Friedman Cosmological Model: Scale Factors for the Application of Formulas and the Density Parameters of the Cosmos

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Abstract. In the course of the history of cosmology, the Friedman-Lemaˆıtre-Robertson-Walker (FLRW) metric and Einstein's general relativity provided the basis for many theories in cosmology, including the one by James Peebles with the cosmological constant Λ, cold dark matter This theory, which describes the evolution of the universe, is known as the ΛCDM model, and the Mori-Zwanzig equation, the beginning point for scientists at the University of Münster's Institute of Theoretical Physics and Center for Soft Nanoscience to begin research that will give a strong theoretical foundation for characterizing systems with a large number of particles and a small number of detected particles. Although the first two the Friedman-Lemaˆıtre-Robertson-Walker (FLRW) metric and Einstein's general relativity are a century away from being formulated, we explain why we interpret the influence of the scale factor on the Friedman cosmological model, and the role it plays in the various equations and how it works by reviewing these milestones in cosmology.

Keywords: Friedman Cosmological Model · Scale Factors · Density Parameters

1 Introduction

One of the most well regarded theories in modern physics is Albert Einstein's general theory of relativity. It has recently been related with two of the past five Nobel Prizes in Physics: in 2017 for the detection of gravitational waves and in 2020 for the finding of the black hole at the heart of our galaxy. This hypothesis primarily describes and explains the universe's expansion since the Big Bang. The pace of expansion is determined by the total energy in the universe, which includes dark matter, dark energy, conventional matter, and other forms of stuff. Dark matter and dark energy, according to the Lambda-CDM cosmological model, play the most important role in the expansion of the universe, aside from visible matter (ordinary matter) [1].

A cosmological model is a geometric representation of the universe that attempts to explain the origins of its existing features and describe its progression across time. It must, of course, explain the findings and be able to make predictions that can be checked

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with future observations. The present model is based on the general theory of relativity, which offers the best agreement on large-scale behavior at the moment.

The distribution of matter in the cosmos is always assumed to be homogeneous in cosmological computations. This is because including the positions of each star would make the calculation excessively complicated. In reality, the universe is not uniform: there are stars and planets in some places, while nothing exists in others.

2 Big Bang and the History of the Universe

In cosmology, by far the most widely accepted explanation is the Big Bang theory, starting with the Big Bang, the universe began to expand, at first only some lighter atoms, such as hydrogen, helium, etc., then, the material under the action of gravity condensed into blocks, known as nebulae, nebulae continue to gather, there will be the familiar stars and stellar systems, the second half of the periodic table of elements began to form. The Big Bang models are inseparable from the history of the universe, and they will help me to visualize this process [2].

2.1 Proposal of the Thermal Explosion Theory

Currently, the standard model of the universe is the "hot big bang" model, and in order to understand why cosmologists accept the hot big bang model, we must understand the history of his birth and the associated formulas. It is mentioned that the properties of the universe change with time. The discovery of the cosmic microwave background provided a decisive basis and experimental results for the observation of the Big Bang model.

According to the model, the universe grew from its original hot and dense condition to its current comparatively cold and fragile state, which is still happening today. The big bang model and the steady-state model battled for dominance in the 1950s and 1960s. The steady-state model's proponents argued that the continuing generation of the matter was more feasible than the sudden creation of the whole cosmos billions of years ago in a single "big bang.“ The continual generation of matter, according to critics of the steady-state model, contradicts the conservation of mass energy. The steady-state model eventually fell out of favor as observational tools evolved, revealing evidence that the ideal cosmological model was no longer valid., due to the fact that as observational means advanced, evidence was observed that increasingly showed that the perfect cosmological principle was not true [2].

2.1.1 HUBBle's Law and Big Bang Model for the Universe

In cosmology, there is a notion that the measurement of galaxy redshifts naturally leads to a Big Bang explanation of the universe's evolution.

