disapproval}|\text{generally feasible}) \times p(\text{generally feasible}) + p(\text{disapproval}|\text{very infeasible}) \times p(\text{very infeasible}) = ay + bw + cx
\]

From this, it can be concluded that the probabilities of various situations when the decision makers get a favorable conclusion when making a decision on the elective subjects are as follows:

\[
p(\text{very infeasible}|\text{approval}) = \frac{ax}{p(\text{approval})}
\]

\[
p(\text{generally feasible}|\text{approval}) = \frac{bz}{p(\text{approval})}
\]
\( p(\text{very infeasible} | \text{approval}) = \frac{cy}{p(\text{approval})} \)

In the same way, decision makers can also obtain the probability of various situations when they disapprove of the conclusion, as follows:

\( p(\text{very infeasible} | \text{disapproval}) = \frac{ay}{p(\text{disapproval})} \)

\( p(\text{generally feasible} | \text{disapproval}) = \frac{bw}{p(\text{disapproval})} \)

\( p(\text{very infeasible} | \text{disapproval}) = \frac{cx}{p(\text{disapproval})} \)

Through the above analysis, we have calculated all the probabilities needed to make the subject selection decision, as shown in Fig. 3. Next, decision makers need to consider how to assign values to possible outcomes. The ultimate goal of decision makers when making decisions on elective subjects is to improve the score of the college entrance examination. Assuming that choosing a very feasible elective subject will increase the total score of the college entrance examination by \( \alpha \); choosing a generally feasible elective examination subjects will increase the total score of the college entrance examination by \( \beta \); choosing a very unfeasible elective subject will reduce the total score of the college entrance examination by \( \gamma \); at the same time, when looking for a consultant for consultation, the time consuming has the effect of reducing the college entrance examination score by \( \delta \) (where \( \alpha \geq 0, \beta \geq 0, \gamma \leq 0, \delta \leq 0 \)).

According to the above assumptions, the decision maker must first make a decision at node A. If the decision maker chooses this elective subject, the result he will face is as follows:

Expectations for choosing this elective subject at point A:

\[
\text{NPV}(A) = p(\text{very infeasible} | \text{approval}) \times (\alpha + \delta) \\
+ p(\text{generally feasible} | \text{approval}) \times (\beta + \delta) \\
+ p(\text{very infeasible} | \text{approval}) \times (\gamma + \delta) \\
= \alpha p(\text{very infeasible} | \text{approval}) \\
+ \beta p(\text{generally feasible} | \text{approval}) \\
+ \gamma p(\text{very infeasible} | \text{approval}) + \delta
\]

Expectations for not choosing this elective subject at point A:

\[
\text{NPV}(\overline{A}) = \delta
\]
In the same way, the expectations for choosing this elective subject at point B:

\[
NPV(B) = \left[ p(\text{very infeasible}|\text{disapproval}) \times (\alpha + \delta) + p(\text{generally feasible}|\text{disapproval}) \times (\beta + \delta) + p(\text{very feasible}|\text{disapproval}) \times (\gamma + \delta) \right]
\]

Expectations for not choosing this elective subject at point B:

\[
NPV(\overline{B}) = \delta
\]

In the same way, the expectation of choosing the elective subject at point C:

\[
NPV(C) = a\alpha + b\beta + c\gamma
\]

Expectations for not choosing this elective subject at point C:

\[
NPV(\overline{C}) = 0
\]
Therefore, are decision makers weighing the need for consultation? We can analyze it through the expectations of three points A, B, and C. For selection counseling, the expectations at points A and B are as follows:

**Expectations at point A**

\[ NPV(A_{\text{max}}) = \max\{NPV(A), NPV(\overline{A})\} \]

**Expectations at point B**

\[ NPV(B_{\text{max}}) = \max\{NPV(B), NPV(\overline{B})\} \]

From this, the expectations of decision makers for consultation can be derived:

\[ NPV(\text{consultation}) = p(\text{approval}) \times NPV(A) + p(\text{disapproval}) \times NPV(\overline{A}) \]

And the decision maker’s expectation of not consulting NPV, that is, the expectation of choosing the elective subject at point C:

\[ NPV(\text{no consultation}) = NPV(C_{\text{max}}) = \max\{NPV(C), NPV(\overline{C})\} \]

Based on the above analysis, when

\[ NPV(\text{consultation}) > NPV(\text{no consultation}) \]

decision makers should choose to consult with professional consultants.

While conducting consultation, if the conclusion given by the consultant is approval:

\[ NPV(A) > NPV(\overline{A}) \]

the decision makers should choose the subject, and vice versa;

if the conclusion given by the consultant is disapproval:

\[ NPV(B) > NPV(\overline{B}) \]

the decision makers should take the subject at that time. subjects, and vice versa.

When there is no consultation, and the decision-maker realize \( NPV(C) > NPV(\overline{C}) \), the decision-maker should choose this subject, and vice versa.

**5 Conclusions**

Based on the SEU model axioms, this paper builds a decision tree model for the subjects selected for the comprehensive reform of the college entrance examination, and calculates the probability of each node when the subjects are selected by Bayes’ theorem, providing decision makers with a basis for decision-making on subjects selected for the college entrance examination.
References


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