A Bi-Level Decision Model with the Hybrid Strategy for Vendor-Managed Inventory in Supply Chain Network

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Abstract

In inventory optimization and supply chain coordination, one challenge for vendors and retailers is how to keep the inventory level as low as possible to cut cost. Vendor-managed inventory (VMI), as an efficient coordinated system, has aroused great concern in both research and commercial communities. Considering the VMI problem in single-vendor multi-retailer supply chains, this paper discusses a hybrid inventory management strategy combing vertical replenishment and lateral transhipment to reduce the inventory cost while increase the profits of retailers. To coordinate the optimization problem between the vendor and retailers, a bi-level inventory management decision-making model is developed to describe the relationship between vendor and multi-retailers, and the associated mathematical programming is proposed to determine the order price for vendor and inventory for retailers simultaneously. Based on hierarchical particle swarm optimization and fuzzy evaluation theory, an improved algorithm is then introduced to solve our mathematical programming problem and get Pareto front consisting of optimal solutions in a shorter time. These numerical results show that the bi-level decision-making model developed has some practical implications for the supply chain coordination, and the strategies with lateral transhipment and vertical replenishment are feasible and work better in inventory management.

Keywords: *Inventory management, Lateral transhipment, Vertical replenishment, Supply chain coordination.*

1 INTRODUCTION

With the increasing goods variety demand of consumers, vendors and retailers are facing new challenges from inventory management and supply chain coordination, such as quick delivery, convenient return, instant exchange, and so on. Thus, an efficient supply chain system has become more and more important in the fierce business competition. Typically, inventory management is considered a key issue in any efficient supply chain system, since inventory cost often accounts for a large proportion of companies' total costs [1].

The vendor-managed inventory (VMI) has already attracted much attention from researchers, due to its better performance in reducing inventory and eliminating the bullwhip effect in supply chain management [5]. Roughly speaking, the VMI is divided into two types:

centralized and decentralized. For the centralized VMI, the vendor manages all operating players. Yet, the decentralized format gives each retailer the power to make its own decision based on the local conditions and other retailers' decisions [3]. Obviously, the decentralized format is more suitable for flexible market demand than the centralized format. So, researchers always focus on how to manage the vertical replenishment coordination between vendors and retailers in the decentralized VMI [6]. Along with the deepening of the research, some studies [2] [4] find that the cost from the vertical replenishment coordination can be partly reduced by the hybrid coordination of integrating vertical replenishment and lateral transhipment.

In vertical replenishment coordination, the vendor is responsible for the replenishment of seven distributors and accepting their returns. In the hybrid coordination of

vertical replenishment and lateral transhipment, however, one distributor, who has excess inventory, can transfer goods to another distributor, who is out of stock, except for returning the vendor. If the vertical replenishment strategy is referred to as the single-line type, the hybrid strategy is the multiple-line type. In contrast to the former, the latter saves the return cost from the distributor with excess stock. Moreover, the distributor with short stock avoids shortage penalty and saves the replenishment transportation cost from the vendor.

Therefore, lateral transhipment is an efficient way to improve inventory management with the help of sharing horizontal information between retailers. This paper aims to design a decentralized VMI system with both vertical replenishment and lateral transhipment by information sharing and allowing transhipments from one retailer to another. The contributions of this paper are threefold.

1) A bi-level decision-making model is developed for the corresponding decentralized VMI programming problem, which allows us to examine how the decision-makers coordinate with each other in the VMI system.

2) To solve the proposed programming problem, we introduce an improved hybrid heuristic algorithm combining hierarchical particle swarm optimization and fuzzy evaluation theory.

3) The performance of the whole supply chain for one vender and several retailers is improved considering the hybrid strategy from the comparison with numerical studies.

2 PROBLEM DESCRIPTION AND MATHEMATICAL MODEL

Considering a single-vendor multi-retailer supply chain, this section first describes the VMI problem, and then develops a bi-level decision model with lateral transhipment and vertical replenishment.

