# Research on Stochastic Differential Game of Multi-Party Cooperation in Informatization Construction of Rural Enterprise

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**ABSTRACT**

Aiming at the multi-party coordination problem in Informatization Construction of Rural Enterprise, combined with the market-oriented characteristics of rural enterprise, a stochastic differential game model is constructed. The results show that: under decentralized decision-making, there is a threshold for the relationship between government effort and the coefficient of action on managers; under centralized decision-making, the efforts of both the government and Staffs are greater than those under decentralized decision-making, while managers need certain conditions to strengthen its expectations of a shared vision; When meeting the threshold for managers to work harder, the overall optimal benefit under centralized decision-making is far greater than the sum of the benefits of all parties under decentralized decision-making, resulting in a strong synergistic gain effect.

**Keywords:** *Informatization Construction of Rural Enterprise; multi-party coordination; stochastic differential game*

## 1. INTRODUCTION

With the rapid development of science and technology in our country, we have entered the information age. The informatization of enterprises has become a trend, and rural enterprises are no exception. It is an important support and guarantee for rural productivity, promoting agricultural development, peasant prosperity, and rural prosperity." Staffs bring land into cooperatives and develop cooperative management, which can reduce the conflict of interests caused by the separation of contract rights and management rights [12]. However, in the actual operation process, due to the relative separation of management rights and ownership caused by the dispersion of equity [7], the difficulty in measuring the degree of effort in the actual operation process, and the lack of a strict supervision and restraint mechanism [3] and other reasons, the managers of collective economic organizations are prone to produce opportunistic behavior. At the same time, rural enterprises have market-oriented characteristics, which cannot be separated from the macro-control and guidance of the government, establishing asset supervision and management platforms, improving market transaction rules, and improving operating mechanisms. Therefore, the informatization of rural enterprises is inseparable from the joint efforts of farmers, managers and governments. Thus, exploring the sustainability of tripartite synergy is of great significance to the success of informatization, to stimulate the vitality of rural enterprises, and to help rural revitalization.

In the related research on Informatization Construction of Rural Enterprises. Yue (2022), through empirical evidence, found that enterprise informatization has a positive effect on the quality of enterprise development. Wan (2022) [8] uses the literature analysis method to design a framework model that can accurately evaluate the effectiveness of enterprise informatization strategic planning, improve the level of enterprise informatization, and strengthen the market competitiveness of enterprises. Wan (2022) [9] found that there is a positive correlation between the application of enterprise blockchain and the efficiency of the internal capital market. Furthermore, Relevant studies have shown that enterprise informatization can also help drive value chain improvement [6], improve enterprise capacity utilization [11], optimize enterprise production business

processes [4], and improve managers' strategic decision-making capabilities [1], etc. Although the above studies have carried out detailed research on many aspects of enterprise informatization, there is no research on rural enterprise informatization.

However, there are few studies on the informatization of rural enterprises, For example: Li (2020) studied the impact of informatization on leisure agriculture and found that informatization is an important source of power for the innovation performance of leisure agricultural enterprise, and informatization has a positive role in promoting the growth of leisure agricultural output performance; Wu (2022) [10] found that the dilemma of rural e-commerce development under the digital economy can be solved through informatization construction.

Referring to the abundant research results, few scholars have studied Informatization Construction of Rural enterprise from the perspective of the collaborative efforts of the participants and the co-creation of the economy, and no scholars have applied differential game theory and synergy theory to the research of rural collective property rights reform. In view of this, this paper builds a tripartite stochastic differential game model of members, managers and governments of collective economic organizations based on the perspective of the collaborative relationship of the subjects, to analyze the optimal strategy of the three-party effort level under the dynamic framework was explored.

## 2. MODELLING

Assumption 1: The benefit of rural enterprise:

$\Pi(t) = \Theta(t)Q - C$, $C$ is expenses in the Information construction. $\Theta(t)$ is the degree of enterprise informatization, $Q$ is the enterprise total assets. Income is distributed in proportion to equity. The shareholding ratio of Staffs is $\alpha$, the manager is $1 - \alpha$.

