Analysis and Comparison of Capital Asset Pricing Model and Arbitrage Pricing Theory Model

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Abstract. Establishing an optimal investment risk structure and maximizing returns while bearing less losses have long been an important goal of investors under certain market risk levels. In 1965, William Sharpe developed the capital asset price model on the cornerstone of Markowitz's mean-variance model, which is an idealized description of looking for portfolio loans according to expected returns and standard deviations. Solved the expected rate of return on a single asset. Its simple logic and intuitiveness make it easier to measure the relationship between risk and expected return. Subsequently, Stephen Ross published an article entitled "The Arbitrage Theory of Capital Asset Pricing" in the Journal of Economic Theory in 1976. He combined the factor model with the no-arbitrage condition to obtain a linear relationship between the expected return and the systematic risk brought by various macroeconomic variables, that is, the arbitrage pricing theory. Both CAPM and APT are the core theories discussed in modern portfolio theory in the dynamically developing capital market, both express the relationship between expected return and risk, and focus on how to reasonably price risk. At the same time, both models have certain drawbacks. This article will gradually analyze the CAPM model and APT, compare the differences between the two, and summarize their advantages and limitations.

Keywords: Capital asset price model, arbitrage pricing theory, multi-factor model, comparison, mathematical finance

1 Introduction

1.1 Introduction to CAPM

William Sharpe (1965) developed the capital asset price model based on the mean-variance model proposed by Markowitz (1952) in Portfolio Selection, which is an idealized description of an asset portfolio that expresses the relationship between expected return and risk in a linear relationship. The model inherits the assumptions of portfolio theory: the securities market is efficient, which means the information is completely symmetric; investors are free to borrow or lend capital at the risk-free interest rate; the total investment risk is represented by the variance or standard deviation, and the sys-

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tematic risk can be represented by the beta coefficient. In addition, investors are required to be rational and to make investment decisions based on the Markowitz portfolio model; the securities market is friction-less, even without taxes and transaction costs.[1] Other than that, there are also implicit assumptions: the distribution of returns on each security is normally distributed; investors can hold any part of a security in a portfolio. The expression of the CAPM model is:

$$E(r_i) = r_f + \beta_{im}[E(r_m) - r_f] \quad (1)$$

$E(r_i)$ represents the expected return of the stock; r_f is the risk-free interest rate of the market; $E(r_m)$ is the expected market return of the market portfolio m; β_{im} is the systematic risk degree of the asset i in the market m. To represent the level of systematic risk associated with this asset, the formula $\beta_i = Cov(r_i, r_m)/Var(r_m)$ can be applied, where β_i is the ratio of the covariance of the i-th market portfolio to the variance of the portfolio return.[2] Then $E(r_m) - r_f$ is the market risk premium, The above formula expresses a straightforward linear connection, which also characterises the CAPM model's ability to streamline a laborious pricing procedure. It is clear that the asset portfolio's return is divided into two components: the risk-free interest rate and the risk-compensation of risky assets. Unsystematic risk is enough to be diversified by a portfolio with many different assets. Therefore, in this case, the beta coefficient deserves attention since it may show us the size of the systematic risk of a particular asset in comparison to the average risk of the market portfolio. So, investors usually need to calculate the β coefficient to help classify different assets to make investment choices: when $\beta < 1$, it means that the systematic risk of the asset is less than the average risk of the market portfolio. When $\beta > 1$, it means that the systematic risk of the asset is greater than the average risk of the market portfolio. When $\beta = 1$, the asset's systematic risk is equal to the portfolio's average risk, its return is equal to the market return, or it will experience the same loss as the market. The vast majority of assets have a beta coefficient larger than zero, however, the possibility that some assets may have a negative beta coefficient cannot be completely ruled out, which would mean that their return would be contrary to the market's average return.[3] According to the research, as the beta coefficient increases, the degree of compensation required due to non-diversifiable risk is also higher. As a result, high risk must be accompanied by high returns.[4]

CAPM equation can be seen as a relation between assets' risk premia and their betas, i.e., security market line

