Application of Mean-Variance Model in the U.S. Capital Market

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Abstract. Portfolio optimization is a popular procedure that is widely used in the financial industry. This paper conducts asset allocation analysis for diversified assets, including iron and steel industry, technology, healthcare, information industry and energy areas. There are five assets selected from the different areas which perform well in recent years. This paper uses three methods, namely Mean-variance analysis, CAPM and FF3F model, to find the portfolio optimization. Also, this paper uses the weights to analyse the performance of portfolio in different methods. The result shows that, in the FF3F model, 'LMBEX' contains the largest weight in both maximum sharpe ratio portfolio and minimum variance portfolio, while in the CAPM, 'ADX' and 'LMBEX' account for the largest weight in maximum sharpe ratio portfolio and minimum variance portfolio, respectively. This research may be useful to the potential investors who interested in steel, technology, healthcare, information, and energy industries.

Keywords: Mean-variance · capital market · FF3F · sharpe ratio

1 Introduction

In 1952, American economics Markowitz first proposed the use of mathematical models, which is called mean-variance models, to analyse investment portfolios. As investment profits increase, investment portfolios inevitably become one of the hot spots for investors [13]. To maximizing returns and minimizing risks is the major target for every financial investor [1]. So how to increase returns and reduce risks has always been a significant topic and goal of public in financial studies [8]. Therefore, how to balance return and risk in the investment portfolio is of great concern. And, for investors, a portfolio is a better choice than a single asset because "diversification can decrease the risk" [9] and also achieve a more robust configuration.

Research on portfolio optimization has grown significantly in recent years. Some researchers have studied a variety of fields to analyse the market trend. For example, Asawa [1] studied portfolio optimization about some machine learning such as clustering based, and Support Vector Machines based and get the useful result for researchers to achieve the near perfection. Dai, Zhu and Zhang [2] analysed the portfolio selection among crude oil, gold and stocks market. Also, Evrim Mandacı and Kırkpınar [3] used the portfolio optimization to analyse the oil markets. According to the research from

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Xu, Ren, Dong and Yang [14], they did a similar investigation. Under the background of the pandemic, researchers studied how to use portfolio selection to select Electronic Company Stocks to achieve the optimal portfolio [16]. In the real estate market, Yilmaz, Korn and Selcuk-Kestel [15] researched the optimal portfolio in the housing markets. During the period of COVID-19, some researchers use portfolio to think about the connection among a number of different fields, such as fine wine, copper, shipping and commodities and so on. And they get the result is that fine wine can help investors rebalance the portfolio during this particular period [11].

However, it may be noticed that portfolio management among certain asset and mutual funds is quite limited. Thus, this raises the interests to make in-depth investigations on this issue. This paper selects two stocks and three mutual funds as research targets, they are: United State Steel Corporation (USSX34.SA), Ebay (EBAY), Goldman Sachs Large Cap Core Fund investor class (GSPTX), Brandywine-Global-Dynamic US Large CAP value fund Class I (LMBEX) and Adams Diversified Equity Fund (ADX). The process of researching can be summarized as follow. First, this paper chooses these five companies which are all from different fields for the closing prices during the period between January 2017 and January 2022. And the data needs to be sorted out in the specified period of time. Second, this paper previously considers using Mean-variance analysis to find the portfolio, but this method has a relatively high bias by using expected return to calculate. So, this paper considers and compares the methods between CAPM and the CAPM-based Fama-French Three Factors Model (FF3F). Third, this paper thinks about using CAPM to find the portfolios which are the maximum sharpe ratio portfolio and minimum variance portfolio. But this model is too realistic as it just has one factor needed to be considered, so a multifactor model is tried to use in this paper. Fourth, the Fama-French Three Factors Model is calculated through the same steps as CAPM's to find both the maximum sharpe ratio portfolio and minimum variance portfolio and think that this is a more accurate and realistic method than CAPM.

The following is a summary of the remaining parts. Section 2 shows the data, Sect. 3 depicts the methods, Sects. 4 and 5 show the results and conclusion, respectively.

2 Data

The data used in this paper comes from Yahoofinance (https://finance.yahoo.com/). This paper selected the following 5 companies which are US steel, EBAY, GSPTX, LMBEX and ADX for their closing prices in five years from January 2017 to January 2022. This paper considered to choose these data as they have a relatively high beta, and they were all performance well during the past 5 years. Also, this paper chooses these companies from different fields in order to diversify the assets and avoid high correlation. Some descriptive statistics of these assets are shown in the Table 1.

From the data showing above, this paper finds that the highest average return is appeared in 'GSPTX', while the lowest average return is 0.0119 from 'LMBEX'. Comparing their variances, 'ADX' has the largest variance while 'US steel' has the lowest. In addition, the lowest max return and highest min return are both in 'ADX'. But the highest max return is shown in 'US steel' and the lowest min return are in 'GSPTX'.