Assumption 2: The degree of enterprise informatization $\Theta(t)$ evolves over the entire time period according to the following dynamic equation:

$$\begin{cases} d\Theta(t) = \begin{bmatrix} \gamma_X X(t) + Y(t) + \\ \gamma_Z Z(t) - \delta\Theta(t) \end{bmatrix} dt + \sigma(\Theta(t))dz(t) \\ \Theta(0) = \Theta_0 \geqslant 0 \end{cases} \quad (1)$$

$X(t)$, $Y(t)$, $Z(t)$ are the efforts of Staffs, managers, and the government. According to the principal-agent theory, manager is the main influencers on the efficiency of asset operation and his influence coefficient on asset operation efficiency is one. $\gamma_X$, $\gamma_Z$ are the influence coefficient of the efforts of Staffs and the government on the operation efficiency of collective assets. $\delta$ is external risk coefficient, which means the influence coefficient of external risk on asset operation efficiency. $\sigma(\Theta(t))$ is the

random interference coefficient, $z(t)$ is the standard Wiener process.

Assumption 3: There is a certain interaction between the effort levels of the subjects, $Y(t) = k_1 X(t) + k_2 Z(t) + k_3(1 - \alpha)$, $k_i$ is the interaction coefficient, that is, the influence coefficient of Staffs, government, and shares on managers, $X(t) = k_4 Z(t)$, $k_4$ is the coefficient of influence of effort for the government on the effort of Staffs. $k_i$, $X(t), Y(t), Z(t) \epsilon [0,1]$, $i = 1, 2, 3, 4$.

Assumption 4: $\pi_X$, $\pi_Y$, respectively, are the basic income of Staffs and managers, mainly referring to wage income, employment opportunities and wages of Staffs, etc., affected by the development of economic cooperatives, and managers' wages are set by contracts in advance.

Assumption 5: The costs of the three parties are $\frac{c_X X(t)^2}{2}$, $\frac{c_Y Y(t)^2}{2}$ and $\frac{c_Z Z(t)^2}{2}$, respectively. $c_j$ are the cost coefficients, that is, the influence coefficient of effort on the cost, $j = X、Y、Z$. The government supervises managers and formulates a dynamic reward and punishment system. The reward is $Y(t)\bar{l}$, and the penalty is $(1 - Y(t))\bar{D}$. $\bar{l}、\bar{D}$ is, respectively, the upper limit of the reward and punishment.

In the infinite time zone, based on the principle of individual rationality, the objective benefit function of each party is:

$$\max_X \left\{ J_X(\Theta_0) = \int_0^\infty e^{-\rho t} \left[ \alpha\Pi(t) - \frac{X^2(t)c_X}{2} + \pi_X(1 + Y(t)) \right] dt \right. \quad (2)$$

$$\max_Y \left\{ J_Y(\Theta_0) = \int_0^\infty e^{-\rho t} \begin{bmatrix} (1 - \alpha)\Pi(t) - \frac{Y^2(t)c_Y}{2} + \\ \pi_Y + Y(t)\bar{l} - (1 - Y(t))\bar{D} \end{bmatrix} dt \right. \quad (3)$$

$$\max_Z \left\{ J_Z(\Theta_0) = \int_0^\infty e^{-\rho t} \begin{bmatrix} \beta\Pi(t) + (1 - Y(t))\bar{D} + \\ Z(t)\pi_Z - Y(t)\bar{l} - \frac{Z^2(t)c_Z}{2} \end{bmatrix} dt \right. \quad (4)$$

Formulas (1)-(4) define a tripartite stochastic differential game with three control variables, $X(t), Y(t)$, $Z(t)$ and a state variable $\Theta(t) \geqslant 0$. Since the control effect of the feedback control strategy is more ideal than that of the open-loop control in economic analysis [5]. Therefore, this paper will adopt the model established by the feedback control strategy research. The time t will be omitted below without causing confusion.