Starting with the Big Bang, assuming that the relative velocities between galaxies are constant from past to present, then at a given time (Hubble time) all the mass in the universe will be gathered in a small volume. The resulting Big Bang model can be defined as a model of the expansion of the universe from its initial highly dense state to its present low-density state [2].

We will use a (scale factor) as the coefficient of the expansion of the universe, which is equal to 1 at the present time, and we can also use it as a coefficient of time, written a(t), telling us how a depends on time t. Thus, the Hubble constant is:

$$\mathbf{H}_0 = \frac{\dot{\mathbf{a}}}{\mathbf{a}} \tag{1}$$

$$\boldsymbol{v} = \frac{\boldsymbol{dr}}{\boldsymbol{dt}} = \boldsymbol{H}_0\boldsymbol{r} \tag{2}$$

The features of the cosmos develop with time under the Big Bang model, with the average density decreasing and the average distance between galaxies increasing.

Hubble's law appears to make sense in the Big Bang model, were homogeneous and isotropic expansion results in a steady drop in the density of the universe from its original elevated value in the Big Bang model.

Herman Bundy, Thomas Gold, and Fred Hoyle first proposed the steady-state model in the 1940s, and the steady-state model can also explain Hubble's law, which states that there are also no privileged moments in time, and similarly, no privileged positions in space.

As a result, a steady-state universe is one in which the fundamental properties of the universe, such as average density and Hubble constant H_0, remain constant over time [2].

In 1965, the famous Cosmic Microwave Background (CMB) was discovered by Arno Penzias and Robert Wilson, which describes an isotropic background of microwave radiation and provides a strong guarantee for the proof of the Big Bang theory. It is called the "cosmic microwave background" because the average photon energy of the CMB corresponds to a wavelength that lies exactly in the microwave region of the electromagnetic spectrum [3].

The CMB is a remnant of the tremendous heat that was present during the early days of the universe. They were subsequently broken up into photons and electrons. When the temperature becomes low enough, the ions and electrons combine to form neutral atoms. The universe cools down enough for atoms to form, but the wandering photons do not interact with hydrogen atoms but begin to move in a straight line [4].

The universe becomes more transparent, and the temperature of the cosmic background radiation has decreased 1090 times since the universe became transparent, due to the fact that the Hubble constant $\frac{\dot{a}}{a}$ has also increased 1090 times since then. In this model, we can conclude that the universe could be extremely dense and hot at the beginning and has been expanding and cooling since then [3].

2.2 The History of the Universe

Is it well known that our universe is not infinitely large, nor infinitely old, it possesses a specific age, and size. Here, I will discuss the history of the formation of the universe and the process of its formation, as well as their respective derivation process and development, which will help us to have a more intuitive understanding of our universe, and set the stage for what I will mention later.

2.2.1 The Age of the Universe

In cosmology, the age of the universe is simply proved to be proportional to the inverse of the Hubble constant.

Therefore, the precise determination of this constant is a key issue in cosmology. Recent measurements suggest that the age of the universe will be about 6 billion years. This means that this value includes 50 to 100 km/s/MPc.

But we know that the age of the universe must be older than the oldest stars in the universe. The age of these stars is estimated to be between 13 and 16 billion years.

Hubble and Spitzer satellites give the consequent results which provided that $H = 67.15 \pm 1.2$ km/s/Mpc, so we know the age of the universe would be expected 13.8 billion years [5]. We also came out with an identical result in the ΛCDM Model of Cosmology later.

2.2.2 The Brief History of the Universe

Our universe began 13.7 billion years ago with the Big Bang, a massive explosion. Observations by NASA's Cosmic Background Explorer provide powerful evidence that our universe did indeed come into existence as an explosion.

A period of darkness ensued, and the universe did not appear until the first massive objects filled with light some hundreds of millions of years later. The detected light is thought to have originated in visible and UV light then was stretched or redshifted during its long journey to lower-energy infrared wavelengths that reached us through expanding space. Light from our very young universe travels much farther and is stretched to lower energy microwave wavelengths as it reaches us.

