# Assets Portfolio Analysis of the Firefighter Case

Bohua Miao(✉)

School of Computer Science, Wuhan University, Wuhan, Hubei, China
2019302100095@whu.edu.cn

**Abstract.** Portfolio optimization has a pretty significant position in the financial field. The purpose of this paper is to make the portfolio decision and the asset allocation by using four stocks and a firefighter's pension that can be viewed as a long-term bond. First this paper aims to do the allocation analysis for the four fields. Then this paper will also discuss the portfolio decision under two different options with and without pension benefits. The data are collected and calculated by using the mean-variance analysis to get the performance of the portfolio. At last, by using the Fama-French three-factor model, the expected returns and Sharpe Ratio are calculated and then do the regression to determine which option is better. The results and the analysis show that, the Fama-French three factor can be more reasonable to do the calculation in the two options, and the 'GOOGL' has the largest proportion in the four stocks. Moreover, even though the portfolio expected return of the option without pension is higher, the Sharpe Ratio of the option with pension is higher, which means it is a better choice. These findings might be helpful to the investors related to the similar conditions.

**Keywords:** Fama-French three factor model · Pension · Portfolio management

## 1 Introduction

Since the 1950s ushered in what Merton Miller called "The big bang of finance", Markowitz proposed Modern Portfolio Theory (mean-variance analysis) in 1952, followed by Sharpe and Li Ntner proposed the Capital Asset Pricing Model [1] and Fama proposed Efficient Markets Hypothesis, and as a result, the research on the optimization of the investment portfolio have been quite a hotspot in the financial field. The significance of the portfolio lies in diversifying risks. A portfolio is to integrate the portfolio of stocks, funds, bonds and other products held by investors or institutions to invest to minimize risks. The portfolio fully embodies the concept of risk diversification, invests funds in different investment projects, and then makes appropriate allocations in specific projects according to the actual situation [2]. In view of this situation, to satisfy the demand that the investors cannot tolerant high risks as well as they detest the loss in the investment, what they intend to do is balancing the risk and the returns to obtain higher returns under the lowest risks that can be controlled. Therefore, the research on how to choose the portfolio to increase the returns and lower the risks can be the central direction on financial sector so that a more plausible portfolio will combine all these conditions to buffer the risks and returns [3].


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Currently, it can be found that there has been numerous research and studies related to portfolio choices. However, few studies focused on making portfolio decision when considering the industries of semiconductor, artificial intelligence, mobile phone and clothing at the same time, by the way, the pension will be included into the portfolio choices at last. So in this case, what will be discussed is that it can be pretending that there is a firefighter who will retire and facing two options, one of which is the buy-out option that give up the pension benefits and will receive a sum payment of 1.45 million dollars then use the money and the saving of the firefighter to invest and another is that take the pension benefits and invest in the portfolio chosen by using the saving to supplement the pension benefits. Most scholars' scope of research are quite extensive. For example, Stojanovic [4] discussed that Optimal portfolio theory, which is certainly one of its most important areas of research. And Eyal did the research that related to the portfolio decision when situated in the non-stationary marketing environment [5]. Moreover, Evans [6] studied this topic by finding the optimal combination of funds based on the entire portfolio's composite return and risk. And Perold [7] talked about large scale portfolio optimization. Meanwhile, some scholars did the research just in certain situations, like Cai, Teo, Yang, Zhou [8] and Peng, Yang [9]. Quite a lot of researchers focused on the techniques used to do the portfolio choices, such as Gupta, Mehlawat and Saxena [10]. And at the same time, some of them payed attention to certain fields, for instance, Zhou and Luo studied the portfolio of Internet financial assets [11]. The attention will be put on the portfolio choice on certain four different industries and consider the situation that make portfolio choice with and without pension. First of all, four stocks are chosen from four different industries, which refers to mobile phone industry, clothing industry, semiconductor industry and artificial intelligence industry, and these messages will be managed for making portfolio decision, which should be done by extracting the closing prices from May 1st, 2017 to January 1st, 2022 and then calculating the returns of these four stocks by using the closing price of this month to divide the closing price of last month and then the results minus 1. Then calculate the excess return by using the respective return minus the risk-free rate (the data of market return rate, SMB, HML and RF have been found and download from the website). Secondly, the correlations between these four stocks are calculated to determine the relationship between these stocks. Thirdly, the covariance of the four stocks and the covariance of pension and four stocks are calculated respectively in order to prepare for later use. Fourthly, running the regression on the four stocks with Fama-French three factor model are needed to calculate the betas. After that, it is time to allocate the different weights of the four stocks to maximize the portfolio return by calculating the maximal Sharpe Ratio. To get the result wanted, before the process, it is necessary to calculate the average of the Mkt-RF(E(Mkt-RF)), average of SMB(E(SMB)), average of HML(E(HML)) and the average of RF in the last five years. Fifthly, use the data that have been calculated to maximize the Sharpe Ratio to get the best allocation of these stocks by using the function of solver in Excel. Then it is essential to take the other option into consideration, which includes the four stocks as well as the pension that can be seen as a kind of long-term bond and its weight is constantly 0.71. The process is almost same as that in the last step with the help of solver in Excel to maximize the Sharpe Ratio and allocate the assets including the four stocks and the pension. Finally,

