Analysis for a Sort of Higher Education Cost from the View at Constrain of Cobb-Douglas Function

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Abstract. The problem of higher education cost has attracted extensive attention from the government and academia. On the basis of fully considering reality restrain condition of shortage of funds and talents, we put forward an optimization model for university running efficiency under limited fund, where we taking quadratic cost function as the objective function and taking Cobb-Douglas production function as the constraint condition for the financial support of colleges and universities. By using Lagrange multiplier function, we have obtained the optimal solution of this optimization model, and bring forth a proposal to improve the efficiency of colleges and universities.

Keywords: higher education · optimization model · quadratic cost function

1 Introduction

In recent years, great changes has took place in Chinese higher education system. Under the guidance of the goal of building a world-class university, investment in higher education is increasing substantially. Therefore, it is an indisputable fact that there is more and more fierce competition among colleges and universities, meanwhile, almost all of universities have made an extraordinary effort to promote the cost-effectiveness [1–3].

The cost-effectiveness has attracted more and more attention of scholars and managers, therefore, some new research methods came into being, and one of them is called as Multi organization theory. An important conclusion of this theory can be expressed in the following way, that is, the total cost of a single organization is less than the sum of the costs of production in different organizations if the single organization produces two or more products. This conclusion actually shows that there is scope economy in this mode of production [4, 5]. Baumol et al. (1982) [6] pioneered a whole set of cost-effectiveness tools to analyze multi-production organization. His research work is pioneering and fruitful, and stimulated the research interest of subsequent scholars. At the moment, there are three most commonly used forms of multi-production cost function in the study about multi-production organization and its applications, they are quadratic cost function and other kinds of cost function [7–12].

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There are three main research methods to investigate the higher education cost. (1) The method of theoretical induction. To mitigate the problems related to low fertility and a rapidly aging population, Tanaka (2019) [13] discussed the Japanese government plans to implement a free tuition policy for higher education for students from less affluent households. Guzman (2021) [14] tried to describe step by step and in a simple way the development and implementation of a low-cost experimental equipment based on Arduino for use in higher education. Meanwhile, Robinson (2021) [15] focused on affordable out-of-state tuition fee reductions. (2) The method of statistical estimation. Focusing on three aspects of cost and higher education, they are costs of tuition, living expenses and travel expenses, Henderson (2021) [16] adapted the language of cost and price, commonly used in discussions of social mobility, arguing for the importance of considering place and geographical mobility for higher education as part of the balancing of financial and social risks, benefits and investments that structure higher education decisions. (3) The method of regression model. Coelho (2021) [17] implemented through a survey applied to 301 students of a Higher Education Institution, and analyzed the data through Structural Equation Modeling (SEM). Banfield (2021) [18] built a so-called shared-cost-profit model to address both issues of sustainability and value. The COVID-19 pandemic has accelerated this crisis as institutions across the country have been forced to move to a fully or partially online model of instruction. In an effort to reduce costs, institutions have increased the number of low-paid, part-time adjunct faculty teaching introductory courses and have promoted the use of free and open online educational resources (OER). However, both solutions lack sustainability and do little to solve the "value" question of higher education.

There is a general problem of rigid expenditure ratio in university expenditure, meanwhile, proportion of variable cost is low, while proportion of fixed cost is high. Therefore, the administrative cost of colleges and universities is high, so that historical burden is heavy. Cost control theory over the world has become mature and stereotyped after 1980. The theory includes target cost control and cost of operation. Though there are quite a lot of mature theories on enterprise cost control, its research in the field of education is rare [19]. It is inspired by the new perspective and inspiration provided by the existing literature, we put forward an optimization model for university running efficiency under limited fund, where we taking quadratic cost function as the objective function and taking Cobb-Douglas production function as the constraint condition. By using Lagrange multiplier function, we have obtained the optimal solution of this model, and put forward the optimization measures to improve the efficiency of colleges and universities.