$E(r_i) - r_f = [E(r_m) - r_f]$The risk premium of an asset is proportional to its beta, i.e., proportional to its covariance with the market (security market line, SML):

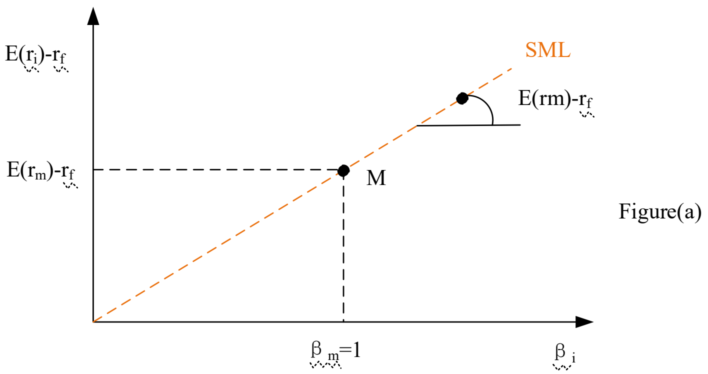

Fig. 1. Security market line

The ordinate of the coordinate axis is the expected return, and the abscissa is the systematic risk measure β. The SML is a straight line with the risk-free rate as the intercept and the market risk premium as the slope. The expected rate of return at point M of the market portfolio is $\beta_m = 1$. When the beta value is high, the expected rate of return from investing in the security is higher; when the beta value is low, the expected rate of return from investing in the security is lower. In effect, the security market line shows that an investor's return is proportional to the risk that the investor faces. When the market is in equilibrium, the risks and returns of any asset or investment portfolio correspond to the securities market line, which means that its actual market price is equal to the theoretical market equilibrium price. However, in the event of market disequilibrium, the points outside the security market line represent the anticipated return-risk combinations.[5]

1.2 Introduction to APT

Stephen Ross published an article entitled "The Arbitrage Theory of Capital Asset Pricing" in the Journal of Economic Theory in 1976. To achieve a linear relationship between the expected return and the systematic risk brought on by various macroeconomic factors, he combined the factor model with the no-arbitrage condition, resulting in the arbitrage pricing theory.[6] The existence of APT is based on three assumptions: the return of any security can be described by a factor model; there are enough securities in the market to diversify risks; the capital market is in a state of equilibrium, and there is no arbitrage opportunity.

APT assumes that a security's rate of return is affected by an unknown number of unknown factors, which are independent of each other. The purpose is to identify these factors and identify the sensitivity of security returns to changes in these factors. According to the number of factors, it can be divided into one-factor model and multi-factor model.

The expression for the one-factor model is:

$$r_i = E(r_i) + \beta_i F + e_i \tag{2}$$

$E(r_i)$ represents the expected rate of return of asset i; F represents the deviation of the public factor from its expected value; β_i is the sensitivity of the security to this factor, which is the degree of influence of factor F on the rate of return of asset i; e_i represents A company-specific disturbance term that is unpredictable, it is a random variable with an expected value of 0. Therefore, a theoretical model of one-factor arbitrage pricing can be built, that is, a portfolio p with a fully diversified unsystematic disturbance term ei consisting of n stocks, and its return rate can be expressed as:

$$r_p = E(r_p) + \beta_p F \tag{3}$$

If F is the market factor m, which is measured by the market risk premium, then when there is no systematic risk, that is, F=0, the diversified portfolio return should be equal to the risk-free return:

$$r_p = r_f + \beta_p [E(r_m) - r_f] \tag{4}$$

By inference, it is clear that the expression is the same as the capital asset pricing model when the market is the only factor. Analyzing APT in an one-factor model can more clearly observe the relationship between arbitrage and equilibrium, and facilitate the direct comparison of APT and CAPM.

However, in real economic situations, there are often more than one factor that affects expected returns. Therefore, it will be more realistic and explanatory to analyze the returns of securities using the multi-factor arbitrage pricing theory. By analysing two-factor model:

$$r_i = E(r_i) + \beta_{i1} F_1 + \beta_{i2} F_2 + e_i \tag{5}$$

In the two-factor model, F1 and F2 are the two factors that affect the security return, respectively, and βi1 and βi2 represent the sensitivity to the individual factors of security i.

