Dynamic Penalty Evolutionary Game and Computer Simulation of Cooperative Promotion in Supply Chain

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Abstract. "Management dilemma" is frequent in supply chain cooperation promotion. By constructing an evolutionary game model with the manufacturer and the retailer as the game players, the above-mentioned problem is dynamically analyzed, and the evolution path is simulated by MATLAB. The results show that no stable strategy is adopted in the evolutionary system and cooperative promotion is in a circular state when the manufacturer adopts static punishment. When the manufacturer adopts the dynamic penalty regulation policy, the evolutionary system stabilizes with the mixed strategy. At the same time, the manufacturer's stronger punishment can improve the retailer's choice probability of promotion strategy, but the promotion subsidy cannot improve the cooperative promotion.

Keywords: supply chain cooperation, retailer promotion, evolutionary game

1 Introduction

As the retailers downstream in a supply chain are closer to the market terminal, in order to solve the problem of the "last mile" of product sales, encouraging retailers to carry out promotion is the main solution. Cooperative promotion in the supply chain refers to the management process in which the manufacturer, as the upstream of the supply chain, shares the retailer's promotion cost or gives the retailer a promotion subsidy to encourage the retailer to promote [1]. There is information asymmetry in the supply chain cooperative promotion between the manufacturer and the retailer. Whether the retailer is willing to cooperate with the manufacturer is influenced by many factors. In order to maximize the interests, the supply chain cooperative promotion faces a "management dilemma," namely: a circle of "no subsidies – no promotion – subsidy – promotion - no subsidy". The analysis of the factors affecting supply chain promotion cooperation and its mechanism is important for improving the efficiency of the manufacturer's supply chain promotion supervision and the performance of product sales. This has been a continuously concerned by the industry and scholars.

There has been plenty of research on cooperative promotion in the supply chain. This paper briefly reviews the supply chain organization forms. The Stackelberg game is widely used in the research methods of supply chain cooperative promotion. First,

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scholars have carried out in-depth research on the impact of pricing and promotion forms in a simple supply chain (composed of a manufacturer and a retailer) on supply chain decision-making [2-5]. Evolutionary game theory has been used in the research of retailers' s promotion opportunism [6]. With the deepening of research, scholars begin to pay attention to the retailer's promotion and multi-channel problems in complex channel mode (one manufacturer and multiple retailers), the competition relationship between embedded retailers, and the influence of online and offline promotion behaviors on supply chain decision-making [7-10]. At present, scholars pay attention to multi-channel and feedback channels, etc., on which variables such as efforts on carbon emissions, equity concern, and goodwill are embedded to further expand the research, with the research method of the Stackelberg game frequently used [11-14].

To sum up, progress has been made in supply chain promotion cooperation, but there are great differences in the research conclusions due to different research scenarios, which weakens the guiding significance for practical management. In this paper, an asymmetric evolutionary game model is constructed in which the manufacturer and the retailer are the two sides of the game, to study the mixed stable strategy of the manufacturer to realize the promotion game through dynamic punishment and supervision measures, so as to improve the management dilemma in the supply chain cooperation promotion, and to discuss the strategy to improve the cooperation efficiency of the supply chain from the perspective of the manufacturer.

2 Evolutionary Game Model of Cooperative Promotion between Manufacturers and Retailers

2.1 Game Description and Basic Assumptions

This paper studies the dynamic evolutionary game of cooperative promotion between the retailer and manufacturer which are two bounded rational agents. There are two strategies for manufacturers to choose from. One is the subsidy strategy, that is, the manufacturer provides certain subsidies to the retailer's promotional behavior to encourage the retailer to carry out promotional activities; the other is the non-subsidy strategy, that is, the manufacturer does not provide subsidies to the retailer. Retailers have two strategic choices, namely: promotion and no promotion. This means whether the retailer offers discounts or non-price promotions to increase short-term sales. Manufacturers, as upstream companies, supervise retailers' promotional activities, and if they are found to be receiving promotional subsidies and fail to carry out promotions, they will be penalized.

