Portfolio Optimization with or Without Safe Asset

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Abstract. Portfolio optimization is one of the most common and essential technique in measuring the plausibility of the designated combinations of the assets. Optimal Portfolio are well diversified to decrease the non-price risk and the unsystematic risk of the assets, which maximizes the returns of the stocks and protects the investors from the underperformances of certain assets. This paper engages in portfolio optimization through the asset allocation for different types of equities: Exchange-traded funds (ETF), mutual funds and stocks. First, there are five assets chosen from the market and their closed price are elicited as their daily returns. Second, using Fama–French 3 factor model (FF3F), the researchers can calculate the expected returns and possible risks of the portfolio. Third, they then utilize the built-in Solver function in Excel to generate a maximum value for the Sharpe ratio by putting various weights on different assets in that portfolio.

Keywords: Portfolio Optimization · Fama–French 3 factor model (FF3F) · Capital Asset Pricing Model (CAPM)

1 Introduction

There is a long history of investing when the Code of Hammurabi in 1700BC provided an official structure for investing: "an approach for the pledge of collateral by codifying debtor and creditor rights regarding pledge land". Such behavior can be view as a trade-off between "immediate consumption" and "deferred consumption", where investors compared the instantaneous benefits of consuming today against the future payoffs of the equity they invested into [1]. In the recent decades, as the exchanges and acquisition of information could never been easier, people engaged in investment behavior more frequently, and to maximize their utility in investment, they would choose assets with the optimal balance between returns and risks: they are trying to find the investment that brought them huge returns and moderate risks. As Somerville states in his paper, risk aversion is a vital factor in financial investment, which depicts a trade-off between risk and return in financial activities [2]. In the light of this, the focus of this paper is about portfolio optimization in investing behavior; that is, by using empirical studying and incorporating historical data, the researchers will be able to generate an optimal portfolio for the rational investors.

During the recent decades, research about optimal returns and portfolio optimization is abundant, but most of them are focused on single-type asset; for example, La Porta

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investigated the expectations and cross-sections of stock returns, or Leim Chin analysed the portfolio optimization of specific types of stocks returns with the reference of index LQ45 [3] [4]. Other paper, on the other hand, focused on applying multifactor model in the real-life situation; for example, Fama and French discussed the anomalies of multifactorial models in empirical data for their three-factor and five-factor models in investing, and Griffin demonstrated the difference between country-specific and global versions of FF3F model through time series analysis [5] [6]. Shedding lights on the optimization problem not only on specific types of stocks or just the model itself, but in a broader context such as combination of various asset types might bring new innovations to the research of this field. The importance of the study can be articulated in various aspects. First, the portfolio optimization provides a rather comprehensive and accurate measure of the returns of the stocks and risks, which benefits those investors who would like to maximize risk and returns trade-off. The optimizing process using ordinary least squared regression (OLS) and multiple factors to precisely predict the expected returns for the portfolio; thus, the investing managers can combine risky assets with a risk-free asset to discover balance between those two factors [7]. Second, although there are papers focusing on portfolio optimizing, there are few papers concentrating on diversified assets using mutual funds, ETF and stocks. By incorporating different types of assets into the portfolio, one can manage the risk by diversifying the types of assets and lower the correlations between different sorts of equities.

The process of conducting this study can be summarized as follow steps. First, the researcher gathers the data of five distinct ETFs, mutual funds and stocks from Yahoo Finance from January 1, 2017, to January l 1, 2022; Second, by applying the correlation function, the researcher is able to see the relationship between different assets and adjust the data set if one sees a very high correlation coefficient. Third, as the Fama–French three factor (FF3F) coefficients are added into the model, one can then calculate the betas and expected returns for different assets. Fourth, using expectation-variance analysis, the conductor can optimize the Sharpe ratio for the portfolio using the Solver function. Fifth, comparing the Sharpe ratio for different portfolios can help generate an optimal portfolio under some constraints. The betas of the stocks are generated via ordinary least squared regression (OLS), and the optimal Sharpe ratio's portfolio using the OLS data will help the researcher to generate more accurate results.