2 Model Analysis

We referenced the cost quadratic function, following Chen Lin (2020) [20]. Thus, our function is of the following form:

$$C_1(p, q) = \alpha_0 + \alpha p + \beta q + \frac{1}{2}\delta p^2 + \frac{1}{2}\gamma q^2 + \rho p q \tag{1}$$

$$s.t. \quad a \ln p + b \ln q = \ln c_0 \tag{2}$$

where α_0 stands for the amount of initial total cost, α stands for the amount of total cost change which caused by the price of the input product increasing by one unit; β represents the amount of total cost change which caused by the price of the output product increasing by one unit; δ represents the price convexity of input products; γ represents the price convexity of output products; ρ represents the price relationship coefficient between input product and input product; c_0 stands for the sum of financial support that universities can provide for all input and output production and external resources of universities; a stands for elasticity of the input product; b stands for elasticity of the output product.

By using Lagrange multiplier method, we push out the following result:

Proposition 1. There at least exist an optimal solution for the optimal problem (1) and (2).

Proof: Select Lagrange multiplier function as follows,

$$L = \alpha_0 + \alpha p + \beta q + \frac{1}{2}\delta p^2 + \frac{1}{2}\gamma q^2 + \rho pq + \lambda(a \ln p + b \ln q - \ln c_0) \tag{3}$$

Then one-order condition can be calculate as following:

$$\frac{\partial L}{\partial p} = \alpha + \delta p + \rho q + \lambda \frac{a}{p} = 0 \tag{4}$$

$$\frac{\partial L}{\partial q} = \beta + \rho p + \gamma q + \lambda \frac{b}{q} = 0 \tag{5}$$

$$\frac{\partial L}{\partial \lambda} = a \ln p + b \ln q - \ln c_0 = 0 \tag{6}$$

Now we just need prove that there at least exist a solution in the equation set (4) (5) and (6) in the first quadrant. By eliminating parameter λ from Eq. (4) and (5), we can obtain following quadratic curve:

$$bp(\alpha + \delta p + \rho q) = aq(\beta + \rho p + \gamma q) \tag{7}$$

Notice that the discriminant $\Delta > 0$ about quadratic curve (7), hence Eq. (7) represents an hyperbola in the geometry. Due to $a > 0$, $b > 0$, logarithm curve (6) has to intersect with quadratic curve (7) in the first quadrant. We can use intermediate value theorem to prove this fact.

Select a adequacy point in the logarithmic curve (6), and found a tangent line as follows:

$$\Gamma : y = -x + k, \quad k > 0 \tag{8}$$

Plugging (8) into (7) gives

$$b\alpha_1 x^2 + \rho(b - a)x(-x + k) + b\alpha_2 x - a\beta_1(-x + k)^2 - a\beta_2(-x + k) = 0$$

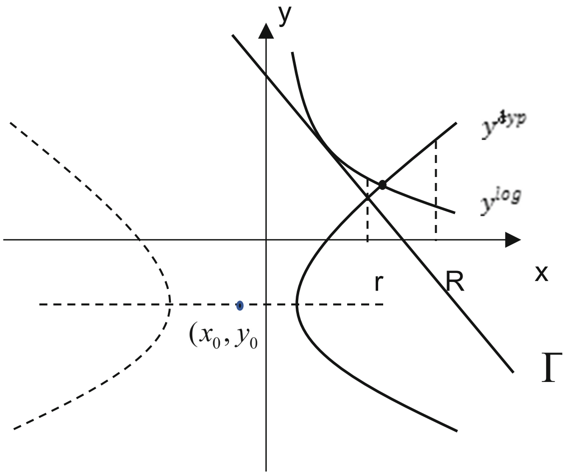

Fig. 1. Curve $y^{\log}(x)$, $y^{\text{hyp}}(x)$ and tangent line $\mathrm{Y}(x)$

That is

$$Ax^2 + Bx + C = 0 \tag{9}$$

where

$$A = b\alpha_1 + \rho(b-a) - a\beta_1 > 0$$

$$B = \rho(b-a)k + 2a\beta_1 k + a\beta_2$$

$$C = -a\beta_1 k^2 - a\beta_2 k < 0$$

It is obviously that $A > 0$, $C < 0$, it means that there must exists an unique real root for quadratic equation in one variable (9). We denote this real root as r.