In addition, economic variables such as exchange rate changes, interest rate fluctuations, term premiums, price factors, and industry production growth rates can all become risk factors. From this, a multi-factor model with multiple sources of risk can be derived:

$$r_i = E(r_i) + \beta_{i1} F_1 + \beta_{i2} F_2 + \cdots + \beta_{in} F_n + e_i \tag{6}$$

According to the derivation method of the single-factor arbitrage pricing theoretical model, the multi-factor arbitrage pricing theoretical model can be obtianed as following:

$$r_p = r_f + \beta_{i1} [E(r_{m1}) - r_f] + \beta_{i2} [E(r_{m2}) - r_f] + \cdots + \beta_{in} [E(r_{mn}) - r_f] \tag{7}$$

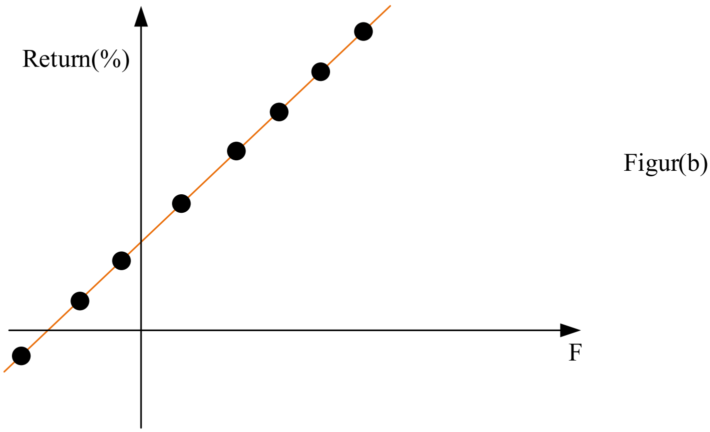

Fig. 2. Return versus systematic risk

The formula for return versus systematic risk for a well-diversified portfolio can be represented by the straight line in Figure b. Referring to the assumption made at the beginning, ATP assumes that there are no arbitrage opportunities in the market, because when there are arbitrage opportunities, every investor in the market seizes the opportunity to have as many positions as possible to obtain risk-free returns. As more arbitrageurs enter the market, this limitless investment size will vanish until there is no longer a return differential between the two portfolios, at which point the arbitrageur's trading space will also vanish. Eventually, the price of the security will reach equilibrium. As the result, in the market equilibrium, the returns of all fully dispersed portfolios are completely determined by systematic factors, and they will all be on the same line (security market line) in Figure b, otherwise there will be arbitrage opportunities.

2 The connection between the CAPM model and the APT model

CAPM and APT are the core theories discussed in modern portfolio theory in the dynamically developing capital market, both express the relationship between expected return and risk, and focus on how to reasonably price risk. Both of them could be applied to capital budgeting, investment performance analysis, and securities valuation. Systematic risk and unsystematic risk are the two forms of risk that CAPM and APT identify when it comes to risk classification.[7]

The CAPM model just requires that one risk factor be taken into account. It is a special case of APT, which is also called the one-factor model. Since market portfolios are used to calculate security returns, the only variables that might have an impact on security returns are market risk and macroeconomic conditions. The expected return of a particular security or portfolio is therefore dependent on the beta coefficient. It can be said that APT, as a multi-factor model, has a wider scope of application and stronger practicability.[8] If only one risk factor condition is involved, its specific situation is consistent with the CAPM model.