The earliest items are a mystery to academics (stars or quasars). The first stars we discovered, referred to as the third family of stars, were, nonetheless, larger and brighter than any stars in our local universe. Originally, these stars were clumped together in little galaxies. A few billion years after the Big Bang, these tiny galaxies had formed into full-fledged galaxies. At the centers of large galaxies, the earliest quasars are commonly discovered [6].

3 Friedmann Cosmological Model and Density Parameters

A model of the universe governed by the Friedmann equations, which describe how the universe expands or contracts is usually characterized by the "Friedmann model". Together with two very significant hypotheses, the solutions of these equations Einstein field equations, form the foundation of the evolution and structure of our universe. And they also emphasize the universe's homogeneous and isotropic nature, and together the above assumptions are known in cosmology as the "cosmological principle" [7].

From his theory of gravitation, also known as general relativity, Einstein wrote the equations that govern the matter-filled universe. However, in his view, the universe had to be static. Therefore, he facilitated his approach to obtaining this result by introducing a term known as the cosmological constant into the equations.

Shortly after him, the Dutchman de Sitter, the Russian Friedman, and the Belgian Le Maitre proposed a non-static universe as a solution to Einstein's equations of relativity.

Friedmann's model, on the other hand, depends on the density of matter inside the universe, while de Sitter's universe is the equivalent of an empty universe. These models have been the basis of the current cosmological models.

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The first hypothesis of the cosmological model is that there is no reason for the Earth to be at the center of the universe, or in any other privileged location, based on cosmological principles. The universe is regarded as follows under this hypothesis:

homogeneous, which presents the same properties everywhere in the cosmological range, and it has another characteristic: indistinguishability of coordinates, where one cannot distinguish the difference at each coordinate without a reference;

Isotropic, in which the vectors acting at each point in the universe are in the same direction, in other words, we cannot distinguish the difference in different directions without the influence of external forces (e.g., under the action of gravity).

The second required assumption we need is the universality of the laws of physics. And that in cosmology these laws are always the same everywhere and cannot be changed [5].

3.1 Friedman-Lemaître-robertson-Walker Metric

If we consider two events, one occurring at the spacetime location (t, r, θ, ϕ), and another occurring at the spacetime location (t + dt, r + dr, θ + dθ, ϕ + dϕ). Their coordinate lengths for the two projections in 3D polar coordinates are r and R as shown in Fig. 1, respectively [2].

Thus, in a three-dimensional polar coordinate, the distance between two locations is:

$$d\ell = R^2 d\theta^2 + r^2 d\phi^2 \tag{3}$$

$$\sin\theta = \frac{r}{R}, R\sin\theta = r, dr = R\cos\theta d\theta \tag{4}$$

We bring it in to get:

$$R^2 d\theta^2 = \frac{dr^2}{1-(\sin\theta)^2} \tag{5}$$

If $\frac{1}{R^2} = K$, K is curvature constant.

$$d\ell^2 = \frac{dr^2}{1-Kr^2} + r^2 d\phi^2 \tag{6}$$

We can also write it as:

$$d\ell^2 = \frac{dr^2}{1-Kr^2} + r^2 d\Omega^2 \tag{7}$$

When we try to bring in the scale factor a(t), we find that the metric becomes:

$$d\ell^2 = -dt^2 + \left[\left[a(t)d\Omega^2\right]\right] \tag{8}$$

It could be also written as:

$$d\ell^2 = -dt^2 + a(t)^2\left[\left[dr^2 + S_k(r)^2 d\Omega^2\right]\right] \tag{9}$$

Here we introduce the function $S_k(r)$ which can be written as:

$$S_k(r) = \begin{cases} R\sin\frac{r}{R} & (K = 1) \\ r & (K = 0) \\ R\sinh\frac{r}{R} & (K = -1) \end{cases} \tag{10}$$

We assume that the universe is completely homogeneous and isotropic, so everything we need to know about its geometry is contained in the curvature constant K (K = + 1, 0, or -1 for the three spatial states, respectively), if K $\neq$ 0, the a(t) scale factor, and the present-day radius of curvature R_0.