# 3. MODEL SOLVING

## 3.1 Decentralized decision-making

Under the premise of individual rationality, Staffs, managers and the government do not consider the sustainability of cooperation and pursue the maximization of personal interests. Aiming at the optimal control problems of the three, the Hamilton-Jacoby-Bellman (HJB) equations for the optimal returns of the three are constructed respectively [2]:

$$\rho V_X(\Theta) = \max \left\{ \begin{array}{c} \alpha\Theta Q - \alpha C - \dfrac{c_X X^2}{2} + \pi_X(1+Y) + \\ V_X{}'(\Theta)\left(\gamma_X X + Y + \gamma_Z Z - \delta\Theta\right) + V_X{}''(\Theta)\dfrac{\sigma^2(\Theta)}{2} \end{array} \right\} \quad (5)$$

$$\rho V_Y(\Theta) = \max \left\{ \begin{array}{c} (1-\alpha)(\Theta Q - C) - \dfrac{Y^2 c_Y}{2} + \pi_Y + \\ Y\bar{l} - (1-Y)\bar{D} + V_Y{}''(\Theta)\dfrac{\sigma^2(\Theta)}{2} \\ V_Y{}'(\Theta)\left(\gamma_X X + Y + \gamma_Z Z - \delta\Theta\right) \end{array} \right\} \quad (6)$$

$$\rho V_Z(\Theta) = \max \left\{ \begin{array}{c} \beta(\Theta Q - C) + (1-Y)\bar{D} + Z\pi_Z \\ -Y\bar{l} - \dfrac{Z^2 c_Z}{2} + V_Z{}''(\Theta)\dfrac{\sigma^2(\Theta)}{2} \\ + V_Z{}'(\Theta)\left(Y + \gamma_X X + \gamma_Z Z - \delta\Theta\right) \end{array} \right\} \quad (7)$$

$V'(\Theta), V''(\Theta)$ respectively represent the first derivative and the second derivative of $V(\Theta)$.

$$\left\{ \begin{array}{l} V_X^* = \left\{ \begin{array}{c} \dfrac{\alpha Q}{\rho+\delta}\Theta + \dfrac{\pi_X - \alpha C}{\rho} + \dfrac{\alpha Q\left(\gamma_X X^* + \gamma_Z Z^*\right)}{\rho(\rho+\delta)} \\ + \dfrac{\left[\pi_X(\rho+\delta) + \alpha Q\right]Y^*}{\rho(\rho+\delta)} - \dfrac{c_X\left(X^*\right)^2}{2\rho} \end{array} \right\} \\[3em] V_Y^* = \left\{ \begin{array}{c} \dfrac{(1-\alpha)Q}{\rho+\delta}\Theta + \dfrac{(1-\alpha)Q\left(\gamma_X X^* + \gamma_Z Z^*\right)}{\rho(\rho+\delta)} \\ + \dfrac{\left[\left(\bar{l}+\bar{D}\right)(\rho+\delta) + (1-\alpha)Q\right]Y^*}{\rho(\rho+\delta)} + \\ \dfrac{\pi_Y - \bar{D} - (1-\alpha)C}{\rho} - \dfrac{c_Y\left(Y^*\right)^2}{2\rho} \end{array} \right\} \\[4em] V_Z^* = \left\{ \begin{array}{c} \dfrac{\beta Q}{\rho+\delta}\Theta + \dfrac{\beta Q\left(\gamma_X X^* + \gamma_Z Z^* + Y^*\right)}{\rho(\rho+\delta)} + \\ \dfrac{\left[\pi_Z Z^* - \left(\bar{l}+\bar{D}\right)Y^*\right]}{\rho} + \dfrac{\bar{D} - \beta C}{\rho} - \dfrac{c_Z\left(Z^*\right)^2}{2\rho} \end{array} \right\} \end{array} \right. \quad (8)$$

From the above, the following conclusions can be drawn:

Proposition 1: In the case of decentralized decision-making, the best effort level of Staffs is:

$$X^* = \frac{k_1\pi_X(\rho+\delta) + \alpha Q\left(\gamma_X + k_1\right)}{c_X(\rho+\delta)}$$

Proposition 2: Under decentralized decision-making, the optimal effort level of the managers of economic cooperatives is:

$$Y^* = \frac{\left(\bar{l}+\bar{D}\right)(\rho+\delta) + (1-\alpha)Q}{c_Y(\rho+\delta)} \quad .$$