Table 1. Descriptive statistics of the selected assets

	'US steel'	'EBAY'	'GSPTX'	'LMBEX'	'ADX'
Mean	0.0156	0.0139	0.0175	0.0119	0.0144
Variance	0.0315	0.0080	0.0109	0.0042	0.0033
Max	0.6591	0.3250	0.5852	0.1994	0.1796
Min	−0.3146	−0.1294	−0.3783	−0.1907	−0.1241

3 Methods

3.1 Mean-Variance Analysis

The mean-variance analysis is a useful tool used by investors to measure investment decisions in order to achieve the maximum expected return or minimum risk. The mean-variance analysis is known to be most fundamental infrastructure in financial economics [6]. According to Ismail and Pham [5], using Markowitz's efficient portfolio strategy is a popular approach because of its straightforward calculation method. The following are the relevant formula:

$$\sum_i Weight_i = 1 \tag{1}$$

The *Weighti* means the weight of different assets in the portfolio.

$$\mathrm{E(R_P)} = \sum_{i=1}^{n} w_i r_i \tag{2}$$

The Rp means the Portfolio return, so the E(RP) stands for the portfolio's expected return. The weight of total capital invested in asset i is denoted by *wi*, while the expected return for asset i is expressed by *ri*.

$$Variance\ of\ Portfolio = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Cov(r_i, r_j) \tag{3}$$

Cov(*ri*,*rj*) shows the variance-covariance between expected return for asset i and expected return for asset j.

$$\text{Sharpe ratio} = \frac{\mathrm{E}(R_P) - R_f}{\sigma_P} \tag{4}$$

The Sharpe ratio is created to help investors find the comparison between return and risk. The portfolio's anticipated return is E(RP), the risk-free rate is represented by *Rf* and the standard deviation is σ*P*.

3.2 Capital Asset Pricing Model

One of the initial and most important models for describing the relation of risk and return is CAPM [10]. According to Levy [7], CAPM is regarded as one of the core backbones in financial field. This model now still "remains central to economic theory" [7] and also be widely used in the area of corporate finance practicing from the research of Graham and Harvey [4]. Capital Asset Pricing Model is a one factor model which just take market portfolio into consideration. The related equation is showing below:

$$\mathrm{E}(R_i) = R_f + \beta_i(\mathrm{E}(R_m) - R_f) \tag{5}$$

$\mathrm{E}(R_i)$ means the anticipated return, β_i means investment's beta value, the market portfolio's return is represented by R_m, the expected return of the market is $\mathrm{E}(R_m)$. And R_f stands for risk-free interest rate. β_i is positive in almost all situations, and it can be thought of as the sensitivity of an asset to market fluctuations. It means that the asset has higher risky when the beta is greater than one. And the beta of less than one indicates that the asset has lower risky relative to the whole market.

3.3 Fama-French Three-Factor Model

This model developed in 1992 is a natural extension of the CAPM model and has multiple factors which is different from the CAPM model. It expends on CAPM by adding another two risk factors. When evaluating a portfolio's performance compared to market returns, the Fama-French three-factor model is always applied [12]. The market value factor and book-to-market ratio factor are taken into consideration aiming to producing an effective tool for evaluating the portfolio. The model can be shown as:

$$\mathrm{E}\left(R_i - R_f\right) = \beta_i\left(\mathrm{E}(R_m) - R_f\right) + s_i(\mathrm{E}(\mathrm{SMB})) + h_i(\mathrm{E}(\mathrm{HML})) \tag{6}$$

The R_i means the portfolio's total return, R_f means the risk-free return rate, $\mathrm{E}\left(R_i - R_f\right)$ means the anticipated excess return. R_m is the market portfolio's return, $\mathrm{E}(R_m)$ stands for the anticipated return of a market, $\mathrm{E}(R_m) - R_f$ shows the abnormal return on the market portfolio. SMB and HML refers to two different types of firm's condition, i.e., market capital and size, respectively. β_i, $s_i$and h_i are the factor coefficients.

4 Results

Using the correlation function, this paper finds the appropriate model as the correlation between any two equities are moderate which are all less than 0.5. So, it shows that there is less correlation between the different stocks and mutual funds. Also, from the data below, the correlations have both positive and negative numbers, which means that the relationship between the two assets have positive or negative situations. Furthermore, all the correlations are non-zero, which illustrates that all the assets are inter-correlated (Table 2).

Using the CAPM model, this paper can find the maximum Sharpe ratio portfolio, and get the result in Table 3.

The largest weight which is 0.7238 appears in 'ADX', but 'US steel' has the smallest weight −0.0502. The 'ADX' gets this weight probably because it is focusing on information technology and energy industry which has a better development trend these years. And the reason for 'US steel' weight probably has to do with the current outlook

Table 2. The correlation between the equities.