the comparison between the two options can be seen by comparing the Sharpe Ratio of them. As a result, what can be inferred is that the Sharpe Ratio of the option with pension is higher, which means it is the better choice.

The rest of this paper is organized as follows. Section 2 shows the data, Sect. 3 describes the methods. Section 4 depicts the results and Sect. 5 is the conclusion.

## 2 Data

The data in this article is derived from Yahoofinance (https://finance.yahoo.com/). And in order to discuss the relationship between these four industries, which refer to the clothing industry, semiconductor industry, artificial industry and mobile phone industry, five representative companies (NIKE, MRVL, GOOGL, AAPL) are selected for the research, and their closing prices are collected as well, from May 1st, 2017 to January 1st, 2022. At the same time, in order to mimic the pension asset, the data related to pension should be included. In this case, Florida bonds can be used as a proxy for the risk characteristic of the pension, and the monthly earnings are 0.62%. To do further research, the data collected is cleaned to match the time and in order to investigate more, Fama-French three factor model is considered and the data of the three factors should be included. So, finally 513 data are collected and some rudimentary information about these data is shown in Table 1.

Through visualizing the data, it can find that the return of the firefighter pension is constant, which is 0.62% monthly. And the 'MRVL' has the highest average return, while the 'AAPL' has the highest median. Moreover, the 'NIKE' has the lowest median and mean at the same time. When it comes to the variance, 'GOOGL' is the lowest and the 'MRVL' is the highest. What's more, 'AAPL' has the lowest min return while 'MRVL' has the highest max return.

**Table 1.** Descriptive information of the selected assets and pension

| | NIKE | MRVL | GOOGL | AAPL |
|---|---|---|---|---|
| mean | 0.0208 | 0.0334 | 0.0211 | 0.0328 |
| variance | 0.0052 | 0.0103 | 0.0044 | 0.0074 |
| max | 0.1519 | 0.2579 | 0.1590 | 0.2144 |
| min | -0.1217 | -0.1839 | -0.1324 | -0.1840 |
| median | 0.0117 | 0.0237 | 0.0201 | 0.0540 |
| | Expected Returns of pension | | | |
| | monthly | 0.0062 | | |
| | annual | 0.0775 | | |

# 3 Methodology

## 3.1 Mean-Variance Analysis

The mean-variance analysis is a mathematical method, which is the investment value analysis based on the relationship between expected return and variance of portfolio return. The investor needs to choose the optimal portfolio from all possible portfolios at the beginning and they have two main goals: the highest possible rate of return and the lowest possible risks. The best achievement should be to balance these two mutually constrained goals. According to the Schmidt [12] and Rubinstein [13], the mean-variance method can be done to help do the portfolio decision.

$$\sum_{i=1}^{n} \omega_i = 1 \tag{1}$$

In this formula, $\omega_i$ is the weight of the $Asset_i$ in the portfolio. And the return as well as variance of a portfolio will be calculated in the following steps.

$$r_p = \sum_{i=1}^{n} \omega_i r_i \tag{2}$$

Where $r_i$ is the return of $Asset_i$, where $r_p$ is the return of the portfolio.

$$E(r_p) = \sum_{i=1}^{n} \omega_i E(r_i) \tag{3}$$

Where $E(r_i)$ is the expected return of $Asset_i$, where $E(r_p)$ is the expected return of the portfolio.

$$Var(r_p) = \sum_{i \neq j} \omega_i \omega_j Cov(r_i, r_j) \tag{4}$$

Where $Cov(r_i, r_j)$ is the covariance between $r_i$ and $r_j$, and where $Var(r_p)$ denotes the variance of the portfolio.