2.1 Problem description

Retailers face dynamic demand at different periods. At the beginning of each period, retailers offer their orders to the vendor based on historical sale data. The vendor is responsible for product supply and distribution. To meet the product orders for retailers, the vendor has its warehouse. To provide efficient service for customers, retailers can adopt lateral transhipment from adjacent retailers when their warehouses are out of stock. At the end of each period, retailers return surplus products to the vendor, if any. The entire process is depicted in Figure 1.
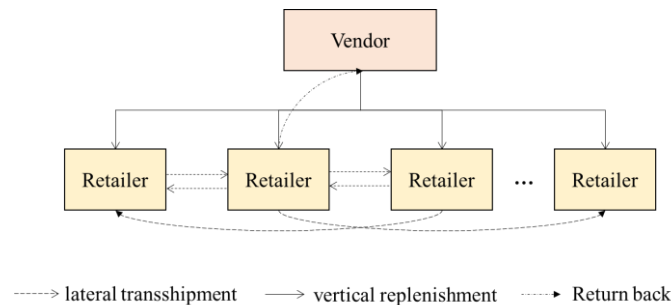

Figure 1: The VMI for single-vendor multi-retailer

In this VMI system, the vendor determines its inventory level to minimize its total costs. Whereas, retailers pursue to maximize profits by the choice of their inventory levels. For the vendor, the costs include the products' transportation and inventory cost and the returning cost from retailers. For each retailer, the revenue comes from three parts --- sale of the products to customers, lateral transhipment to adjacent retailers, and the products returned to the vendor; the costs include the cost of products purchased from the vendor, the penalty cost of product shortage, the transportation cost of lateral transhipment to adjacent retailers, the order cost of lateral transhipment from adjacent retailers, and the inventory cost as well. Different from centralized inventory control, the decision variable controlled by the vendor is the order price of per product to retailers, and the decision variables determined by retailers are the order size of shipments by the vendor and the lateral transhipment between them. Thus, the VMI system here can be regarded as a bi-level

decision-making problem, where the vendor is the leader and retailers are followers. Expecting the following reaction of retailers, the vendor sets the order price at different stages. Retailers make the best favourable decision to respond to the price set by the vendor.

In order to simplify the problem, several assumptions are given.

(1) The inventory capacity of the vendor and retailers is limited.

(2) The order demand for retailers is known at the beginning of each stage.

(3) The retailer receives and sends transhipment to the vendor or other retailers at the beginning of each stage, and the excessive products are returned to the vendor at the end of the stage.

(4) The transhipments time between the vendor and one retailer, or one retailer and another retailer is omitted.

(5) The lateral transhipments occurs only when the inventory level of the retailers exceeds the customer demand.

(6) The order price for all retailers is the same at the same stage.

2.2 Bi-level decision-making model

The bi-level decision-making model is formulated and the description of parameters and variables is shown in Appendix.

$$\min f^{(1)} = \sum_{t=1}^{s} C_{vt} = \sum_{t=1}^{s}\sum_{i=1}^{n} c\_v \times I_{it} + \sum_{t=1}^{s}\sum_{i=1}^{n} c\_tc_i \times I_{it} \quad (1)$$
$$+ \sum_{t=1}^{T}\sum_{i=1}^{n} c\_rc_t \times \max\{(I_{it} + \sum_{j=1,j\neq i}^{n} T_{jit} - D_{it} - \sum_{j=1,j\neq i}^{n} T_{ijt}),0\}$$

s. t.

$$I_{max} \geq \sum_{i=1}^{n} I_{it} \quad (1.1)$$

$$\max f_i^{(2)} = \sum_{t=1}^{s} P_{it} = \sum_{t=1}^{s} D_{it} \times ps_{it} + \sum_{t=1}^{s} c\_lt_t \times (\sum_{j=1,j\neq i}^{n} T_{ijt}) \quad (1.2)$$
$$+ \sum_{t=1}^{s} c\_rc_t \times \max\{(I_{it} + \sum_{j=1,j\neq i}^{n} T_{jit} - D_{it} - \sum_{j=1,j\neq i}^{n} T_{ijt}),0\}$$
$$- \sum_{t=1}^{s} (I_{it} + \sum_{j=1,j\neq i}^{n} T_{jit} - \sum_{j=1,j\neq i}^{n} T_{ijt}) \times c\_r_i - \sum_{t=1}^{s} I_{it} \times c_t - \sum_{t=1}^{s} (c\_lt_t \times \sum_{j=1,j\neq i}^{n} T_{jit})$$
$$- \sum_{t=1}^{s} c\_pc_t \times \max\{0, d_{it} - D_{it}\} - c\_l_{ij} \times \sum_{j=1,j\neq i}^{n} T_{ijt}$$

s. t.