3 Comparing CAPM model and APT model

Although both CAPM and APT show the relationship between expected return and risk in a linear form, they essentially have different modeling thinking angles. Markowitz's mean-variance model is the foundation of the CAPM, which is the outcome of market equilibrium under mean-variance preference. It focuses on maximizing returns on the basis of controlling risks, or avoiding risks to the greatest extent on the basis of controlling returns. In general, CAPM examines how assets are valued when all investors make comparable investments and the market eventually adjusts to an equilibrium with static characteristics. APT is based on the theory of no-arbitrage equilibrium, relying on a multi-factor model, deriving returns from the process of generating stock returns, and using the concept of arbitrage to describe the formation of equilibrium, which is a dynamic process. In order to generate risk-free profits, investors create positions as large as possible through the arbitrage portfolio when there is an arbitrage opportunity in the market. As this situation continues to evolve, the supply and demand among securities change accordingly. The APT model focuses on how the asset is valued when there is no risk-free arbitrage in the final market and it achieves equilibrium.[9] From non-equilibrium to equilibrium, from the existence of arbitrage opportunities to the process of no-arbitrage equilibrium, CAPM depends on a large number of investors to make small adjustments to their positions. By contrast, APT theoretically only requires one arbitrageur to maintain the market without arbitrage state because it is a risk-free arbitrage opportunity.

The assumptions of the CAPM model and the APT model are also different. Compared to the APT model, CAPM's assumptions are very strict, which results in it being limited to a "single investment period". CAPM requires a portfolio based on an efficient market to complete the analysis and ignores taxes and transaction costs, assuming that the market is frictionless. Moreover, in terms of restrictions on investors, all investors are required to be risk-averse, and to have the same view of the security evaluation and economic situation, which is called the consensus expectation assumption. Based on the above assumptions, the APT model does not have these constraints and does not clearly specify investors' risk appetite, and does not require investors to plan and implement investment strategies within a single investment horizon. Additionally, CAPM stresses that market portfolios must be an effective portfolio, but APT does not especially emphases the importance of market portfolios. APT does not analyze a single investment period, and there is no tax problem. Investors can freely borrow and lend funds at risk-free interest rates, which is more realistic.[10] Although the CAPM model's strong fundamental assumptions make the mathematical formulation of the model easier to understand, these criteria cannot yet be satisfied, even on the assumption that the securities market is getting increasingly more developed. In contrast, the assumptions and conditions of the arbitrage pricing model are less restrictive, but the mathematical expression has a very significant complexity and is more comprehensive and adaptable.

In terms of risk interpretation, the CAPM model describes security risk by relying solely on a security's beta coefficient in relation to the market portfolio. Although this will inform investors of the size of the risk, it will not pay attention to where the risk comes from. As a single index model, CAPM disregards the influence of factors beyond the market, believing that only complete market forces influence the return of stocks. In contrast to the APT model, it acknowledges that security returns are linearly related to a group of indices that indicate various factors (such as market factors, inflation, industry factors, interest rate changes, etc.) [11] that affect stock returns. The APT model thereby broadens investors' horizons of thought since it gives them an analytical instrument for locating the origin of security risk as well as the ability to assess risk at various levels.

4 Conclusion

Through the above analysis and comparison of CAPM model and APT model, it is clear that both models have some shortcomings. However, both theoretically and practically play an irreplaceable role in considering the "reasonableness" of different securities prices. The biggest advantage of CAPM is its simplicity and clarity, so it is more convenient to use and is widely used in the calculation of various asset prices (such as human capital pricing, insurance rate calculation, securities market and real estate investment, etc.). However, due to a series of strict assumptions in the model, and when considering systematic risk, only the market portfolio is concerned. This is an example of the actual economy being simplified and the existence theory being abstracted, but it is also lack of comprehensiveness. Therefore, while choosing stocks, investors should consider both the macroeconomic climate and the company's own development in addition to the beta coefficient. But CAPM models are still useful when studying the impact of the overall economy on individual stocks. As opposed to this, the multi-factor APT model explains the risk of securities using a variety of factors, and the beta coefficients of securities returns to various macro factors vary, which is more accurate. As a result, APT may be seen as a specific instance of CAPM, while CAPM can be seen as a supplement to and modification of APT. However, the APT model also has certain drawbacks: it cannot clearly point out the relevant risk factors and risk premium; and in the calculation process, with the continuous increase of the number of risks, the profit analysis of arbitrage gradually becomes complicated, and the difficulty of related operations increase as well. Both models, in a word, embody the core ideas of contemporary financial theory. The CAPM model and the APT model for venture capital offer only limited and profound insights to investors doing venture capital operations.

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