Analysis of Volatility Derivatives

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Abstract. It has become popular to trade in volatility derivatives because of its direct exposure to volatility. The research topic of this thesis is the qualitative analysis and comparison of the existing valuation models of volatility derivatives. The research is based on existing relatively new academic research on volatility pricing models. Through research, it is concluded that the models and their extensions proposed by some scholars in recent years greatly reduce the error between the theoretical value and the market value, making the valuation of volatility derivatives more accurate. These models include the MJD model with the jumping process, the Hurston model and GARCH model with random fluctuations, and the generalized LOU model with time-varying center trend and random fluctuation.

Keywords: VIX, Volatility Trading, Volatility Derivatives, Pricing Model.

1 Introduction

The core of financial research is how to get the trade-off between the risk and return. People have proposed many measures to operate the risk part. Since the middle of the last century, the standard deviation of asset returns has been the most commonly used risk measurement standard. Volatility is a famous abbreviation of this standard deviation. In the past few centuries, the development and expansion of financial markets have led to great changes in the role of volatility. It is not only a part of pricing theory, but also has developed into an asset class. Since the mid-1990s, there has been a subset of derivative securities that have strengthened the regulation of volatility, which is called volatility derivatives. According to Carr and Lee, the return of volatility derivatives largely depends on the volatility of underlying assets [1]. They have upgraded the role of volatility from the traditional nonlinear yield pricing to a more important role in determining the yield of derivative securities. Prominent examples of these derivatives include variance futures and swaps, or VIX options. This thesis has a strong correlation with previous academic research and is a summary based on previous research. The research method of this thesis is to refer to many existing relatively new academic studies on volatility pricing models, list their results, and summarize the results based on these conclusions. This thesis will study on volatility derivatives, including the background information, volatility trading, volatility derivatives pricing and modelling. The

H. Mallick et al. (Eds.): ICEMCI 2022, AEBMR 231, pp. 1379–1384, 2023.
https://doi.org/10.2991/978-94-6463-098-5_156

aim is to evaluate whether the volatility derivatives' valuation is correctly understood by using the existing models. Based on previous research results, this paper presents several popular pricing models of volatility derivatives for the academic community in a more systematic and comprehensive way. Through the comparison between the models, people will have a clearer understanding of the advantages and disadvantages of different models.

2 Volatility Trading

A volatility derivative is a derivative security. Its return profile is determined by the volatility of the underlying asset. Examples are variance and volatility swaps and VIX futures and options. In the past few years, there have been more varieties of volatility derivatives. Today, corridor variance swaps, gamma swaps and conditional variance swaps are several other instruments that can be observed compared with standard swaps. In the late 20th century, volatility derivatives began to appear. The first contract to see the light of day around this time was variance swaps, traded by Michael Weber at UBS, the Swiss investment bank, in 1993. Variance swaps were illiquid in their early years but increased dramatically after 1998 and have been a reliable trading tool for millennia. Thanks to the development of "replication theory", which uses ordinary option combinations to properly replicate variance swaps, market volatility has become greater [1]. While these volatility derivatives have risen in popularity since their inception and are now regarded as a suitable asset class and a tradable vehicle, the decline has been there. Extreme levels of volatility nearly killed the volatility swaps market during the 2008 financial crisis [2]. Unforeseeable extremes made it impossible for traders to reliably hedge against volatility. During this time, liquidity became so scarce that deals in this niche area have dwindled dramatically.

The volatility index (VIX) is a volatility symbol. Launched in 1993, the VIX Index has always been a popular indicator of implied volatility, measuring 30 day volatility expectations. It began to replicate the one month implied volatility of the S&P 100 index and expanded to measure the market's expectations of volatility conveyed by the S&P 500 stock index in 2003. Since its launch, the VIX Index has been regarded as the benchmark for global stock market fluctuations. The interest and development of volatility products and the hype of volatility derivatives is based on the launch of the index. To some extent, its popularity is due to the negative correlation between the change of VIX Index and the change of stock price. That investors trade options on the S&P 500 index to buy protection in the event of market fluctuations, thereby increasing the value of VIX is the most known explanation. As Andersen and Bondarenko have shown, the VIX Index almost consistently exceeds the realized volatility, because on average, investors are willing to pay a considerable premium to obtain the risk of future stock index fluctuations [3]. For this reason, some critics believe that it is the fear index of the market, although a high value does not necessarily mean negative returns in the future [4].

3 Volatility Derivatives

Volatility derivatives trading has become very popular in risk management because it is directly exposed to volatility. Therefore, with these kinds of financial derivatives, volatility is a tradable market tool today [5].

3.1 The Reasons Why People Trade in Volatility Over Regular Financial Derivatives

Volatility has several attractive characteristics. People can hedge the investment risk of S&P 500 index by using VIX derivative positions. Szado found that this strategy did provide meaningful protection, especially during the economic recession [6]. In addition, investors can obtain the risk of S&P 500 index fluctuation by holding VIX derivatives, without delta hedging their S&P 500 index option position with the stock index. Therefore, it is often cheaper to do VIX call options outside the long price than S&P 500 put options outside the short price. Due to this possibility, VIX option is the only asset with the highest open interest in out-of-price call options [7].