## 3.2 Sharpe Ratio

Research on the Sharp ratio in modern investment theory shows that the size of risk plays a fundamental role in determining the performance of the portfolio. Sharp ratio is one of the three classic indicators that can comprehensively consider both returns and risks. A conventional feature of investment is that the higher the expected return of the investment target, the higher the risk of volatility that investors can tolerate; conversely, the lower the expected return, the lower the risk of volatility. Therefore, the main purpose of rational investors choosing the investment target and portfolio is to pursue the maximum return under fixed risk, or to pursue the lowest risk under fixed expected return. Rational investors will choose and hold effective portfolios, that is,

those that maximize the expected return at a given risk level, or those that minimize risk at the level of a given expected rate of return. For example, Zhang learned about the portfolio optimization with the Sharpe Ratio and Ding discussed about the maximal Sharpe Ratio [15].

$$SharpeRatio = \frac{E(R_P) - R_f}{\sigma_P} \tag{5}$$

Where $E(R_P)$ is the expected return of the portfolio, and $R_f$ is the risk-free rate, while $\sigma_P$ is the standard deviation of the portfolio.

### 3.3 Fama-French Three-Factor Model

In this research, the Fama-French three-factor model will be used because it is the most suitable model to analyze. And according to Peng, this model can be quite useful when analyzing the China's securities market [16]. Moreover, it can be used as well when it comes to other countries' stock market, for instance, India, according to Olive, Rohini, Deepa, Selvanayaki. When it comes to the five-factor model, certainly there are some advantages but in this model the information of the whole industry should be known, which is quite difficult to achieve as a matter of fact as a result of the limitation of available information. And five-factor model shows that there is only competitive relationship between different companies in the same industry. But in reality, there are still many kinds of cooperative relationships between these companies. And when it comes to the CAPM model, which is more accurate in predicting the firm's behavior comparing to mean model, it has quite a lot of restrictions and assumptions so that it is too rigid to help make portfolio decision. Moreover, CAPM model has plenty of limitations for us to use in reality, it has just one single factor so more factors should be added. Using the three-factor model, the investors can not only investigate the portfolio return, but they can also see the correlations between different factors. Less assumptions and restrictions are needed, and more accurate return will be shown. So, the Fama-French three-factor model has been proved to be more reasonable when investors compare the other two model.

In this model, there are three factors. HML is the book-to-return factor, SMB is the market value factor and (Rm-Rf) is market asset portfolio. And the model can be expressed as the following formula:

$$E(R_{it}) - R_{ft} = \beta_i\left[E(R_{mt}) - R_{ft}\right] + s_i^E(SMB_t) + h_i^E(HML_t) \tag{6}$$

Where $R_{ft}$ represents the risk-free rate of return on time t; $R_{mt}$ represents the market rate of return of time t; $R_{it}$ represents the rate of return of asset i in time t; $(E(R_{it}) - R_{ft})$ is the market risk premium, $SMB_t$ is the Small minus Big of the Size factor of time t, and $HML_t$ is the simulated combination rate of return (High minus Low) of the book-to-market factor of time t.

### 3.4 Correlation Coefficient

The correlation coefficient analyzes the linkage between the performance of two different assets from the perspective of asset return correlation. The absolute value of the

correlation coefficient reflects the strength of the correlation between the returns of the two assets. The correlation coefficient can measure the change between the rate of return of any two assets [18].

The correlation coefficient is within the interval [−1, 1]. When the correlation coefficient is −1, it means a completely negative correlation, indicating that the rate of return of the two assets changes in the opposite direction. When the correlation coefficient is +1, it means a completely positive correlation, indicating that the rate of return of the two assets is exactly the same. When the correlation coefficient is 0, it means irrelevant.

The positive and negative correlation coefficients are the same as the positive and negative covariance. The correlation coefficient is positive, which means that the rate of return of the two assets changes in the same direction, and the risk of portfolio offsets is less; the negative value means that the reverse direction changes, and the risk is more offset.

And correlation coefficient can be described as the following formula:

$$r(X, Y) = \frac{Cov(X, Y)}{\sqrt{Var[X]Var[Y]}} \tag{7}$$

Where $Cov(X, Y)$ is the covariance of X and Y, $Var[X]$ is the variance of X and $Var[Y]$ is the variance of Y.

## 4 Results

Using the formula of correlation, the correlation between these four stocks. And to make it clearer, the changes in the four stocks will be shown in a line chart. The results are shown in Table 2 and Fig. 1.

Through Table 2 and Fig. 1, it can be seen that the correlation between NIKE and AAPL is the lowest, while the correlation between NIKE and MRVL is highest, which can also be inferred from the Fig. 1.

The betas are calculated by using the Fama-French Three-Factor model and the regression function in Excel. The results are shown in Table 3.

After doing the regression, the respective expected return can be calculated by using the Fama-French Three-Factor model. The results are shown in the Table 4.

As the results indicate, 'GOOGL' has the highest expected return while 'NIKE' has the lowest expected return.

