Portfolio Optimization with Fama-French Model

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Abstract. This paper explores the method of using Fama-French Three Factor Model and mean variance analysis to optimize portfolios, reaching more accurate predictions, and achieving maximum return and minimum risk. Using historical data of stocks from different industries, three factors from the Fama-French database is applied, and new expected return is calculated. Then mean variance analysis is performed to find the optimum Sharpe ratio weights. From the optimized weights, it can be seen that COST, ROM, and JPM are strong performing stocks. Whereas AAPL and small cap stocks like PERI are considered less favorable by both the CAPM model and Fama-French Model. Reducing weight of these stocks could decrease the portfolio risk, hence lowering variance, reaching higher Sharpe ratio. The results in this paper would be beneficial to public and private investors in different financial markets. As shown in this paper and also other works, the use of Fama-French Model and mean variance analysis could increase profit in most cases.

Keywords: optimization · Fama French · mean variance analysis · portfolio

1 Introduction

Since Fama and French [1] introduced the famous three factor model in 1992, it has been considered a reliable way of measuring and finding profitable stocks and portfolios. As Foye [2] pointed out, Fama French Models could provide a better image of equity returns in Eastern Europe and Latin America. Regular investors in the stock market could utilize the three critical factors to make wiser investment discussions. Through careful calculations with the model, investors could be empowered to find the best stocks with low variance and high return numbers in the market. Low variance and high return numbers in a stock's historical data is generally preferred.

Most works on the issue of the Fama-French Model explored its viability and drawbacks or presented a test in stock markets in different parts of the world. Taneja's study analyzed the Fama French Model using a sample of 187 enterprises over a five-year period to confirm its efficiency and demonstrate that Fama French is a solid predictor that cannot be disregarded in India. [3] In Chiah's paper, Fama-French Model is compared with the Capital Asset Pricing Model (CAPM), proving that it is the better model at accurate predictions. [4] In Lin's study, Lin compared the effectiveness of the three factor and five factor variants of Fama-French Model. Lin found that when applied

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with Chinese equity market data, the three-factor variant of Fama-French Model outperforms the five-factor variant in some ways. [5] Ray and Savin's research provided a technical analysis of the Fama-French Model's robustness, showing that after testing for heteroskedasticity and autocorrelation, the Model's performance is in line with expectations. [6] In Jackson's 20-year study, Fama-French Model is shown to prove its theory in the real estate sector. The performance of lodging real estate investment trusts is highly tied to elements such as market, book-to-value, and size. [7] Another recent study investigates and demonstrates that Fama-French model is applicable to Islamic trust funds throughout a market crash. [8] Yu's study references Fama-French Model's factors, exploring the significance of investor sentiment in the United States stock market. [9]

Yet few shows how it could be best used under the current market, this paper concentrates on applying Fama-French Three Factor Model. By optimizing a portfolio using the model and attributing different weights to individual stocks, highest value of Sharpe ratio could be achieved in the said portfolio. Different weights could also be applied to minimized variance portfolio. As it can be seen in the rest of the paper, portfolio with excellent performing stocks optimized with the Fama-French Three Factor Model shows far more modest return numbers, comparing to simple arithmetic mean. After considering SMB, HML, RF, or in simple terms, the sizes of firms, book-to-market ratios, and excess returns, we could be given more realistic predictions.

This paper is constructed as follows: First, five stocks are chosen from the retail industry, technology industry, advertising industry, banking industry, and one trust fund. The projected return of each stock is then determined using the three variables from the Fama-French database and five years of historical data from these stocks. Lastly, by utilizing mean variance analysis, the highest Sharpe ratio weights are found.

2 Data

Data for this paper is collected from Yahoo Finance (https://finance.yahoo.com/). Five strong performing stocks: AAPL; ROM; PERI; COST; JPM, are chosen to create an example of a diverse portfolio. The proposed portfolio includes three big-cap companies, one trust fund and one small cap company. According to historical data, Costco achieved a 348% five-year return, while having relatively little fluctuations. The only downside of this stock is its high price per share, sacrificing flexibility when trading. AAPL is another classic big-cap stock. Its innovative business grants the public confidence. Although Apple is a high fluctuation high risk stock, it is safe to say that Apple's stock price would continue to grow. Apple achieved a 120% return for the last five years. JPMorgan Chase & Co. is one of the largest banking cooperation in the United States. Its stock chart has a roughly upward sloping line, but its probability of fault is next to none. Hence it could be included in the portfolio. PERI is a relatively good performing small-cap stock. Perion Network is an internet advertising company based in Israel. From all growing small caps stocks, Perion stands out. There are some huge ups and downs, but Perion is in a general direction of growing, hence it makes the into the portfolio. TQQQ is a leading trust fund in the market. Unfortunately, it still has not recovered from the collapse beginning Jan. 2022. But it's performance before is stellar, it is one of the best managed trust funds and will continue to be one in the future.

