Empirical Study on Stock Index Futures Arbitrage and Relationship with Spot Index
Based on CSI300 Stock Index Futures

Yiran Gao(✉)

Xi'an Jiaotong-Liverpool University, Jiangsu, China
1508333633@qq.com

Abstract. As a futures investment tool, stock index futures has been concerned by investors since its introduction and widely used as one of the ways of risk averse. This paper empirically analyzes the relationship between the stock index futures trading and the spot index of CSI 300 using the co-integration theory and Granger causality test. It is found that futures price has interaction with spot index. The ETF fund portfolio is used to track the spot index. And the risk-free arbitrage strategy is formulated between futures contracts and spot index. This paper intends to provide support for investors to invest rationally in stock index futures' arbitrage trading.

Keywords: Stock index futures · Spot index · ADF · Arbitrage · No arbitrage interval

1 Arbitrage Principle and Arbitrage Method

1.1 Overview of Arbitrage Principle

Arbitrage Pricing Theory (APT) is an asset pricing theory and model under the assumption of market equilibrium proposed in 1960s. According to this theory, on the condition that all investors in the market have the same investment thinking and risk aversion degree, as well as the capital market is fully competitive, if the market does not reach the equilibrium state, then the opportunity of risk-free arbitrage will appear. Since the existence of risk-free arbitrage opportunities is profitable for risk-averse investors, rational investors will chase and make full use of the arbitrage opportunity until it disappears, and the market gradually reaches equilibrium.

From APT, we can conclude that arbitrage refers to a way receiving risk-free profits without paying any additional cost. Arbitrage principle is buying goods with lower prices and selling higher prices one, in order to earn the spread price.

1.2 Four Types of Arbitrage Methods

Common arbitrage methods are divided into calendar spread arbitrage, inter-market spread arbitrage, inter-commodity spread arbitrage and future-spot arbitrage.

F. Balli et al. (Eds.): ESFCT 2022, AEBMR 226, pp. 506–513, 2022.
https://doi.org/10.2991/978-94-6463-052-7_60

Calendar spread arbitrage refers to the arbitrage of buying and selling commodity in different periods. In the futures transaction, if the price is expected to rise in the future and the price bias between the near future and the forward futures contract is large, the price increase of the near future futures would be higher than that of the forward futures, so arbitrage is obtained by purchasing near future contract and selling the forward futures contract.

Inter-market arbitrage refers to the purchase or sale commodity at the corresponding market price in different markets (such as different exchanges) taking advantage of price differences to make a risk-free profit.

Inter-commodity arbitrage refers to the strong correlation between the futures contracts of two different commodities. As market demand changes, the price spread of two commodities deviates from the theoretical equilibrium. In this case, the two futures are operated in the opposite direction, buying the futures with low price and selling the futures with high price could obtain profits.

This paper focuses on the arbitrage of stock index futures and spot index, which promotes market equilibrium and improves market liquidity.

2 Comparative Analysis of Stock Index Futures and Spot Index

This paper compares the relationship between China Security Index(CSI) 300 futures and spot index, designs the arbitrage strategy based on daily closing prices of 117 trading days from 4/1/2022–30/6/2022 (from WIND). CSI represents the CSI 300 index; IFS represents the continuous contracts of the month (immediate futures); and IFL represents the continuous contracts of next month (distant futures).

2.1 Price Action Relationship

Figure 1 shows the price bias between the futures' daily closing price of CSI300 index and the price of CSI300 in 30/6/2021–30/6/2022. As well as Fig. 2 represents the bias between CSI300 and next month's continuous contract. It can be seen that the spot price is slightly higher than the futures price, fluctuating within 50 for most of the time.