	'US steel'	'EBAY'	'GSPTX'	'LMBEX'	'ADX'
'US steel'	1				
'EBAY'	0.3742	1			
'GSPTX'	0.2303	0.4699	1		
'LMBEX'	0.0319	−0.0498	−0.0451	1	
'ADX'	0.2611	0.1512	−0.0324	−0.0417	1

Table 3. Results for maximum Sharpe ratio portfolio optimization under CAPM model

	'US steel'	'EBAY'	'GSPTX'	'LMBEX'	'ADX'
Weight	−0.0502	−0.0095	0.0211	0.3148	0.7238
Expected return new			0.0145		
Variance			0.0025		
Standard deviation			0.0499		
Sharpe ratio			0.2914		

Table 4. Results for minimum variance portfolio under CAPM model

	'US steel'	'EBAY'	'GSPTX'	'LMBEX'	'ADX'
Weight	−0.0892	0.0174	−0.2550	0.7439	0.5829
Expected return new			0.0125		
Variance			0.0021		
Standard deviation			0.0463		
Sharpe ratio			0.2706		

for this industry, which has received a great impact by the emergence of new materials. Also, this paper finds the result of minimum variance portfolio (Table 4).

The largest weight 0.7439 appears in 'LMBEX' which shows that this equity has the smallest variance. It has this weight maybe because it is mainly focusing on the financial services and healthcare, which are both relatively stable development of the industry and do not have too many uncertainties. And 'US steel' has the smallest weight which can infers that it has a higher volatility. One of the possible reasons for its smallest weight is that new materials dealt a huge blow to the iron and steel industry. The estimation results of Fama French 3 factor model are shown below in Table 5 and 6. Respectively.

Then this paper can find the maximum Sharpe ratio portfolio, and the result is showing in Table 7.

Table 5. Estimations of FF3F parameters

	β_i	s_i	h_i
'US steel'	1.6566	0.5975	0.8850
'EBAY'	1.0112	0.6486	−0.2013
'GSPTX'	1.4044	−0.0568	0.0182
'LMBEX'	0.9600	−0.0502	0.5268
'ADX'	0.9116	−0.1875	0.0761

Table 6. Standard error and P-value

	β_i	s_i	h_i
Standard error	0.4460	0.7668	0.4972
P-value	0.0005	0.4393	0.0808

Table 7. Results for maximum Sharpe ratio portfolio under FF3F model

	'US steel'	'EBAY'	'GSPTX'	'LMBEX'	'ADX'
Expected return	0.0165	0.0084	0.0105	0.0092	0.0070
Weight	0.0369	−0.0204	−0.1190	0.6925	0.4100
Expected return new			0.0085		
Variance			0.0027		
Standard deviation			0.0517		
Sharpe ratio			0.1466		

As the Table 7, 'LMBEX' has the largest weight, which is 0.6925, but 'GSPTX' has the smallest weight which is −0.1190. The 'LMBEX' (Brandywine global) has the weight probably because it focuses on the financial sectors and healthcare which are the areas of public interest. 'GSPTX' is concentrates on the technology sector, so it has the smallest weight may be because of the current emphasis on environmental protection and safety.

Also, this paper calculated the minimum variance portfolio, and the result is showing in Table 8.

From the Table 8, 'LMBEX' has the largest weight, which is 0.7439, but 'GSPTX' has the smallest weight which is just −0.2550. In the minimum variance portfolio, the 'LMBEX' (Brandywine global) has this weight probably because this mainly focuses on the field in financial services and healthcare which has areas of relatively stable development, and it also means that Brandywine global has the smallest volatility. 'GSPTX'

Table 8. Results for minimum variance portfolio under FF3F model

	'US steel'	'EBAY'	'GSPTX'	'LMBEX'	'ADX'
Expected return	0.0165	0.0084	0.0105	0.0092	0.0070
Weight	−0.0892	0.0174	−0.2550	0.7439	0.5829
Expected return new			0.0070		
Variance			0.0021		
Standard deviation			0.0463		
Sharpe ratio			0.1313		

focuses on technology, but the possible reason leading to its lowest weight may be that the technology is a fast-changing fields, which means it is volatile.

5 Conclusion

According to the current information statistics, most of the research on the portfolio is based on the analysis of specific industries or even hot sectors. In this paper, the research aims to help investors make the best investment decision in diversified fields, such as stocks of steel and iron industry and online sales platform, and mutual funds of health-care, industry, and technology sectors. This paper lists three analysis methods to find the optimal portfolio, which are Mean-variance analysis, Capital Assets Pricing Model and Fama-French 3 Factor model. In the latter two models, this paper chooses to find maximum sharpe ratio portfolio and minimum variance portfolio, respectively. In the CAPM, this paper finds the areas related to information technology and energy technology, as well as financial services and healthcare accounted for a larger proportion, while in the FF3F model, the fields related to financial services and healthcare are even more prominent. However, there is no denying that these methods still have shortcomings, for example, there are more factors affecting the portfolios that are probably not considered.

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