**Table 2.** The correlations between the four stocks

| | NIKE | MRVL | GOOGL | AAPL |
|---|---|---|---|---|
| NIKE | 1.0000 | | | |
| MRVL | 0.4920 | 1.0000 | | |
| GOOGL | 0.3675 | 0.3701 | 1.0000 | |
| AAPL | 0.3624 | 0.4367 | 0.4531 | 1.0000 |

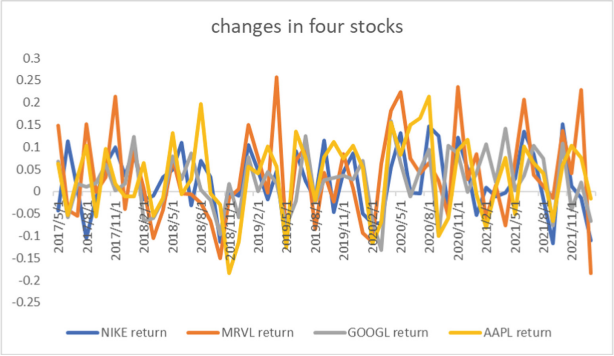


**Fig. 1.** The changes in the four stocks

**Table 3.** The results of betas

| | Mkt-RF | SMB | HML |
|---|---|---|---|
| NIKE | 0.9112 | 0.1594 | -0.3283 |
| MRVL | 1.1700 | 0.1724 | -0.5643 |
| GOOGL | 1.0820 | -0.3280 | -0.0087 |
| AAPL | 1.2386 | -0.4563 | -0.5147 |

**Table 4.** The results of the expected return of the four stocks

| | NIKE | MRVL | GOOGL | AAPL |
|---|---|---|---|---|
| expected return | 0.0063 | 0.0073 | 0.0076 | 0.0067 |

**Table 5.** Covariance of the four chosen stocks

| | NIKE ex | MRVL ex | GOOGL ex | AAPL ex |
|---|---|---|---|---|
| NIKE ex | 0.0051 | 0.0036 | 0.0017 | 0.0022 |
| MRVL ex | 0.0036 | 0.0102 | 0.0025 | 0.0038 |
| GOOGL ex | 0.0017 | 0.0025 | 0.0043 | 0.0025 |
| AAPL ex | 0.0022 | 0.0038 | 0.0025 | 0.0073 |

To make the portfolio expected return as high as possible under the possible lowest risk, the Sharpe Ratio are maximized to construct certain portfolio. The results are shown in the Table 6. Before that, the covariance of the four stocks should be calculated first and the results are shown in the Table 5.

**Table 6.** Results for portfolio without pension by maximizing Sharpe Ratio

| | NIKE | MRVL | GOOGL | AAPL |
|---|---|---|---|---|
| expected return | 0.0063 | 0.0073 | 0.0076 | 0.0067 |
| weight | 0.2530 | 0.0477 | 0.6113 | 0.0880 |
| | Portfolio expected return | 0.0072 | | |
| | sharpe ratio | 0.1122 | | |

**Table 7.** Results for portfolio pension by maximizing Sharpe Ratio

| | pension | NIKE | MRVL | GOOGL | AAPL |
|---|---|---|---|---|---|
| expected return | 0.0062 | 0.0063 | 0.0073 | 0.0076 | 0.0067 |
| weight | 0.7100 | 0.0982 | 0.0046 | 0.1517 | 0.0355 |
| | | portfolio expected return | 0.0065 | | |
| | | sharpe ratio | 0.3430 | | |

From the tables, it is obvious that 'GOOGL' has the largest weight in this portfolio accounting for 0.6113, while 'MRVL' has the smallest weight in the portfolio accounting for 0.0477. The portfolio return is about 0.0072 while the Sharpe Ratio is about 0.1122.

When considering the other option, which refers to adding the pension into portfolio then making the portfolio decision with the four stocks and the pension. This time the weight of the pension is constantly 0.71, and to make the portfolio decision, Sharpe Ratio should be maximized. The results are shown in the Table 7.

The table indicates that in the part of stocks, 'GOOGL' has the largest weight. When it comes to the portfolio expected return, the option with pension is lower than the option without pension, however, the portfolio with pension has higher Sharpe Ratio than the portfolio without pension.

## 5 Conclusion

At present, the most research related to the portfolio decision focus on the whole market or just single specific field. In this study, the portfolio optimization is made based on the four fields-clothing industry, semiconductor industry, artificial intelligence industry and mobile phone industry. And then the options with and without pension are discussed in order to solve the firefighter case by combining the pension with the four stocks. In the paper, the mean-variance analysis is applied to show the performance of the portfolio

and help to do portfolio optimization and the construction of the portfolio. The Sharpe Ratio is applied in order to compare the two options and determine which option is better;

The Fama-French three factor model is used to help calculate and analyze the two options. Finally, the study identities the artificial intelligence industry has the largest weight in the four industries, and the option with pension is better because of its higher Sharpe Ratio, even though the portfolio expected return is higher.

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