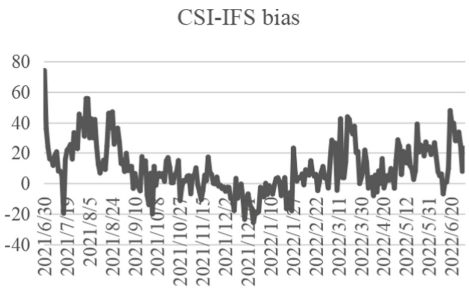

Fig. 1. Daily yield bias between spot and immediate futures

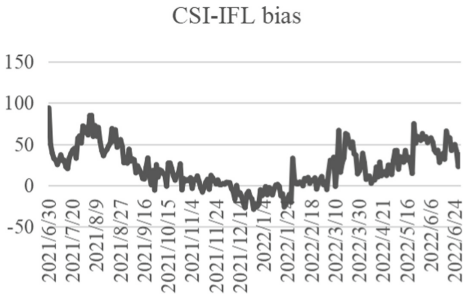

Fig. 2. Daily yield bias between spot and distant futures

Table 1. ADF test of logarithms data

Logarithms	Test statistic	1% critical value
lnCSI	−0.803	−4.035
lnIFS	−0.966	−4.035
lnIFL	−0.965	−4.305

Table 2. ADF test of first order difference data

First-order difference	Test statistic	1% critical value
d(lnCSI)	−11.309	−4.035
d(lnIFS)	−11.409	−4.035
d(lnIFL)	−11.548	−4.035

2.2 ADF Stationarity Test

For subsequent calculation, logarithms are taken for all data, respectively lnCSI, lnIFS and lnIFL. Augmented Dickey-Fuller (ADF) test is to determine stationarity of three groups of time series data. The null hypothesis is that the series has unit root. Results are shown in Table 1.

Since the statistical values are all greater than 1% critical value, the null hypothesis should be accepted, indicating that the original logarithmic sequence is not stationary. Therefore, we should use ADF test to determine the stationarity of first order difference sequences. Three groups of sequences are represented as d(lnCSI), d(lnIFS) and d(lnIFL) respectively. Results are shown in Table 2.

Statistical values of test results are all less than 1% critical value, indicating that these three groups of time series are all first-order integral series.

Table 3. ADF test results of estimated residuals

Estimated residuals	Test statistic	1% critical value
ε1	−5.711	−2.598
ε2	−4.603	−2.598
ε3	−3.604	−2.598

2.3 Co-integration Test

Since the price data of spot and futures are first-order integration, the co-integration test is used to judge the long term equilibrium relationship between the two sequences. Engle-Granger (EG) two-step co-integration test is carried out: A linear regression models among the sequences are established to estimate the residual value as Eq. 1–Eq. 3 [1]. Then judge whether the residual sequence is stable, according to three regression models, the residuals of each equation are estimated. When the time series are all first-order integrals and the residual series are stationary, it can be indicated that the two groups of time series have long-term equilibrium relationship. The results of ADF test for estimated residuals are shown in Table 3.

$$\ln \mathrm{CSI} = 0.1923519 + 0.9773249 \ln \mathrm{IFS} + \varepsilon_1 \tag{1}$$

$$\ln \mathrm{CSI} = 0.4743131 + 0.9439568 \ln \mathrm{IFS} + \varepsilon_2 \tag{2}$$

$$\ln \mathrm{CSI} = 0.2859136 + 0.9661673 \ln \mathrm{IFS} + \varepsilon_3 \tag{3}$$

From Table 3, statistic values are all less than 1% critical value, which reject the null hypothesis. Residual series of each group are stable, indicates there is a long-term equilibrium relationship between each two of the three groups of time series.

2.4 Granger Causality Test

To further determine whether the spot price has an impact on the recent forward futures, and whether the change of futures price leads to spot price changes, Granger causality test is conducted on the three sequences in pairs [2]. First, the original logarithm sequence is used to judge the choice of lag order. The result is shown in Table 4.

By the Akaike information criterion (AIC) principle, the lag order is selected to be 2. Then the VAR model is established to conduct Granger test for each group. The null hypothesis is that the equation can exclude a certain lag variable. Results are shown in Table 5.