According to the game description, the strategy of the manufacturer and the retailer includes subsidies no subsidies, promotion, and no promotion. The game assumptions are as follows:

(1) Retailers carry out sales promotions to improve product purchase rate and product price, thus obtaining greater benefits. Let M be the revenue of the retailer when he carries out the promotion, and m be the revenue when he does not carry out the promotion, so that $M > m$.

(2) The manufacturer gives a promotion subsidy of *V* to the retailer who carries out the promotion activities, and there is no subsidy if the retailer does not carry out the promotion.

(3) The profit of the manufacturer under the condition that the retailer carries out the promotion is *G*, and the profit without promotion is *g*, so *G*> *g*; the retailer has to pay the cost of *C* to carry out the promotion, and no cost means no promotion.

(4) If the manufacturer conducts necessary market supervision, finding that the retailer has accepted the promotion subsidy but fails to carry out the promotion, the fixed penalty for the retailer will be *F*.

To sum up, the payment matrix of the cooperative promotion game between the manufacturer and the retailer is shown in Table 1:

Table 1. Promotion game payment matrix of manufacturers and retailers[Owner-draw]

	Manufacturers	
Retailers	Subsidies *y*	No subsidies 1-*y*
Promotion *x*	*M+V-C*, *G-V*	*M-C*, G
No Promotion 1-*x*	*m+V-F,g-V+F*	*m,g*

2.2 Solution of Evolutionary Game Model

It is supposed that the proportion of retailers who adopt the "promotion" strategy is *x* (0 ≤x≤ 1), and the proportion of those who adopt the "no promotion" strategy is 1−*x*. It is assumed that the proportion of manufacturers who take the "subsidies" strategy is *y* (0 ≤*y*≤ 1) and the proportion of those who adopt the "no subsidy" strategy is 1-*y*.

For the retailers, let the expected payoffs of the "promotion" and "no promotion" strategies be U_{m1} and U_{m2} with an average expected return of $\overline{U}_m$. The expected benefit of choosing the "promotion" strategy is $U_{m1} = y(M+V-C)+(1-y)(M-C)$, and the expected benefit of choosing the "no promotion" strategy is $U_{m2} = y(m+V-F)+(1-y)\times m$, with the average expected return of $\overline{U}_m = xU_{m1}+(1-x)U_{m2}$.

For the manufacturers, let the expected payoffs of the "subsidies" and "no subsidies" strategies be U_{s1} and U_{s2} with an average expected return of $\overline{U}_S$. The expected payoff from choosing the "subsidies" strategy is $U_{s1} = x(G-V-R)+(1-x)(g-V+F-R)$, and the expected payoff from choosing "no subsidies" is: $U_{s2} = xG+(1-x)g$, with the average expected return is $\overline{U}_s = yU_{s1}+(1-y)U_{s2}$ 。

The replication dynamic equations for constructing the manufacturer and the retailer are:

$$\begin{cases} \frac{dx}{dt} = x(1-x)(yF + \triangle m - C) \\ \frac{dy}{dt} = y(1-y)(F - V - xF) \end{cases} \tag{1}$$

There are five equilibrium points in the autonomous system: A1 (0, 0), A2 (0, 1), A3 (1, 0), A4 (1, 1), D (x*, y*), where: $(x^*,\ y^*) = (\frac{F-V}{F}, \frac{C-\nabla m}{F})$. The formula $\Delta m = M - m$ represents that the retailer gains more profit when he conducts promotion compared with not promoting, which is called promotion gain in this paper.

