Markowitz and Index Model Comparison Using Different Stocks

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Abstract. Markowitz Model and Index Model are two dominant portfolio optimization models financiers use in recent years. By using these two models, different possible optimized investment strategies can be created for people to distribute their properties wisely. And in this paper, to run the Markowitz Model and Index Model independently, an investment portfolio with 20 years of daily data of total returns of the S&P 500 Index and ten stocks will be employed. After setting five different constraints which correspond to circumstances in real life more, consequential data of the Markowitz Model and Single Index Model will be produced which include the Minimum Variance point, Maximum Sharpe point, Efficient Frontier, Inefficient Frontier, and Minimum Variance Frontier. These data are indispensable elements to compare Markowitz Model and Single Index Model. Comparing these two models is important because it can be ensured that the results from the two models do not differ a lot and these two models are two believable and useful tools to build the optimized investment strategies for people. And this paper also shows that these two models are useful and meaningful under different circumstances after a long time.

Keywords: Markowitz Model · Index Model · Minimum Variance · Maximum Sharpe · Efficient Frontier · Inefficient Frontier

1 Introduction

Nowadays, more people are concerned about investing their money into stocks, no matter whether they are wealthy or not. This is a good way to earn money if people get enough principal, which is called "money begets money". However, when facing thousands of stocks, investors can easily get confused, become indecisive, or make wrong decisions. Although all investments are risky, there is always a need for investors to find those "safe" stocks. Thus, the portfolio become popular for its diversification and certainty. Different stock portfolios have different combinations and distributions. While various portfolios are created every day, models such as Markowitz Model and Index Model that can test them are also discovered. And it is crucial for all investors who do or do not have the professional knowledge to know whether these models help, or work and which model works better in various circumstances.

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1.1 Related Research

Whitelaw et al. discovered that nations with low COVID-19 per capita mortality tend to share methods such as early monitoring, detection, contact tracking, and stringent isolation in the broader context of the epidemic's globalization. The magnitude of coordination and data management required to effectively implement these initiatives in most successful countries is dependent on the adoption and integration of digital technology into policy and health care. This viewpoint offers a paradigm for using digital technology in pandemic preparation, surveillance, detection, contact tracing, quarantine, and health care [1]. Garfin looked at how society's reliance on technology grew during the COVID19 illness pandemic, and how social and vocational changes may last long after the crisis has passed. As a result, we must make informed decisions about how we might use technology to better our lives, reduce stress, and promote mental health [2].

He et al. discussed developing technologies utilized to combat the COVID-19 threat, as well as the problems of technology design, development, and implementation. It also offers suggestions and advice on how information systems and technology experts might aid in the fight against the COVID-19 pandemic. This study will aid future research and technology development in order to produce better COVID-19 solutions [3]. Almeida et al. analysed the effects of digital transformation on three corporate areas: labour and social relations, marketing and sales, and technology Digitalization is predicted to have a cross-cutting impact on all of these domains, encouraging the development of new digital products and services based on the notion of flexibility. Furthermore, new ways of working will increase demand for new talent, regardless of where people are located [4].

Wu et al. collected data on health information technology to better understand how the Chinese health informatics community responded to the COVID-19 epidemic. To make clinical administration of COVID-19 easier, researchers used mobile and web-based services like Internet hospitals and WeChat, as well as big data analytics, cloud computing, Internet of Things, artificial intelligence, 5G telemedicine, and clinical information systems [5].

The study's objective is to compare ten different firms' stock data using the Markowitz model and the index approach. The underlying data is a roughly 20-year history of daily total return data for ten stocks from various industry groupings three or four, as well as a stock index and a risk-free rate (monthly federal funds rate). The daily data must be aggregated into monthly observations to reduce non-Gaussian effects. All required optimization inputs for the entire Markowitz model and the exponential model are produced based on these monthly observations. Finding the areas of permitted portfolios for the following five situations with extra constraints using these optimization inputs for MM and IM.