Table 1. Descriptive statistics of the selected assets

	AAPL	ROM	PERI	COST	JPM
Mean	0.0328	0.0405	0.0437	0.0219	0.0142
Variance	0.0074	0.0135	0.0484	0.0029	0.0050
Max	0.2144	0.2999	0.6772	0.1450	0.2135
Min	0.1840	0.2485	0.3208	0.1171	-0.2246

Approximately five years of historical data is collected, from 05/2017 to 01/2022. After some simple manipulation, the historical data can be interpreted as average return metrics. Table 1 illustrates different factors of the five chosen stocks.

After close examination of the above data, it is clear that ROM possesses the highest average monthly return, at 4.05%, while JPM has the lowest average monthly return, at 1.42%. In regard to risk, COST holds the lowest variance, at only 0.0029, while PERI has the highest risk, at 0.0484. The four metrics is in-line with the stock charts. AAPL has relatively high return and moderate fluctuations; PERI has the highest return, at a cost of high fluctuations; COST and JPM has the lowest fluctuations and good return, as seen on the stock charts.

3 Methods

Stock price fluctuations can be seen as time series data, random within every single stock, but there exists a correlation in between. Using the three different factors from Fama-French Model, the raw historical data could be processed to present a more realistic past performance. Compared to its preceding industry standard CAPM Model, the Fama-French Model includes a risk factor, which help better visualize the actual variation of a stock. In addition, it also takes the tradeoff between risk and return into account, providing investors with a clearer, more accurate picture of how a stock would perform. As seen in Shaharuddin's study, research evidence suggests that the Fama-French Model can be explained in the market of Islamic unit trust funds, and is valid and useful to investors and fund managers. [8]

Fama-French Three Factor Model

The Fama-French method is divided into three steps:

First, run time-series regressions to estimate beta,

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i\left(r_{m,t} - r_{f,t}\right) + e_{i,t} \tag{1}$$

Where:

$r_{i,t}$ is total return of stock or portfolio i at time t;
$r_{f,t}$ is risk-free rate of return at time t;
$r_{i,t}$ - $r_{f,t}$ is expected excess return;
α_i and β_i are factor coefficients;
$r_{m,t}$ is total return of market portfolio at time t;

$r_{m,t}$ - $r_{f,t}$ is excess return of the market portfolio (index)

Then, using estimated betas, run a cross-section regression of the average excess returns on $r_i - r_f$,

$$r_{i,t} - r_f = \lambda_0 + \lambda_1 \widehat{\beta_i} + e_i$$

$$r_{i,t} - r_f = \lambda_0 + \lambda_1 \widehat{\beta_i} + e_i \tag{2}$$

Using this, the complete formation of the Fama-French Three Factor Model could be formed:

$$r_t - r_{f,t} = \alpha_i + \beta_i \left(r_{m,t} - r_{f,t} \right) + s(SMB_t) + h(HML_t) + e_{i,t} \tag{3}$$

Here SMB_t is the Small Minus Big size premium at time t, and HML_t is the High Minus Low value premium at time t.

Mean Variance Analysis

Mean variance analysis is a process of optimizing portfolio return by adjusting respective weight of each stock, finding the combination of weights that achieves the maximum return metric. As seen in Rubenstein's 2002 study, mean variance analysis has been the standard way of managing portfolio since its inception. [10] The mean variance analysis is carried out with the following steps:

$$\sum Weight_i = 1 \tag{4}$$

Here $Weight_i$ represents the weight of i^{th} asset in the portfolio. The sum of weights within the portfolio should always equal to 1. The return and variance of a five asset portfolio is calculated as follows.