The Jacobian matrix can be obtained from Equation (1):

$$J = \begin{bmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial y} \\ \frac{\partial \dot{y}}{\partial x} & \frac{\partial \dot{y}}{\partial y} \end{bmatrix} = \begin{bmatrix} (1-2x)(yF + \triangle m - C) & x(1-x)F \\ y(1-y)(-F) & (1-2y)(F - V - xF) \end{bmatrix} \tag{2}$$

Friedman has proposed that the stability of the equilibrium point of a differential equation can be obtained from the local stability analysis of the system. When the determinant (*detJ*) of the matrix is positive and the trace (*trJ*) of the matrix is negative, then the equilibrium point is asymptotically stable, i.e. evolutionarily stable strategy [15]. The Jacobian matrix can be obtained from Equation (9):

The determinant of the Jacobian matrix *J* is

$$|J| = \begin{vmatrix} (1-2x)(yF + \triangle m - C) & x(1-x)F \\ y(1-y)(-F) & (1-2y)(F - V - xF) \end{vmatrix} \tag{3}$$

The trace of the Jacobian matrix *J* is

$$tr(J) = (1-2x)(yF + \triangle m - C) + (1-2y)(F - V - xF) \tag{4}$$

The determinant and trace of the Jacobian matrix *J* are denoted by $Det(J) = a_{11}a_{22} - a_{12}a_{21}, Tr(J) = a_{11} + a_{22}$. According to the calculation and analysis in combination with *J* and *det* values, the local stability of the above equilibrium point can be divided into the following cases, as shown in Table 2:

Table 2. Steady state of equilibrium points[Owner-draw]

Equilibrium points (*x,y*)	*Det* (*J*)	*tr* (*J*)
A1 (0, 0)	$(\nabla m - C) \times (F - V)$	$(\nabla m - C) + (F - V)$
A2 (1, 0)	$(\nabla m - C)V$	$-(\nabla m - C) - V$
A3 (0, 1)	$-(F + \Delta m - C) \times (F - V)$	$V + \Delta m - C$
A4 (1, 1)	$-V(F + \Delta m - C)$	$V - (F + \Delta m - C)$

D (x, y)	$-x^*(1-x^*)(F-Vm) \times y^*(1-y^*)F$	0

3 Equilibrium Point and Stability Analysis

According to the calculated values and size comparison relationship of determinant and trace in Table 2, when $0 \le x \le 1$ and $0 \le y \le 1$ $F-V \ge 0$, the $C-\Delta m \ge 0, C-\Delta m \le F$ stability analysis of 4 local equilibrium points and central point are shown in Table 3, and the phase diagram corresponding to Table 3 is shown in Figure 1.

According to the analysis method of Jacobian matrix local stability, when the equilibrium point satisfies determinant *Det(J)*>0 and trace *Tr(J)*<0, it indicates that the system is in the local asymptotic stable state in the dynamic evolution process, this point is regarded as the local evolution stable strategy (ESS) of the system, and the rest are unstable points.

Table 3. Steady state analysis of equilibrium point under static penaltysupervision strategy [Owner-draw]

Equilibrium points *(x,y)*	Det *(J)* symbol	tr *(J)* symbol	local stability
A1 (0, 0)	-	Not sure	unstable
A2 (1, 0)	-	Not sure	unstable
A3 (0, 1)	-	Not sure	unstable
A4 (1,1)	+	+	unstable
D (x,y)	-	0	unstable

It can be seen from Table 3 that there is a center point and four saddle points in the evolutionary system in which the manufacturers and retailers participate. According to the judgment rule of *ESS*, it is obvious that there is no ESS in the evolution system, and its evolution trajectory is a closed trajectory loop circularly moving around the central point *(x, y)*, as shown in FIG. 1. This situation shows that there is no stable point in the retailer's strategy of participating in cooperative promotion under the static supervision of the manufacturer (only a fixed amount of punishment). The system is in the circle of "no subsidies – no promotion – subsidies – promotion – no subsidies. In practice, the manufacturer subsidizes the retailer's promotion activities, and the retailer is profitable to carry out the promotion activities. The manufacturer observes that after the retailer carries out the promotion activities, to maximize his interests, he no longer subsidizes the retailer, and the retailer without subsidies will lose money when carrying out the promotion and chooses the "no promotion" strategy. As a result, manufacturers begin to subsidize retailers, and supply chain cooperation promotion faces the "management dilemma." The non-existence of a stable state in the evolutionary system reflects the repetitiveness of supply chain cooperative promotion management and the complexity of forming a stable cooperative state of supply chain promotion in the evolutionary system.