Proposition 3: In the case of decentralized decision-making, the optimal level of government efforts to carry out macro-control is:

$$Z^* = \frac{\pi_Z + k_2\left(\bar{l}+\bar{D}\right)}{c_Z} + \frac{\beta Q\left(k_2 + k_4\gamma_X + \gamma_Z\right)}{c_Z(\rho+\delta)} \quad .$$

The coefficient of action depends on the situation. The interaction coefficients $k_2$ are proved as follows: $\frac{\partial Z^*}{\partial k_2} = \frac{\beta Q}{c_Z(\rho+\delta)} - \frac{\left(\bar{l}+\bar{D}\right)}{c_Z}$, when $\beta Q > \left(\bar{l}+\bar{D}\right)(\rho+\delta)$, $\frac{\partial Z^*}{\partial k_2} > 0$, the level of government effort increases with, and vice versa.

## 3.2 Centralized decision-making situation

Proposition 4: The overall optimal benefit under centralized decision-making is:

$$V_T^* = \left\{ \begin{array}{c} \dfrac{\gamma_X(1+\beta)QX_T^* + \left[\pi_X(\rho+\delta) + (1+\beta)Q\right]Y_T^*}{\rho(\rho+\delta)} + \\ \dfrac{\left[\pi_Z(\rho+\delta) + \gamma_Z(1+\beta)Q\right]Z_T^*}{\rho(\rho+\delta)} + \dfrac{(1+\beta)Q}{\rho+\delta}\Theta \\ + \dfrac{\pi_X + \pi_Y - (1+\beta)C}{\rho} - \dfrac{c_X X_T^* + c_Y Y_T^* + c_Z Z_T^*}{2\rho} \end{array} \right\} \quad (9)$$

Proposition 5: The optimal effort level of Staffs, managers, and the government is:

$$\left\{ \begin{array}{l} X_T^* = \dfrac{k_1\pi_X(\rho+\delta) + (1+\beta)Q\left(\gamma_X + k_1\right)}{c_X(\rho+\delta)} \\[1.5em] Y_T^* = \dfrac{\pi_X(\rho+\delta) + (1+\beta)Q}{c_Y(\rho+\delta)} \\[1.5em] Z_T^* = \dfrac{\left(k_2\pi_X + \pi_Z\right)(\rho+\delta) + (1+\beta)Q\left(k_4\gamma_X + k_2 + \gamma_Z\right)}{c_Z(\rho+\delta)} \end{array} \right. \quad (10)$$

Putting formula (20) into formula (1), we get:

$$d\Theta(t) = \left(\Omega_T - \delta\Theta(t)\right)dt - \sigma(\Theta(t))dz(t), \Theta(0) = \Theta_0 \geq 0 \quad (11)$$

Proposition 6: The expectation, variance and stability of collective asset operating efficiency under centralized decision-making are:

$$\left\{ \begin{array}{l} E\left(\Theta_T(t)\right) = \dfrac{\Omega_T}{\delta} + \left(\Theta_0 - \dfrac{\Omega_T}{\delta}\right)e^{-\delta t} \\[1.2em] \lim\limits_{t \to \infty} E\left(\Theta_T(t)\right) = \dfrac{\Omega_T}{\delta} \\[1.2em] D\left(\Theta_T(t)\right) = \dfrac{\sigma^2\left[\begin{array}{c}\Omega_T - 2\left(\Omega_T - \delta\Theta_0\right)e^{-\delta t} \\ + \left(\Omega_T - 2\delta\Theta_0\right)e^{-2\delta t}\end{array}\right]}{2\delta^2} \\[2em] \lim\limits_{t \to \infty} D\left(\Theta_T(t)\right) = \dfrac{\sigma^2 \Omega_T}{2\delta^2} \end{array} \right. \quad (12)$$