$$I_{it} + \sum_{j=1,j\neq i}^{n} T_{jit} = \sum_{j=1,j\neq i}^{n} T_{ijt} + D_{it} + \max\{0, I_{it} + \sum_{j=1,j\neq i}^{n} T_{jit} - \sum_{j=1,j\neq i}^{n} T_{ijt} - D_{it}\} \quad (1.2a)$$

$$l_t \leq {D_{it}}\big/{d_{it}} \leq 1 \quad (1.2b)$$

$$I_{it} + \sum_{j=1,j\neq i}^{n} T_{jit} - \sum_{j=1,j\neq i}^{n} T_{ijt} \leq I_i \quad (1.2c)$$

$$\sum_{j=1,j\neq i}^{n} T_{ijt} \begin{cases} =0, & d_{it} \geq I_{it} \\ >0, & d_{it} < I_{it} \end{cases} \quad (1.2d)$$

$$T_{ijt} \times T_{jit} = 0, \quad \forall i,j,i\neq j \quad (1.2e)$$

$$c\_rc_t = \alpha \times c_t \quad (1.2f)$$

$$c\_pc_t = \beta \times c_t \quad (1.2g)$$

$$c\_lt_t = \gamma \times c_t \quad (1.2h)$$

$$ps_{it} = \varepsilon \times c_t \quad (1.2i)$$

$$I_{it}, T_{ijt}, T_{jit}, D_{it}, d_{it}, \quad \forall i,j,i\neq j \quad non-negative\ integer \quad (1.2j)$$

The vendor's objective function (1) means the cost needed by the vendor, including the cost of inventory, transportation, and returns from retailers. The constraint (1.1) implies that all products sold to retailers are not more than the vendor's maximum inventory capacity. The retailers' objective function (1.2), as a constraint of the vendor, means the profit earned by retailer R_i. The constraint (1.2a) implies that the number of products owned by the retailer, including the products provided by the vendor and the products from the lateral

transhipments of other retailers, is equal to the number of products sold to customers, lateral transhipments to other retailers, and returned to the vendor. The constraint (1.2b) represents the lower and upper limits of service level for retailers. The constraint (1.2c) implies that the products from the vendor and other retailers are less than the retailers' maximum inventory. The constraints (1.2d) and (1.2e) guarantee lateral transhipments. The constraint (1.2d) ensures that the lateral transhipments occurs only when the inventory level exceeds the customer demand. The constraint (1.2e) ensures that the lateral transhipments from retailer R_i to R_j and the lateral transhipments from retailer R_j to R_i cannot happen at the same stage. Constraints (1.2f), (1.2g), (1.2h), and (1.2i) represent the value of return cost, penalty cost, lateral transhipments cost, and sale price, respectively. The constraint (1.2j) is the non-negative constraint to I_{it}, D_{it}, T_{ijt}, T_{jit}, and d_{it}.

In the bi-level decision-making system, like the Stackelberg model, members make their decisions in sequence --- from the top-level vendor to the lower-level retailers. The decision made by the vendor aims at minimizing the cost. The decision results will affect the objectives of retailers. In view of different decisions made by the top-level vendor, the lower-level retailers will adopt different reactions. Thus, although the vendor has priority in making decision, it has to take into account the following reaction of retailers at making their own inventory decision. All in all, the bi-level decision-making model is a dynamic interaction process among members until equilibrium is achieved.

3 SOLUTION APPROACH

To solve the bi-level decision-making problem, we introduce an improved heuristic algorithm named the hierarchical particle swarm optimization (HPSO). Specifically, the solution of the problem is transformed into solving the lower and upper optimization problem, given upper and lower decision variables. That is, two standard particle swarm optimization algorithms are applied to solve the upper and lower problem, respectively. The non-dominated sorting idea is then used to address the multi-objectives for upper and lower decision-makers, and the bi-level programming problem is solved based on the interaction game between lower and upper decision-makers. Finally, for the Pareto solutions obtained by HPSO, the fuzzy evaluation theory is employed to judge the satisfaction degree of decision-makers for different solutions, and choose the better solution for upper and lower decision-makers, consequently.