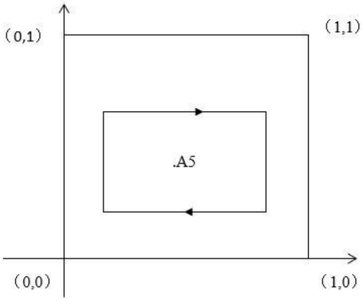

Fig. 1. Evolution phase diagram of the game under the manufacturer's static penalty supervision strategy [Owner-draw]

To reflect the evolution path of the system more intuitively, MATLAB is used to simulate the system. Based on the conditions of satisfying the evolution system $F - V \geq 0$, $C - \Delta m \geq 0, C - \Delta m \leq F$, the initial values of external variables are set as F=8, V=5, C=5, and Δm=3 to simulate the evolutionary system. as shown in Fig. 2.

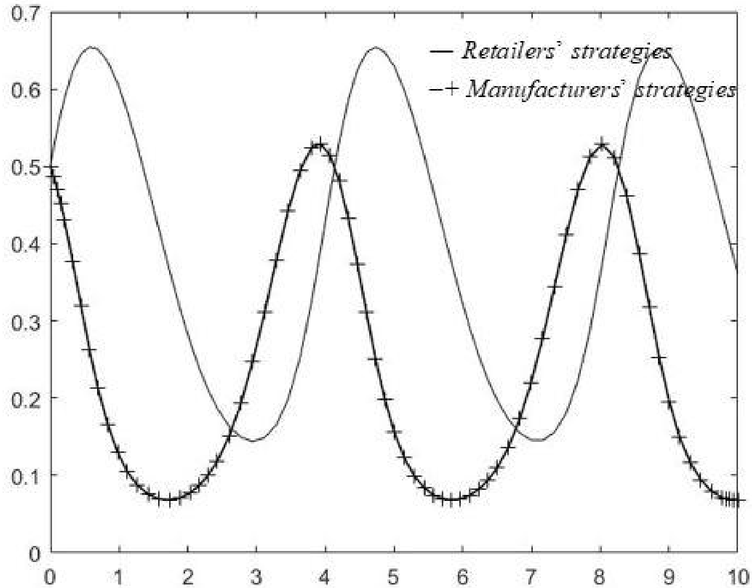

Fig. 2. Evolution simulation diagram of the game under the manufacturer's static penaltysupervision strategy [Owner-draw]

Fig. 2 shows the game evolution trend curve of the system when both the manufacturer and the retailer take *(x, y) = (0.5, 0.5)* as the initial probability, indicating that the game process of the manufacturer and the retailer in the supply chain cooperation promotion shows a periodic behavior mode. As shown in Fig. 2, the strategies of both the manufacturer and the retailer fluctuate repeatedly, which indicates that the promotion strategy selected by the manufacturer and the retailer is always fluctuating. This corresponds to the stability analysis result of an evolutionary game in Table 3, which reflects the repeatability of the supervision process of cooperative promotion of the supply chain of the manufacturer. In practice, it is manifested as the manufacturer's "free rider" strategy and the retailer's s promotion "opportunism" tendency. When the manufacturer is sure that the retailer will carry out the promotion, he will adopt the "free rider" strategy not to subsidize the retailer. In the short run, the manufacturer may gain benefits, but in the long run, it will harm the overall interests of the supply chain. Similarly, once the retailer gets the manufacturer's promotion subsidy, he chooses the "opportunism" strategy of "no promotion" to maxim benefit. In order to make the evolution process of mixed strategies converge effectively, further research on the adjustment scheme is needed.