3.1.1 ΛCDM Model of Cosmology

The model is a mathematical modeling of Big Bang cosmology that comes from the Friedmann-Lemaitre-Robertson-Walker (FLRW) equations and general relativity.

We must also be aware of the mass density parameter, which measures the ratio between the density of the cosmos under investigation and a certain density, known as the critical density c, which is connected to the Hubble constant. In our opinion, the value of this parameter is zero (i.e., now).

According to the CDM model, the cosmos is made up of photons, neutrinos, conventional matter (baryons, electrons), and cold dark matter, as well as "dark energy," which is commonly thought to exist in the form of a constant vacuum energy density called the cosmological constant (Λ). It is the cause of the Hubble expansion's apparent acceleration, with cold dark matter playing just a gravitational role.

In addition to this, ΛCDM further specifies that our cosmic space is flat [9].

z is the redshift constant, obtained by measuring the distance by which the wavelength is elongated divided by its original length $z = \frac{\lambda_o - \lambda_e}{\lambda_e}$, where λ_o is the wavelength we observe, and λ_e is the wavelength just emitted. After understanding the above cosmological notation, we can consider the ΛCDM model (Lambda-CDM Model) to derive the observable length of the universe [10]:

$$t = \int_{z_{ini}}^{z_0} \frac{dz}{H(1+Z)} \tag{11}$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2} + \frac{\Lambda c^2}{3} \tag{12}$$

ρ_{crit} is the critical density, taking the geometry of the universe as an example, if the universe is flat, then the density of this matter and substituting into the Friedmann

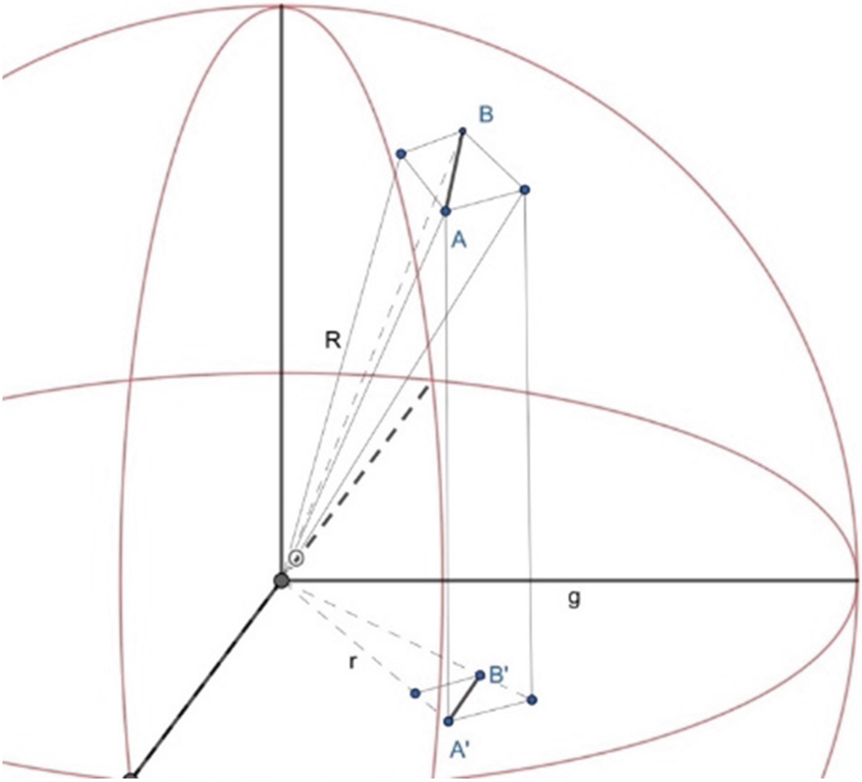

Fig. 1. The Friedman-Lemaˆıtre-Robertson-Walker Metric [8] (Source: Juruo RD (January 28, 2020, at 20:41) How is the FRW gauge obtained? https://www.zhihu.com/question/366697774/answer/988070547. Licensed by Juruo RD)