The HPSO, as an iterative and nested optimization algorithm, is designed based on game theory. Since the decision-maker makes current strategies based on historical strategies, its PSO is applied to simulate the strategy selection process due to the better function

approximation ability, which can be seen as a group optimization algorithm. During the solution process, the two optimization problems involved in bi-level programming are solved iteratively. On the one hand, the optimal solution for the upper problem is solved based on the solutions obtained in the lower problem. On the other hand, the optimal solution for the lower problem is solved based on the solutions obtained in the upper problem, and the solutions obtained in the lower problem can be viewed as the input of the upper problem. Like this, the optimal solution for the upper and lower problems is generated with the iteration of the population. Just as shown in Figure 2, the architecture of HPSO is described.
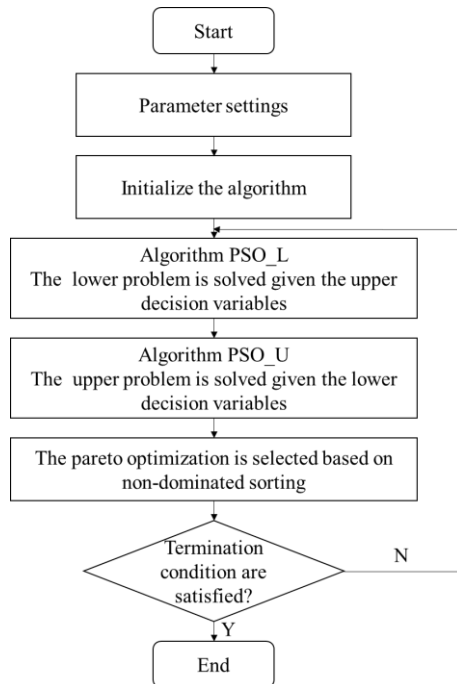

Figure 2: The architecture of HPSO

Step 1 Parameter settings. In this step, the parameters applied in HPSO are determined first, including inertia weight w, population size pop, acceleration factor c_1 and c_2, the maximum number of iterations max_iter, and so on.

Step 2 Initialize the algorithm. The initial population for PSO_L (PSO for the lower problem) is generated randomly based on the value range of decision variables for the upper problem. Besides, the initialization process for PSO_L is also performed.

Step 3 Optimization procedure. In this step, two standard PSO, including PSO_L and PSO_U (PSO for the upper problem), are conducted to solve the upper problem and lower problem respectively with the solutions sharing during the process.

Step 4 The end. The HPSO is stopped when the termination conditions are satisfied. Otherwise, goes to Step 3.

4　COMPUTATIONAL STUDY

To test the effectiveness and performance of the proposed bi-level decision-making model and improved algorithms, we make use of them to solve a numerical example. Besides, the numerical result with vertical replenishment strategy only is also conducted to compare it with the hybrid strategy. All numerical experiments are done on a computer equipped with Intel Core i7-1065G7 processors running at 1.3 G Hz and with up to 32 GB of RAM.

4.1　Case study

Considering a VMI case, there are a vendor and 6 retailers, and the whole process consists of four stages. The values of input parameters for the vendor and retailers are shown in Table 1.

Table 1 The input parameters for the vendor and retailers

Parameter	Value	Parameter	Value
s	4	γ	1.3
I_{max}	582	ε	1.5
I_i	100	α	0.9
I_t	0.95	β	1.2
cv	0.4\$	d_{it}	$d_{it} \in [90, 100]$

The developed HPSO is applied to solve the problem. The Pareto fronts are obtained with 3,397.20 s of search time. Meanwhile, according to the fuzzy evaluation theory and the membership function, we get the satisfaction of the vendor and six retailers with these Pareto solutions, and two solutions are shown in Figure 3 and Figure 4.

As for these Pareto results, firstly, we consider a situation where the inventory supply is equal to the total customer demands. In other words, the inventory delivered to retailers just meets their customer demand, and the partial shortage faced by a single retailer can exactly be modified by lateral transhipments from other retailers.

As for Solution in Figure 3, although the total inventory supplied by the vendor also meets all customers' demands, the retailers are faced with a mismatch between supply and demand. At the moment, lateral transhipments occur. By comparisons, it is found that the total cost of the vendor decreases with the number of stocks transferred laterally. The cost-saving lies in avoiding the cost of the return, since each retailer can adjust the inventory to customer demand by lateral transhipments. For retailers, the total profit is reduced with T_{ij}, because they need to bear additional transhipments costs and transportation costs. The other aspect of the comparative

analysis is based on the changes in the vendor's order price or the retailer's ordering cost.