The concrete structure is shown below. Section 2 depicts the data with their sources and types, Sect. 3 shows the methodologies of generating the results, Sects. 4 and 5 present the results of the data running and discussion of possible problems and concerns during the analysis. Section 6 is the conclusion.

2 Data

The data in this study is mainly elicited from Yahoo-finance (https://finance.yahoo.com/). The five columns of data are selected from different mutual funds, stocks, and ETF in order to diversify the asset and decrease the risk of the portfolio. Among them, ANNPX, VUG are mutual funds which put lots of weights on technology stocks; CSX and NEX are stocks in railroads and oil; and XSVM is an ETF that put most of the funds in financial service. The Table 1 shows the correlations between the five assets.

Table 1. Correlation of the five assets

	ANNPX	*VUG*	*CSX*	*XSVM*	*NEX*
ANNPX	1	0.579	0.355	0.468	0.328
VUG	0.579	1	0.695	0.637	0.343
CSX	0.355	0.695	1	0.604	0.343
XSVM	0.468	0.637	0.604	1	0.555
NEX	0.328	0.343	0.369	0.555	1

Table 2. Descriptive statistics of the selected assets

	ANNPX	VUG	CSX	XSVM	NEX
mean	0.0165	0.017	0.015	0.016	0.017
variance	0.005	0.003	0.006	0.006	0.067
min	−0.209	−0.109	−0.184	−0.308	−0.749
max	0.264	0.154	0.216	0.237	0.983

By monitoring the correlation between different assets, the researchers can generate a more plausible portfolio with the equities which have low correlations between each other. After determining a rather moderate-correlation portfolio, it is beneficial to investigate into the expected returns for the five assets.

In the Table 2, it can see that GTLLX has the highest returns among the five assets, while its risk is also higher than the equities that have lower returns. On the contrary, XSVM has the lowest return while its risk is not the lowest, so it is predictable that holding less XSVM will generate a higher Sharpe ratio.

3 Methods

3.1 Expected Return and Risks

The expected return commonly refers to the gains which investors would like to get according to the historical performance of an asset. In this case, historical rates of return are the closed price of the asset from a specific range of time. The process of calculating expected returns is that one uses potential outcomes by the chances of their occurrences multiply by their individual results, as shown in the formula below [7]. Therefore, expected returns can be an indicator to determine the future performance of a stock. The sum of the returns and probability is calculated as the expected value (EV) which is shown below in Eq. (1):

$$Expected\ return = \sum_{i=1}^{N} Return_i \cdot Probability_i \tag{1}$$

In the formula, "i" symbols the number of assets. Based mainly on the historical data, the expected return can only set reasonable expectations of the certain asset, but it cannot generate absolute prediction of the future performance. Therefore, the expected return can be treated as an equally weighted average, but not a predictor for that financial product.

When evaluate the performance of one portfolio, there are commonly two indicators, i.e., expected return and standard deviation. Traditionally, the standard deviation can be calculated by the equation in (2).

$$standard\ deviation = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - x)^2} \tag{2}$$

In the formula above, N represents the sample size, x_i measures the returns of the assets in the portfolio, and x is the average returns of the asset [8].

3.2 Capital Asset Pricing Model (CAPM)

The CAMP model has a long history in describing the whole market and one certain asset [9]. With the introduction of the CAPM model, the expected return is calculated based on a mathematical model shown below. Using the ordinary least-squared regression, the researchers can get the beta value of the stocks and multiply it by the expected market returns, and it will output the expected return of the certain assets. The following equation demonstrates the calculation process of expected returns given its risk as follows:

$$Expected\ Return_i = R_f + \beta_i\left(ER_m - R_f\right) \tag{3}$$

Where "i" represents the individual assets starting from i = 1, R_f is risk-free rate of the asset, β_i symbolizes beta of the investment calculated using OLS regression, and ($ER_m - R_f$) is the market risk premium (difference). In the CAPM model, investors expect to be compensated for risk and the time value of money; otherwise, they will be less motivated to hold that asset. One important parameter, i.e., beta, should be focused carefully. When the value of beat is greater than 1, a higher risk of the asset than the market can be achieved. However, the lower risk normally comes with lower returns, so it will not be sufficient for choosing only high or low beta's financial products. The market risk premium is then multiplied by the beta of a stock, yielding the projected returns from the market above the risk-free rate. CAPM is more accurate in predicting the firm's behavior than simply using mean and variance for the stocks; however, with too many restrictions and assumptions, CAPM is too rigid and idealized for investor's behavior. In addition, the CAPM model has a lot of limitations for the research to utilize in a real world setting because it is a single factor model but attributes all risk to one factor, which can be problematic and results in huge biases. Therefore, this paper asks for multifactor models to better model the companies' performances. It is supposed that returns on a security come from multiple common factors, the "idiosyncratic" returns, as a result, will not be independent across firms [9]. Therefore, more factors are needed.

Table 3. FF3F coefficient for five stocks

Coefficient	ANNPX	VUG	CSX	XSVM	NEX
HML	0.147	0.281	0.098	0.769	3.024
SMB	−0.085	0.127	0.113	0.964	1.599
RMt-Rft	0.948	1.068	1.135	1.010	2.021

3.3 Fama-French Three-Factor Model

However, the above mentioned CAPM some deficiencies, thus, the subsequent researchers base the CAPM to formulate the Fama–French three-factor model. Compared to CAPM requiring many idealized assumptions over the investors, FF3F model generates more accurate results for expected returns for the stocks without assuming the rationality of the investors. The formula to predict a certain type of stock is the following:

$$R_{it} - R_{ft} = \alpha_{it} + \beta_1 \left(R_{mt} - R_{ft}\right) + \beta_2 SMB_t + \beta_3 HML_t + \varepsilon_{it} \tag{4}$$

In the formula, "t" represents the values of different parameters at a time t; SMB_t stands for market value factor while HML_t accounts for the book-to-market factor at time t [10].

3.4 Sharpe Ratio

$$Sharpe\ Ratio = E(Rp - rf)/\delta p \tag{5}$$

In the formula above, Rp represents expected return of the portfolio, r_f equals average of risk-free rate of the asset, and δp represents the standard deviation of the portfolio. In Table 4, Rp is shown as ER_portfolio in row 3, column 1. And rf is the risk-free rate: 0.0009 derived from by taking the average of the risk-free rate in the empirical Fama–French 3 factor sheet. Finally, δp is the standard deviation of the portfolio, which is shown as the "Standard deviation" in row 5, column 1 in Table 4. The goal of the portfolio design is to maximize the Sharpe Ratio; that is, to maximize firefighter's utility. By achieving this, the researchers will use the Solver function, a built-in function in Excel that redistributes weights on different assets and returns a maximum or minimum value for the term that researchers want. In this case, the target term is the Sharpe ratio of the portfolio.

4 Results

After applying FF3F model into the data for five different stocks, the three coefficients for the three factors used to generate expected returns for the assets are showed as followed:

In the graph, it can be concluded that the HML coefficient for five assets is similar and positive correlated to the shock returns. On the contrary, SMB factor for ANNPX is negative comparing to the other four assets whose SMB coefficient are all positive, meaning that the ANNPX asset is negative affected by the small companies outperforms.

Table 4. Statistical results for portfolio without the state pension

	ANNPX	VUG	CSX	XSVM	NEX
ER	0.008	0.007	0.009	0.012	0.028
Weight	0.066	0.217	0.153	0.502	0.063
ER_portfolio	0.011				
variance	0.005				
Standard deviation	0.069				
Sharpe ratio	0.153				

Table 5. Statistical results for portfolio with the state pension

	ANNPX	VUG	CSX	XSVM	NEX	pension
ER	0.008	0.007	0.009	0.012	0.028	0.006
weight	1	0.0007	0	0.0002	0	−0.0009
ER_portfolio	0.008					
variance	5.2E-07					
Standard deviation	0.0007					
Sharpe ratio	9.507					

The market risk premium coefficient are ranging from 0.948 to 2.021, which is moderate; thus, the risk-premium of the asset will have a positive correlation with the expected asset returns. The data in Table 3 then are used as the betas in formula 4 for generating the expected return for individual asset. Then, the expected returns of the individual equity with the empirical fixed risk-free rate and the variance of the portfolio can help conducting an optimal Sharpe ratio weight for different assets in the portfolio. The results are showed in Table 4.