equation we get:

$$\rho_{crit} = \frac{3H_0^2}{8\pi G} \tag{13}$$

The critical density of the universe is composed of DM (dark matter) + DE (dark energy) + BM (baryonic matter) + R (radiation). We know that the flatness of the universe can be used to infer the mass-energy density in the universe.

Then we combined $H = H_0\sqrt{\Omega_m a^{-3} + \Omega_{rad} a^{-4} + \Omega_\Lambda}$ [11] with $t = \int_{z_{ini}}^{z_0} \frac{dz}{H(1+Z)}$, so we got the age of the universe, t ≈ 13.8 billions of years.

$$t = \frac{1}{H_0} \int_{z_{ini}}^{z_0} \frac{dz}{(\Omega_m a^{-3} + \Omega_{rad} a^{-4} + \Omega_\Lambda)^{\frac{1}{2}}} \approx 13.8 billions \tag{14}$$

3.2 Density Parameters

The ratio of the actual (or observed) density to the critical density is known as the Density Parameter. The energy density of any component may be written in the form

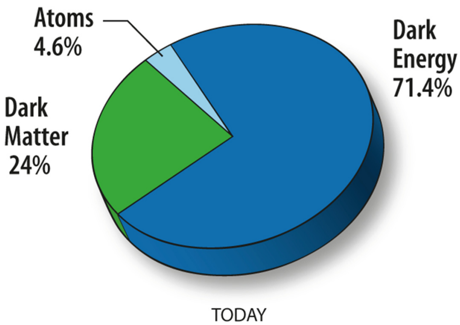

Fig. 2. Components of the mass of our universe [12] (Source: NASA / WMAP Science Team (August 4th, 2013) Universe Content -WMAP 9yr. https://wmap.gsfc.nasa.gov/media/121236/index.html, Licensed by NASA)

$\varepsilon = nE$, where n is the particle density and E is the average energy of each particle. The dependency of the number density as the universe expands is $\mathrm{n} = \mathrm{a}^{-3}$, which indicates that the number density of particles is proportional to the scale factor, assuming that particles are neither generated nor destroyed.

The energies of photons or other massless particles have a dependence of $\mathrm{E} = \frac{hc}{\lambda} \propto \mathrm{a}^{-1}$, where h is Planck's constant with a value of 6.626176×10^{-27} J per second, because their wavelengths λ expand with the expansion of the universe. Thus, for photons and other massless particles, $\varepsilon\mathrm{r} = \mathrm{nE} = \mathrm{n}\left(\frac{hc}{\lambda}\right) \propto \mathrm{a}^{-3}\mathrm{a}^{-1} \propto \mathrm{a}^{-4}$ [2].

The theory that photons are neither created nor destroyed, however, is incorrect. With a temperature of $\mathrm{T}_0 = 2725.5$, the current energy density of the cosmic microwave background is, thus we can infer:

$$\varepsilon_{CMB} = \alpha T_0^4 = 4.175 \times 10^{-14} Jm^{-3} \approx 0.2606 MeVm^{-3} \tag{15}$$

So we got the parameter density of CMB:

$$\Omega_{CMB} = \frac{\varepsilon_{CMB}}{\varepsilon_c} = \frac{0.2606 MeVm^{-3}}{4870 MeVm^{-3}} \approx 5.35 \times 10^{-5} \tag{16}$$

4 Components of the Universe

By making observations, it was discovered (in several different ways) that all the sum up to 1, implying that the universe's total energy density is exactly equal to the critical density. This is very incredible.