4 Governance and Simulation of Supply Chain Joint Promotion under Manufacturers' Dynamic Penalty Policy

The reasonable manufacturers' supervision of the retailers' promotion activities will not only affect the manufacturers' supervision enthusiasm but also affect the retailers' supply chain cooperation promotion investment enthusiasm. The calculation and simulation results of the first and second parts prove that there is no evolutionarily stable strategy between the manufacturers and the retailers under the static reward and punishment measures, so it is necessary to study the influence of the dynamic supervision measures of the manufacturers on the behaviors of both parties in the game.

The dynamic punishment measures taken by manufacturers to retailers are to empower the supervision means and improve the efficiency of supply chain cooperation promotion. Suppose that the probability of the retailer's promotion is x, which is proportional to the manufacturer's expectation of cooperative promotion, then $1-x$ can reflect the incomplete rate of the retailer's failure to achieve the manufacturer's cooperative promotion. When the retailer chooses to maintain the status quo and the manufacturer adopts the regulatory strategy, let the manufacturer's penalty to the retailer change from a fixed constant F $F(X)=(1-X)H$ to H>V+C, where H is the manufacturer's maximum penalty that is the sum of the promotion subsidies and the promotion cost as the lower limit.

4.1 Stability Analysis of Dynamic Penalty System

F(x)=(1-x) H is substituted for F in Equation (1) to obtain the system replication dynamic equation (5) under dynamic penalty measures.

$$\begin{cases} \frac{dx}{dt} = x(1-x)(yF(x) + \triangle m - C) \\ \frac{dy}{dt} = y(1-y)(F(x) - V - xF(x)) \end{cases} \quad (5)$$

There are five equilibrium points in the autonomous system: A1* (0, 0), A2* (0, 1), A3* (1, 0), A 4* (1, 1), D* (x**, y**), $(x^{**},\ y^{**}) = (\frac{F(x^{**})-V}{F(x^{**})}, \frac{C-\mathrm{V}m}{F(x^{**})})$.

If $0 < \frac{F(x^{**})-V}{F(x^{**})} < 1$, $0 < \frac{C-\mathrm{V}m}{F(x^{**})} < 1$, the determinant of F(x)=(1-x) H Jacobian matrix J is:

$$|J| = \begin{vmatrix} (1-2x)(yF(x) + \triangle m - C) + x(1-x)yF'(x) & x(1-x)F(x) \\ y(1-y)[(1-x)F'(x) - F(x)] & (1-2y)(F(x) - V - xF(x)) \end{vmatrix} \quad (6)$$

The trace of the Jacobian matrix J is:

$$\begin{aligned} tr(J) = {} & (1-2x)(yF(x) + \triangle m - C) + x(1-x)yF'(x) \\ & +(1-2y)(F(x) - V - xF(x)) \end{aligned} \quad (7)$$

Table 4. Steady state analysis of equilibrium point under dynamic penalty supervision strategy [Owner-draw]

Equilibrium points (x,y)	Det (J) symbol	tr (J) symbol	local stability
A1* (0, 0)	-	Not sure	saddle point
A2* (1, 0)	-	Not sure	saddle point
A3* (0, 1)	-	Not sure	saddle point
A4* (1, 1)	+	+	unstable
D * (x**, y**)	+	-	ESS

4.2 Simulation Analysis of Manufacturer's Dynamic Penalty Measures

Based on the evolutionary system condition satisfying $F - V \geq 0$, $C - \Delta m \geq 0, C - \Delta m \leq F$ and the initial values of external variables are set as F=8, V=5, C=5, and Δm=3 to simulate the evolutionary system.