2 Method

2.1 Markowitz Model

In 1952, Markowitz applied for the first time the mathematical concepts of mean and variance of asset portfolio payoffs in his academic paper "Asset Choice: Effective Diversification", which clearly defined investor preferences mathematically [6]. For the first time, the principles of marginal analysis were applied to the analytical study of asset portfolios. This research result is mainly used to help households and companies how to use and combine their funds wisely to achieve the maximum return when the risk is certain. The two core considerations when investing in securities and other hazardous assets are expected return and risk. Market investors are grappling with how to calculate a portfolio's risk and return, as well as how to balance these two indications for asset allocation. Markowitz's mean-variance model was developed in this setting.

The Markowitz model assumes the following conditions:

(1) The investor evaluates each investment option based on the probability distribution of the securities' returns over time.
(2) Based on the predicted return of the security, the investor calculates the portfolio's risk.
(3) Investor's decision is based solely on the risk and return of the security.
(4) The investor expects the most return at a specific level of risk; conversely, the investor expects the smallest risk at a certain level of return.

Markowitz developed the method of calculating the anticipated return and risk of a securities portfolio and the notion of effective frontier based on the following assumptions. The best asset allocation model is constructed using the mean-variance model. The investor can predict the expected return, and the above equation may be used to calculate the proportion of the investor's investment in each investment project that minimizes the overall investment risk.

The minimal variance set is made up of several minimum variance combinations based on different expected returns. For the following four scenarios with extra limitations, we must discover the areas of permitted portfolios which are optimal and satisfied these constraints:

(1) This extra optimization constraint is intended to emulate FINRA Regulation T [8], which permits broker-dealers to permit their clients to have positions that are financed 50% or more by their account equity:

$$\sum |w_i| \leq 2 \tag{1}$$

(2) This extra optimization restriction is intended to emulate some arbitrary "box" weight limitations that the client may specify [8]:

$$|w_i| \leq 1 \tag{2}$$

(3) To illustrate how the region of allowed portfolios in general, and the efficient frontier, look like when there are no additional optimization constraints.
(4) This additional optimization constraint is intended to mimic the normal limits seen in the mutual fund business in the United States [9].

$$w_i \geq 0 \tag{3}$$

(5) Lastly, whether the inclusion of the broad index in our portfolio will have a positive or negative impact, for which an additional optimization constraint is considered [10].

$$w_i = 0 \tag{4}$$

2.2 Data

Nvidia Corporation is a Delaware-based global technology firm headquartered in Santa Clara, California. It is a software and fabless firm that creates graphics processing units (GPUs), application programming interfaces (APIs) for Data Science and High-Performance Computing, and system on a processing unit (SoCs) for the mobile computing and automotive markets. It was formed on April 5, 1993. Nvidia also produces diverse products such as GPU, CPU, DPU, Chipsets, collaborative software, computers, TV accessories, and laptops to meet the need of consumers. It is known as 10 biggest tech companies in the world.

Cisco Systems, Inc. is a global technological company based in San Jose, California. Cisco develops, produces, and distributes networking gear, software, telecommunications equipment, as well as other high-technology services and products, and is critical to Silicon Valley's success.

Intel Business, stylized as intel, is an American multinational corporation and technological firm based in Santa Clara, California.

The Goldman Sachs Group, Inc. is a New York-based American global investment bank and financial services firm. Investment banking, securities underwriting, asset and investment management, and framework of financial are all services provided by Goldman Sachs.

U.S. Bancorp is a Delaware-based American bank holding corporation situated in Minneapolis, Minnesota. It is the parent corporation of U.S. Bank National Association and the country's fifth largest financial organization.

The Toronto-Dominion Bank is a worldwide banking and financial services business based in Toronto, Ontario, Canada.