Denote: $y^{\log}(x)$ as ordinate corresponding abscissa x at logarithmic curve (6); Denote $y^{\text{hyp}}(x)$ as ordinate corresponding abscissa x at hyperbola (7); $\mathrm{Y}(x)$ as ordinate corresponding abscissa x at tangent line (8).

First, let us consider point $x = r$, due to the convexity of logarithmic curve, there exits an inequality for the ordinate at arbitrary point x as follows:

$$y^{\log}(r) > y^{\text{hyp}}(r) = \mathrm{Y}(r) \tag{10}$$

Then, without losing generality, we suppose that $\mathrm{k} - r > 1$ (in the same way we can prove the case $\mathrm{k - r} < 1$). Take $R^a = c_0$, because $|\mathrm{a}\ln\mathrm{R} > \ln\mathrm{c}_0 - \mathrm{b}\ln(\mathrm{k-r})$, in the light of monotone decreasing function $y^{\log}(x)$ and monotone increasing function $y^{hyp}(x)$, we obtain an inequality at point $x = R$ (see Fig. 1):

$$y^{\log}(R) < k - r < y^{\text{hyp}}(R) \tag{11}$$

Associated two inequalities (10) and (11), according to intermediate value theorem, there at least exists an intersection point for the curve (6) and (7) in the interval $[r, R]$.

Proposition 2. There almost exist an optimal solution for the optimal problem (1) and (2).

Proof: According to Rolly theorem and monotone decreasing property of double logarithm curve (6), we can prove the conclusion of Proposition 2 is true.

3 Numerical Example

In this section, we consider a few special case for the Proposition 1 and Proposition 2.

4 Conclusions

It has been racked their brains for many scholars to study the education cost for colleges and universities. However, because the current accounting system for colleges and universities cannot do direct cost accounting, a perfect accounting system that has relatively strong operability doesn't appear till now. The more difficult obstacle to overcome is that, presidents of colleges and universities must face the reality restrain condition of shortage of funds and talents.

According to the restrain condition of Cobb-Douglas production function, all the colleges and universities try do their best to find an optimal way to reach the minimum cost in higher education, such that to maximizing the efficiency of running a school. In the other words, they can design scientific allocation of input and output such that to minimum cost in running a school.

Table 1. Optimal value for discriminant $\Delta = 0$

Objective function	Restrain condition	Optimal value
$0.5\,\delta p^2 + \beta q$	$pq = c_0^2$	$(1 + 0.5\delta)\sqrt[3]{\delta\beta^2 c_0^4}$

Table 2. Optimal value for discriminant $\Delta > 0$

Objective function	Restrain condition	Optimal value
$0.5\,\delta p^2 + 0.5rq^2$	$pq = c_0^2$	$\sqrt{\delta\gamma}c_0^2$

Table 3. Optimal value for discriminant $pq = c_0^2$

Objective function	Restrain condition	Optimal value
$0.5\,\delta p^2 + 0.5rq^2 + \rho pq$ $+\rho pq$	$pq = c_0^2$	$(\sqrt{\delta\gamma} + \rho)c_0^2$

From Table 1 we can see that, the type of university indicated by $\Delta = 0$ belongs to teaching-oriented type. For teaching-oriented universities, they can choose asset allocation in favor of student training, thereby minimizing the overall cost of running a school. The type of university indicated by $\Delta > 0$ (see Table 2) belongs to scientific research and teaching universities type. For this kind of universities, because they do not have their own obvious characteristics and advantages, they can choose a compromise between the asset allocation plan of scientific research and teaching universities, and strive to achieve the minimum cost of running a school through equalization and balance in all aspects; The type of university indicated by $\Delta < 0$ belongs to scientific research type. They can choose to prefer scientific research in asset allocation, thereby minimizing the overall cost of running a school. As can be seen from Table 3, the total running cost of scientific research type is greater than that of scientific research and teaching universities type, because the investment of R&D in scientific research and type is greater than that of scientific research and teaching universities type. Meanwhile, the total running cost of teaching- oriented universities is the least due to their least initial cost.

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