$$Portfolio\,Return = \sum (Asset\,Return_i \cdot Weight_i) \tag{5}$$

$$E\left[Portfolio\,Return \right] = \sum Weight_i \cdot E[Asset\,Return_i] \tag{6}$$

Here $E\left[Portfolio\,Return \right]$ represents the expected return of portfolio, $E[Asset\,Return_i]$ represents the expected return of i^{th} asset.

$$Var(Portfolio\,Return) = \sum_{i=1}^{5} Weight_i \cdot \sigma_i + 5(Weight_1 \cdot Weight_2 \cdot Weight_3 \cdot Weight_4 \cdot Weight_5 \cdot Cov_{1,2,3,4,5} \tag{7}$$

Here $Var(Portfolio\,Return)$ is the variance of the portfolio.

$$Cov = \frac{\sum (R_{A-E} - Ave_{A-E}) \cdot (R_{I-M} - Ave_{I-M})}{(Sample\,Size) - 1} \tag{8}$$

Here Cov is the variance-covariance between assets.

$$Sharpe\,Ratio = \frac{E\left(R_p\right) - R_f}{\sigma_p} \tag{9}$$

Here $E\left(R_p\right)$ is the expected return of portfolio, R_f is risk-free rate, and σ_p is the portfolio's standard deviation.

4 Results

Through exhaustive data processing, the optimal Sharpe ratio portfolio weights are found, as seen in Tables 2 and 3.

The classic CAPM model generates rather radical numbers. COST takes up approximately 77% of the portfolio, and JPM has a weight of almost 30%. The rest three stocks have negligible weights. CAPM only considers systematic risk, focusing the relationship between a certain stock's performance and the market risk. Hence the two relatively "stable" stocks in the portfolio are given almost all the weights. COST has been a stock that achieves stable growth, plus it has resisted past market busts fairly well, thus it has been given a undisputable high weight. JPM stock price does not grow as fast as COST, but it has the same quality of being stable, therefore it earned a 30% weight. It is also interesting that ROM is being shorted in this portfolio. ROM's past performance has been strong, except for the market-wide bust in early 2022, for it is behavior comparing to the market returns, it was not favored by the CAPM model.

As can be seen in the data above, COST has a much more modest weight after executing mean variance analysis using the expected return numbers predicted from the Fama-French Three Factor Model. At 48.2%, it still takes half of the portfolio, but more weight is given to ROM, up to almost 40%. JPM achieves a much lesser weight, at 1.7%. AAPL and PERI still has less significant weights in the portfolio, having 6.9% and 3% respectively. Unlike the CAPM model, which only considers one factor, the Fama-French model takes three essential factors into consideration, to give a more accurate prediction on the future.

Table 2. Portfolio weight distribution using CAPM Model

	AAPL	ROM	PERI	COST	JPM
Weights	0.0961	22120.1811	0.0135	0.7720	0.2995

Table 3. Portfolio weight distribution using FF3F Model

	AAPL	ROM	PERI	COST	JPM
Weights	0.0695	0.3997	0.0308	0.4823	0.0176

Table 4. Portfolio statistics using FF3F/CAPM Model

	CAPM	FF3F
Expected Return	0.0011	0.0208
Variance	0.0018	0.0050
Standard Deviation	0.0419	0.0705
Sharpe Ratio	0.0272	0.2946

By applying the weights to the portfolio, we can see the difference in performance in Table 4. The portfolio optimized with Fama-French Model has higher expected return and Sharpe ratio numbers, when compared to the portfolio optimized with CAPM.

5 Conclusion

Most of the current portfolio research focuses on proving the viability and effectiveness of the Fama-French Model, referencing on different markets around the world. This paper explores the effect of Fama-French Model when used on the US stock market. Stocks from the retail industry, technology industry, advertising industry and banking industry are chosen, as well as one trust fund included. After using mean variance analysis to optimize portfolio weights, it is shown that by using Fama-French Three Factor Model, predictions on stocks could be more accurate. In addition, using Fama-French Model to optimize portfolios could achieve weights that have higher returns. From the data results, it is obvious COST, ROM, and JPM are strong performing stocks that are worth investor's attention.

This paper may need more data to further validate its findings, due to some limitations, only five years of historical data was collected for reference, longer periods of data would make the results more reliable. Furthermore, more data from trust funds could be included to explore Fama-French Model's effect in a different yet similar market.

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