Since 1967, the Allstate Corporation has been headquarters in Northfield Township, Illinois, close to Northbrook. It began as a division of Sears, Roebuck, & Co. in 1931 and was split off in 1993. Personal lines insurance is also offered by the corporation in Canada.

The Procter & Gamble Company was formed in 1837 by William Procter and James Gamble as an American global consumer products firm headquartered in Cincinnati, Ohio.

Johnson & Johnson (J&J) is a medical device, pharmaceutical, and consumer packaged products global firm founded in 1886. Its common stock is a component of the Dow Jones Industrial Average, and it is placed 36th on the 2021 Fortune 500 list of the biggest firms in the United States by total revenue.

Colgate-Palmolive Company is a global consumer goods corporation located on Park Avenue in Midtown Manhattan, New York City. It is a manufacturer, distributor, and provider of home, health care, personal care, and veterinary products.

Table 1. Basic features of the Variables

Variable	Average Value	Maximum Value	Minimum Value	Standard Deviation
S&P 500	290.79	392.7	150.9	80.88
AVVV	13,700	17,200	10,100	2350.18
X Variable 2	7,293,067	7,507,400	7,024,200	155,171.53
X Variable 3	193.04	201.4	174	7.30

Also, the Solver and Solver Table in the excel are two important tools to be used during the calculation process to get the result for different constraints. Because there will be different minimum value, maximum value, and increment for different constraints, it is important to set them to corresponding value and get the final number.

3 Results

The weighted coefficients are shown in Table 2. To obtain our initial data, we separated the procedure into four parts. To eliminate non-Gaussian effects, we must aggregate daily data into monthly observations, and then generate the required optimization inputs for the Markowitz model and index model based on this monthly data. The raw data is then used in the second stage to determine the annualized average return and standard deviation; beta and alpha; and residual stand [11].

Then, we need to set five different constraints for both Mark and deviation: Correlation of the stocks of each ten different companies. In the third step, we create a thousand possible portfolios, and through a graph of those data, we could see the overall trend of the Return and Standard deviation of the possible portfolios. The fourth step is the calculation for Markowitz Model. To calculate the return, we used the SUMPRODUCT function, which multiplies ranges or arrays together and returns the sum of products. Standard deviation, on the other hand, would be more complex (Tables 1, 3 and 4).

We need to use the correlation of each stock and the properties of the matrix multiplication to create the standard deviation for a current portfolio for the Markowitz Model. Though the index model has a different function for calculating the return, it turns out that both functions would have rather close numbers, so the return from both ways of the calculator would be valid. Markowitz Model and Index Model. Just like the part introduced before, we need to calculate five constraints of two models by Solver separately. Thus, Markowitz Model is the one being solved first (Fig. 1).

For constraint one that allows the user to specify upper and lower bounds on the weights of the assets, the return must be set to equal to the dummy variable and IMsharpe must smaller or equal to two. Then, run the solver Table that set the minimum value as –0.3, the maximum value as 0.4, and the increment as 0.01. After clicking maximum and minimum in the Solver, we can get the Efficient and Inefficient Frontier under constraint one.

Constraint two allows the users to specify upper and lower bounds on the weights of the assets, the return must be set to equal to the dummy variable and IMsharpe must

Table 2. The weighted coefficients of the selected asset between minimum variance and max Sharpe

	MinVar	MaxSharpe
SPX	0.3837	– 1.0997
NVDA	–0.0297	0.2246
CSCO	–0.029	0.0089
INTC	0.0133	– 0.0817
GS	–0.059	0.1273
USB	–0.003	0.1321
TDCN	0.19415	0.4646
ALL	–0.115	0.079
PG	0.2593	0.535
JNJ	0.1883	0.4272
CL	0.1967	0.183
Return	0.07508	0.16988
StDev	0.10953	0.16476
Sharpe	0.685	1.031