In the Table 4, it can be seen that the portfolio maximizing the Sharpe ratio will put most of the weight on XSVM, the ETF that put most of the weights on financial service. However, according to the Solver function, it can be concluded that to increase the Sharpe ratio, the investor should put nearly no weight on ANNPX and NEX, in which ANNPX is the mutual fund that focuses on technology, and NEX is a United States oil stocks. The remaining weight of the portfolio is distributed into VUG and CSX. Therefore, from the graph, the maximum Sharpe ratio is 0.153 and the standard deviation is 0.069.

Comparing Table 5 and Table 4, it can be seen that the variance of the portfolio is $5.2 * 10^{-7}$ when incorporating the safe asset into the portfolio, which is much lower than the portfolio without the pension; on the contrary, the expected returns of the portfolio in Table 4 is 0.011 and Table 5 is 0.008, which is similar. Therefore, according to the formula for generating the Sharpe ratio, the Sharpe ratio for the portfolio with the safe asset is much higher than the one without safe asset.

5 Discussion

The goal of the paper is to investigate the optimal portfolio through allocating assets. It is recognizable that the Shape ratio with the pension in Table 5 is much larger than the portfolio without pension. The only difference between the two portfolios is that the second portfolio includes a safe asset, Florida state pension. By adding the Florida state pension which yields relative low returns with very small variances, the weights generating by the Solver function demonstrates a huge difference than the one without the safe asset. However, through the graph, it is noticeable that to maximize the Sharpe ratio, the investor should short the Florida state pension rather than holding any, and nearly all the weight is put on the mutual funds that has the highest share of technology firms (ANNPX). Therefore, adding the option for safer asset to buffer the variance of the portfolio might not work in this case.

The second thing worth noticing is that CAPM and FF3F's have different assumptions on risk-free rate. Comparing to FF3F model that does not require strong assumptions on the investors' behavior, CAPM idealized the behaviors of the investor and the financial behavior of the stocks, resulting a 0 in risk-free rate in the CAPM model. By using FF3F data, the model is exposed to more realistic data; therefore, the betas obtained from FF3F model are more accurate.

In this paper, only monthly data for a total of five years of different assets are chosen, which gives only 61 hedge points for the model to run; therefore, the sample size is a little bit small in this paper. To improve this, the researchers can import the data from a longer period and generate constants through ordinary least square regression that fit better to the trend in previous years.

When taking the health condition or life expectation of the investors into consideration, the investment decision might be varied for that investor. If the life expectancy is long, the investor might choose the one with the highest Sharpe ratio's portfolio since it renders the optimal balance between expected returns and risks; on contrast, the investor's life expectancy is relatively short, he might choose the portfolio with the maximum returns and take more risks to exchange for instant benefits since he values the future much less than the one with robust health conditions.

6 Conclusion

To conclude, in this paper, five assets from different types of assets: ETF, mutual funds, and stocks are chosen for optimal portfolio analysis. Then, FF3F model is applied to the data and generate the expected return for different equities, and the solver function yields the optimal weight for different assets. Finally, by adding a safe asset—the Florida state pension, one can compare different asset to see which one yields the highest Sharpe ratio. The results show that keeping the Florida state pension will generate a much higher Sharpe ratio than the portfolio that does not include the safe asset. Therefore, it can be concluded that the safe asset will alter the choice of the weight in the portfolio and generate a much higher Sharpe ratio to the portfolio, and it is optimizing the yields of the combinations.

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