3.2 The Way They Can Be Used by Hedgers and Speculators

- **Hedging Position.** Hedging is an investment. Its process is to reduce the risk of adverse price changes of assets through offsetting investment. There are many hedging strategies. It is a strategy that we can try to long a security and then short the same or similar securities. As part of their portfolio, there are a variety of industries that trade with volatility. Volatility is usually negatively correlated with the level of stocks or indexes, which makes volatility hedging a good strategy. Volatility is known to increase during market crashes, a phenomenon known as Black leverage [8]. It makes hedging in volatility during these periods a possible means that can reduce losses.
- **Speculative Position.** In the financial field, speculation has the risk of significant value loss and the opportunity to obtain significant gains. Through speculation, large gains will offset risks or bring other compensations. Investors take directional positions on the volatility of the underlying asset. If people believe that the current level of volatility is incorrect, then volatility derivatives can be used to make profits. The political or financial turmoil caused by the current debt problem, or the change of belief due to the upcoming elections, are good examples of this kind of situation.

4 Valuation of Volatility Derivatives (Pricing and Modelling)

4.1 Black-Scholes Model

Can a unique price for each given option be determined by the market, and is this price clearly computed? A valid answer to this question is the famous Black-Scholes model.

$$C = SN(d_1) - Ke^{-rt}N(d_2) \quad (1)$$

where

$$d_1 = \frac{\ln\frac{S}{K} + \left(r + \frac{\sigma_v^2}{2}t\right)}{\sigma_S\sqrt{t}} \quad (2)$$

and

$$d_2 = d_1 - \sigma_S\sqrt{t} \quad (3)$$

Where:
C = Call option price
S = Current stock (or other underlying) price
K = Strike price
r = Risk-free interest rate
t = Time to maturity
N = A normal distribution

It has been criticized, nonetheless, due to its drawbacks and dangers. The model itself is straightforward, and the closest thing to the real world is given. The model may not always support the market empirically, and some of the assumptions and simplifications of reality may not be entirely correct. Some of the B-S model's assumptions, however, could be relaxed by performing some extensions [5]. These assumptions include those on efficient markets, liquidity, and the absence of transaction costs.

4.2 Corresponding Extensions

Merton Jump Diffusion Model (MJD). It is debatable, according to Marklund and Karlsson, if the continuity of stock returns is intended to behave in a well-continuous manner governed by the geometric Brownian motion [5]. Stock returns typically exhibit jumps, which indicates that they cover a brief period and are susceptible to significant fluctuations. The Merton Jump Diffusion model is a well-known version of the original model that includes a jump process to address this problem (MJD).

Heston Model and the GARCH Model. One assumption is: there is a constant volatility (σ). To relax this assumption, there is often a shift to stochastic volatility models, that is, volatility is no longer assumed to be constant, but follows a stochastic process given by some dynamic principles. Such models Examples considered to provide better empirical support are Heston model and GARCH.

Lévy Process. Another assumption of great significance in the B-S model is the Gaussian log increment of stock prices. Studies of stock prices have pointed out that due to empirical evidence of heavy tails, the usage of Gaussian models is incompatible, suggesting that it may be more reasonable to use a more general family that eliminates false assumptions to replace Brownian motion, for example, a Lévy process [9].

4.3 SQR, LOU Process and the Extension

So far, the two most obvious mean reversion models proposed for VIX are the square root process (SQR)[10] and the lognormal Ornstein Uhlenbeck process (LOU)[11]. Some authors have studied the empirical performance of these VIX derivatives pricing models. By first estimating the parameters of SQR model from historical VIX data, Zhang and Zhu studied the empirical validity of this model, and then evaluated the VIX futures pricing error implied in these estimates [12]. VIX futures data were also used to evaluate the benefits of adding jumps to SQR diffusion [13]. In addition, they estimated a GBM process. Wang and Daigler used the option data written on VIX to compare the empirical fitting of SQR and GBM models. They also found evidence supporting the GBM hypothesis [14]. The findings confirm that, despite the popularity of SQR in the empirical literature, the Lou model fits better, especially during the crisis[15]. However, both models produced significant price distortions during the crisis, and neither appears to capture the level/slope of the term structure of futures prices or the slope of volatility. The attribute difference of VIX Index under actual indicators and risk neutral indicators means that there is an economically critical systemic risk being priced in the VIX derivatives market.

5 Conclusion

The review results show that none of the existing valuation models of volatility derivatives can accurately determine the value of volatility derivatives, and there will be errors between theoretical estimates and market prices. However, the expansion of the previous model proposed by some scholars in recent years has greatly reduced this error and made the valuation of volatility derivatives more accurate, which are the MJD model with jumping processes, the Hurston model and GARCH model with random volatility, and the generalized LOU model with time-varying central trend and random volatility. This thesis only conducts qualitative analysis, comparison, and summary of several popular valuation models, and does not involve quantitative research. The above model will be expanded and optimized in the future, and constructive suggestions will be put forward. The discrepancies between actual and theoretical derivative prices will be analyzed to validate the models. By combining futures and options data, it will also be possible to evaluate which of those additional features is more relevant for pricing futures and which is more vital for options.

Acknowledgement

First of all, I want to thank my teachers for giving me valuable guidance on my thesis. Secondly, gratitude will be given to my parents and friends. Besides, I express my special thanks to my boyfriend. Without his support and guidance, attempts at any level cannot be satisfactorily completed.

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