Except 0.063 is greater than 0.05 which accepts the null hypothesis, the others all refuse the null hypothesis. The change of the spot price can effectively explain the change of futures, and plays a greater role in distant futures prices than immediate futures. Moreover, distant futures price changes and the immediate futures also affect each other.

Table 4. Lag order for Granger test

Lag	FPE	AIC	HQIC	SBIC
0	1.8e−13	−20.82	−20.7906	−20.7476
1	2.7e−15	−25.0166	−24.8991	−24.7269*
2	2.4e−15*	−25.1499*	−24.9442*	−24.643
3	2.5e−15	−25.1021	−24.8083	−24.378
4	2.7e−15	−25.0427	−24.6608	−24.1014

Table 5. Granger causality test results

Null hypothesis	P statistic
CSI cannot explain IFS	0.007
IFS cannot explain CSI	0.063
CSI cannot explain IFL	0.000
IFL cannot explain CSI	0.004
IFS cannot explain IFL	0.006
IFL cannot explain IFS	0.011

Stock index futures and spot prices are mutually restricted, and at the end of the futures expiration date, the two prices tend to be unified, futures prices converge to spot prices, and stock index futures play a role of price discovery in the spot market.

3 Arbitrage Strategy of Stock Index Futures

3.1 Cash and Carry Arbitrage

Positive arbitrage is a common arbitrage strategy. When the futures price is higher than the spot price at the same time, it sells the stock index futures contract and buys the portfolio of constituent stocks or the fund which represents the stock index, and obtain risk-free profits.

3.2 Reverse Cash and Carry Arbitrage

When the futures price is lower than the spot price, reverse arbitrage operation is carried out. The stock portfolio or fund representing the spot is sold and the stock index futures contract is purchased to obtain risk-free income.

3.3 No-Arbitrage Interval

No-arbitrage interval is a range which reflects the actual price of futures considering the real investment in the market stock index futures trading costs, handling fees, margin

and other issues. The upper bound depends on positive arbitrage strategy. When futures price in the market is higher than this no arbitrage bound, the positive cash and carry arbitrage can be operated. Similarly, the lower bound is estimated by reverse arbitrage, which means when the futures price is lower than the lower bound of the no-arbitrage interval, arbitrage can be carried out by shorting the spot and buying the futures.

4 Empirical Analysis of Arbitrage

When carrying out the arbitrage of stock index futures, the stock index is not the actual trading object. The stock portfolio of the market index or the Exchange Traded Fund (ETF) tracking the information of the index are selected to trade the part of spot index. ETF is an open-end fund with variable share traded on an exchange. It is listed and traded on formal exchanges and has the characteristics of fund diversification and instant trading in the stock market [3]. We choose four floor trading ETFs of the CSI 300 index which has best performance copying the index spot. The trading codes of ETFs are: 510300(a), 159919(b), 510330(c) and 510310(d).

4.1 Correlation Test

The difference between an ETF index fund and the underlying index it simulates is measured by tracking error. Comparing daily return rate of CSI300 and four ETFs in 4/1/2022–30/6/2022, it is clear that four funds had roughly the same volatility as the CSI 300. ETF funds can reflect the stock index well. A combination of the four funds could be a better substitution of the spot index. The daily return of the CSI300 as well as four ETFs are shown in Fig. 3.

In order to establish the regression model, it is necessary that the four ETFs are stationary time series data. Therefore, ADF test is used to determine whether daily closing prices of these ETFs are stable. The ADF test results of the four funds are shown in Table 6.

The test results of the four groups of time series all show stationarity, which can be directly used to construct the regression model.