Table 3. Return for Minimum Variance

	Markowitz Model	Index Model
Constraint 1	20.18%	13.51%
Constraint 2	10%	10%
Constraint 3	7.508%	7.152%
Constraint 4	10%	10%
Constraint 5	8.706%	10%

Table 4. Return for Maximum Sharpe

	Markowitz Model	Index Model
Constraint 1	20.18%	19.72%
Constraint 2	10%	10%
Constraint 3	16.988%	12.868%
Constraint 4	10%	10%
Constraint 5	13.058%	10%

be smaller or equal to 1. Then, run the solver Table that set the minimum value as –0.3, the maximum value as 0.4, and the increment as 0.01. After clicking maximum and minimum in the Solver, we can get the Efficient and Inefficient Frontier under constraint one.

For constraint three, an investor might borrow up to 50% of the purchase price of stocks that can be purchased using a broker or dealer's loan, and the other 50% must be paid in cash, we just need to set dummy variable equals to the return and run the solver table from –0.4 to 0.6 with the 0.05 increment. Then, the efficient frontier will be drawn out. For the inefficient frontier, we should check the maximum, set the range from 0.15 to 0.6 with the 0.01 increment, and run it.

For constraint 4 which is designed in particular U.S mutual fund industry, the standard deviation should be set equal to the dummy variable, then run the solver table from 0.07 to 0.32 with a 0.01 increment. By changing the minimum to maximum in the Solver, we

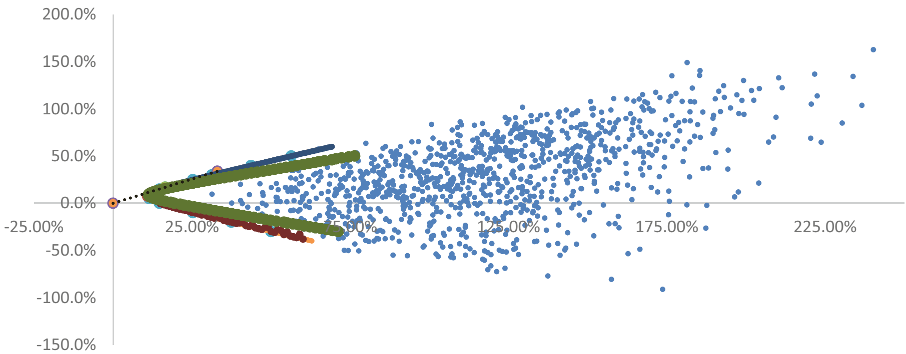

Fig. 1. Markowitz Model

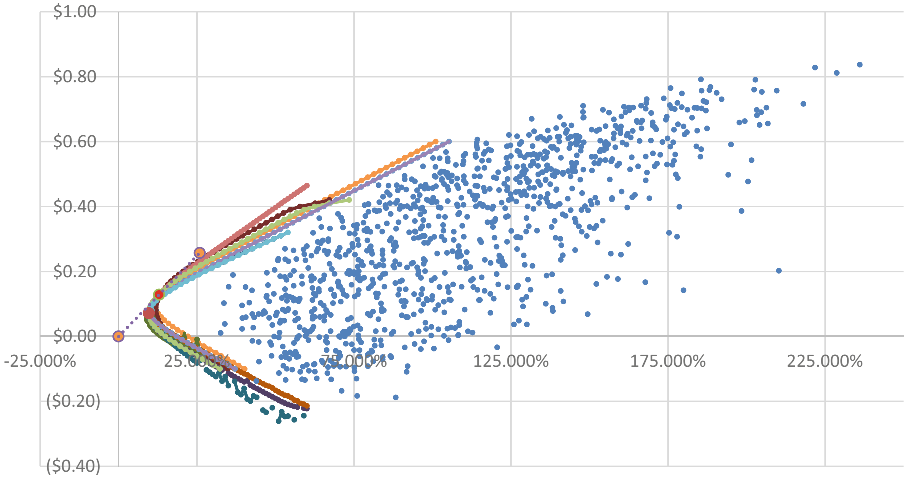

Fig. 2. Index Model

Table 5. Sharpe Ratio for Minimum Variance

	Markowitz Model	Index Model
Constraint 1	0.7625	0.77
Constraint 2	1.016	0.948
Constraint 3	0.685	0.742
Constraint 4	0.868	0.908
Constraint 5	0.779	0.935