Next, numerical simulation is used to explore the relationship between collective asset operating efficiency and its expected value. Let the parameters be: $\Theta_0 = 3$, $\Omega_T = 4$, $\delta = 0.01$, $\sigma = 0.7$, $t \in [0, 2]$. Using the numerical approximation analysis method of Zwillinger (1997) [13] to discretize the formula (12), we get

$$\Theta(t+\Delta t) = \Theta(t) + \left(\Omega_T - \delta\Theta(t)\right)\Delta t + \sigma\sqrt{\Theta(t)}\sqrt{\Delta t}\,\zeta(t) \quad (13)$$

$\zeta(t)$ is an independent and identically distributed standard normal distribution variable, and the step size is $\Delta t = 0.01$. From Proposition 6 and Equation (14), the simulation results can be obtained in Figure 1:
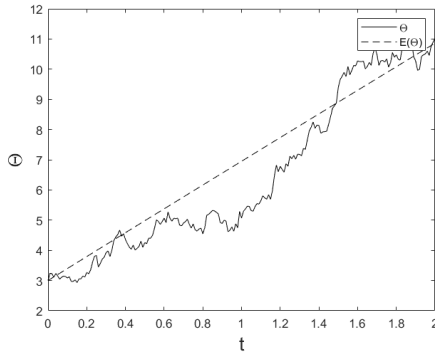


Figure 1: Asset operating efficiency and its expectations over time

It can be seen from Figure 1 that under the influence of external uncertain factors, the asset operation efficiency of economic cooperatives fluctuates around the expected value, and does not stay at the expected operation efficiency.

### 3.3 Comparative analysis

Proposition 6: under decentralized decision-making and centralized decision-making, the best effort levels of the three parties are compared as follows:

$$\begin{cases} X_T^* > X^*, Z_T^* > Z \\ Y_T^* > Y^*, if\ (\alpha + \beta)Q > (\bar{l} + \bar{D} - \pi_X)(\rho + \delta) \end{cases} \quad (14)$$

Proposition 7: Given the operating efficiency $\Theta$ of any asset, the comparison of the optimal returns under the two decision-making modes is:

$$V_T^* > V_X^* + V_Y^* + V_Z^* \quad (15)$$

$$s.t: (\alpha + \beta)Q + \frac{c_Y(\pi_X + \pi_Y)(\gamma + \delta)^2}{\pi_X(\rho + \delta) + (1 + \beta)Q} > \bar{l} + \bar{D} - \pi_X.$$

It can be seen from Proposition 7 that if, the condition that the overall optimal benefit of Staffs, managers, and the government under centralized decision-making is greater than the total benefit of the three parties under decentralized decision-making is weaker than the condition that managers work harder. That is to say, the synergistic gain effect produced by the concerted efforts of the three parties is significant, and the overall benefit is bound to be greater than the benefit generated by the goal of maximizing personal interests.

### 4. CONCLUSIONS

Based on the market characteristics of economic cooperatives, this paper constructs a stochastic differential model for the cooperative governance of the government, managers, and Staffs, and promotes the informatization construction of rural enterprise. The efficiency and stability of asset operation are compared and analyzed. Simultaneously, under decentralized decision-making, the influencing factors of individual effort level are analyzed, and the influencing factors of individual effort level under centralized decision-making are compared and analyzed. The following conclusions are drawn:

(1) Under decentralized decision-making, the efficiency influence coefficient and the proportion of equity held have a positive effect on the enthusiasm of Staffs, and the shares held, government rewards and punishment caps have a positive effect on promoting managers' efforts, and indirect benefits, the efficiency influence coefficient, etc. have a positive effect on the government's willingness to participate, and the government's effect coefficient on managers and their efforts depend on the situation; (2) Under centralized decision-making, the efforts of the government and Staffs are greater than the corresponding values under decentralized decision-making, and collaboration based on common goals generates strong endogenous power. The manager depends on the situation, in order to make the manager continue to work hard, he must increase his expectations for the development of the informatization construction of rural enterprise; (3) Under the condition that managers work harder: the overall optimal benefit under centralized decision-making is far greater than the sum of the optimal benefits of all parties under decentralized decision-making, resulting in a strong synergistic gain effect. This shows that under the action of collective rationality and common vision, it is not only conducive to the sustainability of cooperation and the maximization of overall interests, but also to the development of informatization construction of rural enterprise.

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