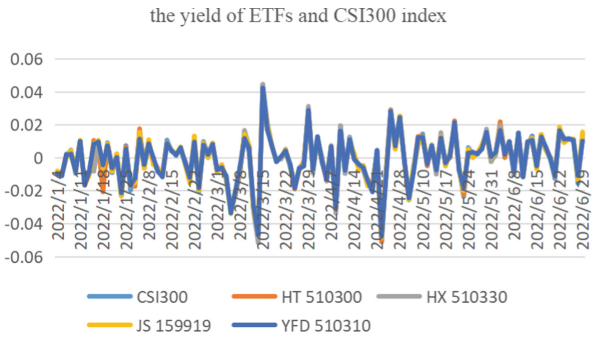

Fig. 3. Daily return of spot and ETFs

Table 6. ADF Test result of four ETFs

ETFs	Test statistic	1% critical value
510300 (a)	−11.331	−4.035
510330 (b)	−11.118	−4.035
159919 (c)	−11.047	−4.035
510310 (d)	−10.996	−4.035

Table 7. Ordinary Least Squares regression result

ETFs	Coefficient	Standard error	t-test value	P statistic
510300(a)	0.193	0.061	3.17	0.002
510330(b)	0.098	0.054	1.81	0.074
159919(c)	0.352	0.072	4.91	0.000
510310(d)	0.387	0.080	4.82	0.000

4.2 OLS Regression of Arbitrage Model

TheN the regression Eq. (4) of CSI 300 return rate to the portfolio proportion of the four ETFs and Table 7 are established.

$$\mathrm{CSI} = 0.193\,\mathrm{a} + 1.098\,\mathrm{b} + 0.352\,\mathrm{c} + 0.387\,\mathrm{d} \tag{4}$$

The ratios of the four ETFs are set at 19.3%, 9.8%, 35.2% and 38.7%, respectively, which can be used as the replacement portfolio of the spot index. The adjusted R square is 0.9921, which indicates that the model is well fitted.

4.3 Construct Arbitrage-Free Interval

The transaction fee of ETF is 0.02% of the transaction volume, the spot impact cost is 0.16%. Futures transaction commission is 0.0023%, futures impact cost is 0.11%, risk-free interest rate is 1.5% and financing interest rate is 5.5%. (Calculated from WIND).

S_t represents the price of CSI 300 index and F_t represents the price of stock index futures contract. Applying the formula of no-arbitrage interval of CSI300 in A-Share market [4], the upper bound of no-arbitrage interval is in Eq. (5), and the lower bound is in Eq. (6).

$$F_t = \frac{S_t * 1.0018 * e^{0.055*\frac{117}{365}} + S_t * 0.0029}{1 - 0.001123 * e^{0.0055*117/365}} \tag{5}$$

$$F_t = \frac{S_t * (e^{0.055*\frac{117}{365}} - 0.9982 * e^{0.055*\frac{117}{365}} - 0.0016) - F_t * 0.001123}{1 + 0.001123 * e^{0.0055*117/365}} \tag{6}$$

5 Conclusion

This paper analyzes the basic principles and types of arbitrage method, then focuses on the relationship between futures and spot index. The ADF test, co-integration test and Granger causality test are used to find that there is a long-term equilibrium relationship between stock index futures and spot index. Spot index and distant futures has a strong price impact on each other, while immediate futures have a small impact on spot. It is also observed that the price of stock index futures converges to the spot index. Finally, this paper makes use of this characteristic and the price bias, takes CSI 300 as an example to construct a no-arbitrage interval within half a year, replaces the spot index with ETF portfolio, and designs two strategies for stock index futures and spot index.

References

1. L. Tan, Y.F. Xu and A. Gashaw, Influence of Exchange Rate on Foreign Direct Investment Inflows, Mathematical Problems in Engineering, 2021, pp.1-12. DOI: https://doi.org/10.1155/2021/7280879.
2. C. Liu, W.J. Zhang, Empirical study on the relationship between stock index futures and spot market in China, Securities and Futures of China, 2012(03), pp. 4-5.
3. S.Q. Bai, W.J. Sun, Empirical research of CSI-300 stock index futures arbitrage strategy, 2014 International Conference on Management Science & Engineering, pp.1326-1332. DOI: https://doi.org/10.1109/ICMSE.2014.6930384.
4. Q. Ting, The study of arbitrage between CSI 300 index futures market and spot market, Nanjing University of Finance and Economics for the Academic Degree of Master, 2013, pp.12-14.