Table 6. Sharpe Ratio for Maximum Sharpe

	Markowitz Model	Index Model
Constraint 1	0.77	0.805
Constraint 2	1.022	0.95
Constraint 3	1.031	0.996
Constraint 4	0.325	0.914
Constraint 5	0.954	1

can get the efficient frontier and inefficient frontier sequentially. For constraint 5 that is designed to test the positive and negative effect after including the broad index into our portfolio, the SPX return should be set to 0 first, and then set the return equals to dummy variable and run the solver table from –0.3 to 0.5. Then, choose maximum we can get efficient frontier, choose minimum we can get inefficient one (Fig. 2).

4 Discussion

Using different constraints, the data of minimum Variance point and maximum Sharpe point will also be different. Additionally, by comparing the return for the minimum Variance point of the Markowitz Model and the return for the minimum Variance point of the Index Model, people can know the riskless model under various constraints. According to this chart, the most obvious observation is that the return of the Markowitz model which equals 20.18% is far greater than the return of the Index Model which equals 13.51%, which means that under constraint one, the Markowitz model is a better and more optimal model to use to form the minimum variance portfolio. And under constraint five, the return of the Index Model which equals 10% is moderately greater than the one of the Markowitz Model which equals 8.706%, which means the Index Model is a better model under this circumstance. However, there are no other prominent hints under the other four constraints. Most time, both models have the same or similar returns.

It is also useful and crucial to compare the Markowitz Model's and Index Model's data of return for the Maximum Sharpe portfolio. People can get the optimal return with a high probability after doing this. In this chart, we can know the Maximum Sharpe portfolio's overall performance is greater than the Minimum Variance portfolio for these ten companies, especially for the constraint 3 and the constraint 5 which have increased 5 to 9%. If investigate the Maximum Sharpe portfolio individually, the Markowitz Model will be the better model to be used under constraint 1 (20.18% > 19.72%), constraint 3 (16.988% > 12.868%), and constraint 5 (13.058% > 10%), while other two constraints will be indifferent with the Markowitz and Index Model.

Besides analyzing the return, the Sharpe Ratio is also a crucial determinant to see if the model works well. Normally, given the risk, a greater Sharpe ratio suggests superior investment success. A Sharpe ratio of less than one is regarded as poor. According to chart 4, the data of the Index Model is generally slightly greater than Markowitz Model except for constraint 2 which its Sharpe Ratio of Markowitz Model is greater than the Sharpe Ratio of Index Model. And the value in this Markowitz Model is greater than one and equals 1.016. Thus, Index Model will be a greater model generally.

In Table 5, Markowitz Model's Sharpe ratio for the Maximum Sharpe portfolio reaches 1 for constraint 2 and constraint 3, which means that under the same risk, Markowitz Model will have a higher return. However, the Index Model will be the better choice for the rest three constraints which have a higher Sharpe ratio (Table 6).

5 Conclusion

According to the above analysis, Markowitz Model and Index Model are two useful models. We took ten big companies that exist in real-life to prove that they can be used practically and get the optimal portfolio after calculations. However, under different circumstances, the results may differ. Investors who are risk-averse can invest in optimal portfolio stocks established by a single index model. After all, the level of risk is lower, whereas investors who are risk-takers can invest in optimal portfolio stocks formed by the Markowitz model because the amount of risk is higher. Future researchers are also expected to apply a variety of analytical techniques that incorporate the concept of risk, as well as a lengthier study period with a daily closing price set at 4 pm as usual.

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