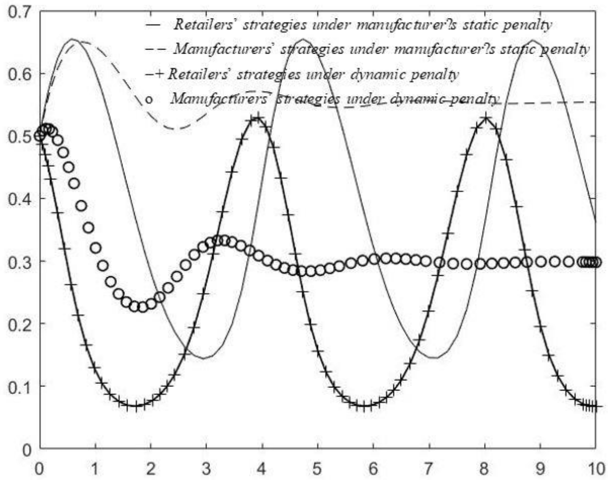

Fig. 3. Evolution Simulation Diagram of Game under Manufacturer's Dynamic Penalty Supervision Strategy[Owner-draw]

With the same initial value x=0.5 and y=0.5, MATLAB software is used to simulate the strategy evolution process of the manufacturer and the retailer in the supply chain cooperation promotion under the manufacturer's static punishment strategy and dynamic punishment measures respectively, as shown in Fig. 3. Under the static penalty measure F=8, the manufacturer and the retailer's choice of strategy fluctuates continuously and cannot form a stable strategy; when the dynamic penalty measure F(y)=(1-y) H is taken, the fluctuation of the manufacturer and the retailer's strategies tend to be stable with the increase of time and the number of games, and finally the manufacturer and the retailer converge to a certain point respectively. It shows that the manufacturer's dynamic penalty measures can improve the retailer's choice of promotion strategy and make the game players reach the evolutionary stable equilibrium quickly, as shown in Fig. 3. When x=0.5 and y=0.5 are taken as initial values, with the increase of time and game times, the evolutionary trace of the promotion game between the retailer and the manufacturer will gradually converge inward to the equilibrium point D*, which also verifies that the evolutionary system of the promotion game between the manufacturer and the retailer in the supply chain has asymptotic stability under the dynamic punishment measures of the manufacturer.

5 Conclusion

In this paper, an evolutionary game model with the manufacturer and retailer as players is built to analyze the "management dilemma" of the retailer and the manufacturer's strategy choice in supply chain cooperation promotion, and simulate the evolutionary path with a numerical method. Based on the research conclusion, the measures and suggestions to promote the cooperation efficiency of the supply chain from the perspective of manufacturers are put forward as follows. (1) It is important to strengthen the supply chain cooperation consciousness of manufacturers, increase the promotion subsidies to retailers, and increase the proportion of manufacturers sharing the promotion cost. The manufacturer can increase the profit by increasing the promotion subsidies to encourage the retailer's promotion activities. The manufacturer wants to increase its profit by reducing the "free-rider" and retailer's promotion activities. The "free-rider" strategy is not conducive to the cooperation of the upstream and downstream members of the supply chain. The manufacturer should reasonably share the retailer's promotion cost, improve the retailer's promotion enthusiasm to ease the channel conflict, and improve the cooperation efficiency of the supply chain. (2) It is important to improve the promotion effect by optimizing the promotion mix and enhancing the retailer's promotion gain. The promotion cost is an important factor affecting the retailer's income, which can optimize the promotion combination to reduce the cost of unit sales volume, and improve the retailer's promotion income. Besides, this also lifts the retailer's enthusiasm for promotion, thus improving the efficiency of supply chain cooperation. (3) Manufacturers should strengthen the market supervision of retailers and reasonably increase the punishment, and strengthen the process control of retailers' promotion activities. Increasing punishment can reduce retailers' opportunistic behavior, but over-reliance on increasing punishment may aggravate channel conflicts. At the same time, it should be noted that the double-margin phenomenon in supply chain cooperation is one of the important reasons for channel conflicts, which will reduce the cooperation efficiency of the supply chain.

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