Furthermore, when the vendor's supply exceeds the customer demand, as shown in Figure 4, the vertical replenishment, lateral transhipment and return to the vendor occur in the whole process. Compared with the situation where supply equals or less than demand, the total cost of the vendor increases, because of the increase in inventory cost, transportation cost, and return cost. For retailers, the total profit is higher compared to the situation of insufficient supply.
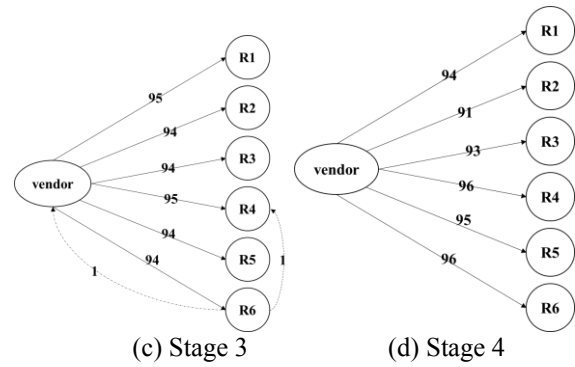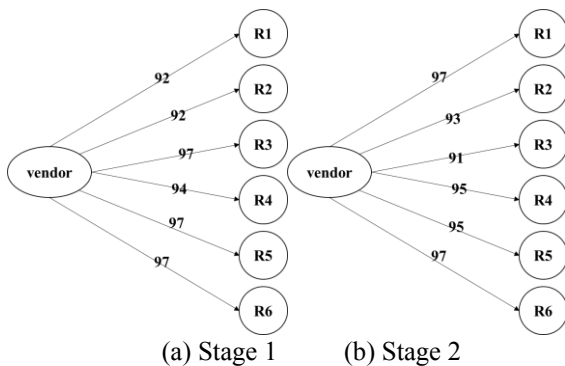

(c) Stage 3   (d) Stage 4

Figure 4: The flowchart for one solution

5   CONCLUSIONS

Facing fierce market competition, successful inventory management in supply chains depends on efficient and effective delivery of products to customers. This makes the adoption of decentralized VIM necessary to decrease costs, match supply with demand and improve customer service level. Therefore, this paper addresses a decentralized VMI problem with vertical replenishment and lateral transhipments, since lateral transhipments has become more and more prominent with the development of IT and information technology. To describe the VMI problem, we develop a bi-level decision-making programming model where one vendor supplies to multi-retailer. Based on fuzzy evaluation theory, an improved hierarchical particle swarm optimization is proposed to seek the optimal solutions on the Pareto front. Then, several numerical results are simulated to illustrate the model and its possible solutions, and from numerical results, some insights and implications are derived. Finally, it can be concluded that the inventory management considering a hybrid strategy of lateral transhipments and vertical replenishment works better in minimizing the supply chain cost and bringing more financial benefits to all players in the whole VMI system.

Besides, our model can be extended in several directions. A multiple inventory coordination scheme (say a three-echelon VMI system) may be investigated in a more hierarchical supply chain structure. Moreover, one may opt to deviate from the underlying assumption and consider a situation involving multi-vendor and multi-retailer, which is closer to the real world.

(a) Stage 1   (b) Stage 2

Figure 3: The flowchart for one solution

(a) Stage 1   (b) Stage 2

APPENDIX

Table A. Parameters and Variables in the model

Parameters	Description
i, j	Indexes of retailers
t	Index of stages
n	Number of retailers

s	Number of stages	c_lt_t	The lateral transshipment cost of per product, γc_t, charged by retailers at stage t
I_{max}	The maximum inventory of the vendor		
I_i	The maximum inventory of retailer R_i	D_{it}	The number of products from retailer R_i to customer or the customer demands actually met by the retailer R_i at stage t
c_v	Inventory cost of per product for the vendor		
c_r_i	Inventory cost of per product for retailer R_i		
c_tc_i	The transportation cost from the vendor to retailer R_i	R_{it}	The number of products returned to the vendor for retailer R_i
c_l_{ij}	The transportation cost of lateral transshipment from R_i to R_j	T_{ijt}	The number of products for lateral transshipment from retailer R_i to R_j at stage t
l_t	The minimum service level of retailers at stage t		
d_{it}	The customer demand for products faced by retailer R_i at stage t	ps_{it}	The sale price of per product for retailer R_i at stage t
α	The proportion of return cost to order price	C_{vt}	The total cost of the vendor at stage t
β	The proportion of penalty cost to order price	P_{it}	The profit of retailer R_i at stage t
γ	The proportion of lateral transshipment cost to order price		
ε	The proportion of sale price to order price		
c_t	The order price of per product charged by the vendor to retailers at stage t		
I_{it}	The order quantity of retailer i or the inventory level kept by vendor that will be distributed to the retailer i at stage t		
c_rc_t	The return cost of per product or the order price of per product returned by the vendor at a discount rate of α at stage t		
c_pc_t	The penalty cost of per product, βc_t, that a retailer paid for its shortage at stage t		

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