The following shows the contributions of the various components now and at the time of the CMB emission (380,000 years after the Big Bang). According to the WMAP science team's observation [12], 23% of the energy of our universe is Dark matter, 72% of that is from dark energy and the rest is normal matter as shown in Fig. 2.

5 Conclusion

Throughout the paper, we first provide a history and explanation of the development of the Big Bang theory and its related developments in order to facilitate an in-depth understanding of the history of the formation of the universe. To prove the hot big bang theory, we use a, which is used as the expansion coefficient of the universe in Hubble's law, to explain why the density of the universe has continued to decrease from a higher value in the hot big bang theory model until today. The Hubble constant (H_0) we use can be interpreted as $\frac{\dot{\mathbf{a}}}{\mathbf{a}}$, and we conclude that the Hubble constant has increased by a factor of 1090 since the universe became transparent, corresponding to a decrease in the temperature of the cosmic background radiation in the CMB by a factor of 1090. Thus, at the current time, we have evidence that the Big Bang theory is valid.

We next characterized the history of the formation of the universe and the formation process, through the process of the thermal big bang theory we proved, we can find that the age of the universe is proportional to the Hubble constant, through satellite observations, we know that $H_0 = 67.15 \pm 1.2$ km/s/Mpc, and combined with the ΛCDM model, from which the age of the universe is deduced to be about 13.8 billion years.

It is well known that the Earth is insignificant and small in the macroscopic perspective of the universe, and there is no reason why for example the geocentric theory of the early hypotheses generally puts the Earth at the center of the universe, or in any privileged region. So we are introduced to homogeneous and isotropic, they are the universals of the laws of physics. These laws are constant everywhere and at all times.

In view of all the results of the observations of the universe, especially the expansion of the universe, we can write down a metric to describe this phenomenon, which is also known as Friedman-Lemaître-Robertson-Walker metric. We first calculated and derived it by assuming the polar coordinates of two objects, and finally arrived at the formula $d\ell^2 = \frac{dr^2}{1-Kr^2} + r^2 d\Omega^2$ for the relative distance between two in the universe, and by replacing the $\frac{1}{R^2}$ in the formula with K, which is the curvature constant, we arrived at the three cases of Friedmann's cosmological model, which change according to the value of K, are divided into $K = +1$ (closed universe), $K = 0$ (flat universe), and $K = -1$ (open universe), so we can see that the dynamics of the entire Friedmann cosmological model is derived from the scale factor, which can even be considered as a record of the evolutionary history of the universe, which also proves that we have never deviated from the topic, discussing the application of the scale factor in the cosmological formulation has been the main object of our discussion.

As a parametric form of the Friedman-Lemaître-Robertson-Walker metric, the ΛCDM assumes that the universe consists of photons, neutrinos, and ordinary matter (baryons, electrons), and cold dark matter. In addition, we also mention in the ΛCDM model a number of parameters and constants that play an important role in cosmology, namely the density parameter (Ω), the redshift constant (z), and the cosmological constant (Λ), which will play a role in the calculation of the age of the universe and their relation to the scale factor $H_0 = \frac{\dot{\mathbf{a}}}{\mathbf{a}} = \sqrt{\frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2} + \frac{\Lambda c^2}{3}}$.

By combining H_0, which represents the Hubble factor, and the formula representing the age of the universe (t), we arrive at an approximate age of the universe of 13.8 billion.

Next, we examined the effect of defining the total density parameter $\Omega = \frac{\varepsilon}{\varepsilon_c}$ and the critical density $\varepsilon = nE$, we can calculate the critical density of the CMB, as well as its total density parameter, the results are respectively $\varepsilon_{CMB} \approx 0.2606 MeV m^{-3}$ and $\Omega_{CMB} \approx 5.35 \times 10^{-5}$.

Lastly, we examined the components of the universe based on Fig. 2 which was posted by NASA.

These data are consistent with the notion that the parameters and constants used in the previous calculation of the CMB total density parameter and critical density.

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