



# Studies on the Modifications and Applications of the Net Present Value and Internal Rate of Return

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**Abstract.** In terms of investment decision making, it is vital for the decision makers, to be able to select the investment project that is the most profitable. Therefore, various models are established including the net present value (NPV) and the internal rate of return (IRR) for the investors to compare different investment projects. In this paper, studies by scholars focusing on the advantages and disadvantages, and the modifications as well as applications of the NPV and IRR models are summarized and presented, along with further prediction of future market situations. Studies have also suggested that the prediction of cost of capital has to be carefully taken into consideration in NPV, and the pitfalls of IRR when encountering non-conventional cash flows which multiple numbers of IRR may be attained. In addition, further studies have shown that the feasibility of NPV and IRR are limited, due to the non-conventional cash flow, thus several modified versions of NPV and IRR were constructed including Max-NPV, Decouple NPV, Modified IRR, and Average IRR. Moreover, this paper analyses the improvements among the amendments upon the two models, by eliminating the problems of inconsistencies of NPV and IRR; case studies are also discussed to clarify the practical uses of the modified versions. Despite the coherent results obtained from the calculations, analysis of individual investment projects should still be done, in order to work out the optimal decision.

**Keywords:** Net present value · Internal rate of return · Applications · Modifications · Max-NPV · Decouple NPV · Modified IRR · Average IRR

## 1 Introduction

Profit maximization is the main aim of private businesses, so it is essential for the businesses to make effective capital investment decisions to make profits from the fierce competition [1]. For example, the construction of railway involves a considerable capital expenditures, therefore, an effective capital budgeting method is required to lower the risk of investment and guide entrepreneurs to make the correct investment decision [2]. Decision making is based on cost–benefit analysis and financial feasibility of the projects

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[1]. There are three common methods which a manager or an analyst can use to assess the worth of projects, IRR, NPV and the payback method [3]. However, IRR and NPV may give different rankings when comparing with different projects, in which deciding the most profitable project remains a controversy. Dorfman in 1981 had dated the debate back to Fisher in 1907 and Boehm-Bawerk in 1889 [4].

This article also reviews the pitfalls of NPV and IRR, including accuracy, strong assumption that are based on which would not be the case of real life, and unable to deal with non-conventional cash flow. Although the internal rate of return is a more favored approach for its convenience, it is overall less reliable than NPV.

As the most antiquated methods to evaluate the value of financial project, NPV and IRR have developed overtime. This article focuses on Decouple NPV (DNPV) and the Average internal rate of return (AIRR). According to the research conducted by Sortino [2], this method is linked with prospect theory. In terms of AIRR, the improved version of IRR, by the study of K.J.S. Satyasai [5], it is a return on the investment, which assumes a particular return on the cash flow stream. In addition, the applications of DNPV and AIRR are also mentioned by quoting the studies of Buss and Rosenblatt.

## 2 Literature Review

### 2.1 The Original NPV

The Economic Theory of the Location of Railways was published in 1887 by the American remarkable engineer, A. M. Wellington. In order to prove the effectiveness of capital expenditures for the construction of railway, Wellington firstly applied the present value to non-financial project [2]. The concept of net present value was proposed in 1907 by Irving Fisher [6]. The establishment and popularity of NPV are mainly due to the achievement of Fisher and Grant in 1950, who is the author of Principles of Engineering Economy [6]. The formula of NPV can be expressed as below:

$$NPV = \sum_{t=0}^T \frac{CF_t}{(1+r)^t} \quad (1)$$

where  $CF_t$  represents the cash flow of time period  $t$ ,  $T$  represents the time period and  $r$  represents the discount rate. The cash flow is expressed as its present value as it is discounted. The net present value adds the present value of each time period up. The discount rate depends on how the firm raises the fund. It can be the opportunity cost of capital when firm borrows money or the expected rate of return when invests other projects [3].

#### 2.1.1 Advantages of NPV

Individuals can benefit from using NPV because this rule displays the present value of money taking into account the time factor expressed as a discount rate [7]. There is an assumption that the money today is more valuable than the money in the future, so NPV rules will help discount the value of inflow in the future back to the value of it today by dividing the discount factor. Also, it can still be used even if multiple investments

are required [7]. In addition, if the discount rate is expected to vary over time due to the fluctuations in the economic situation, the net present value could still be calculated by changing the discount factor in the formula.

### 2.1.2 Disadvantages of NPV

However, NPV method relies on the accurate estimation of the market situation [8], which can be difficult and time-consuming for investors. It can be misleading while making decisions if individuals fail to estimate the interest rate and cash flows in the future. Furthermore, it is not an optimal way to analyze projects with the same NPV but different initial investments [8]. For example, if there are two projects that have the same NPV but are different in sizes. The larger project will be riskier; however, this is not clearly justified by the NPV model.

## 2.2 The Original IRR

The concept of internal rate of return was first implicitly put forward by Irving Fisher in 1907 [9]. The definite mathematical relationships related to IRR were first established by Boulding in 1935 and applied it from financial assets to real assets. IRR is widely used as a tool for financial decision making. It suggests investors should only invest the projects with IRR greater than the discount rate. And the investor would prefer the projects with higher IRR to the projects with lower IRR if they are mutually exclusive [10]. IRR can be used to calculate the theoretical profitability of a financial project. It can be calculated when NPV is equal to 0. The formula of IRR can be seen as following:

$$0 = \text{NPV} = \sum_{t=0}^T \frac{CFT}{(1 + \text{IRR})^T} - CF_0 \quad (2)$$

where the number of time period is represented by  $t$ , the cash inflow exists during time period  $t$  can be represented by  $CF_t$  and the initial investment is represented by  $CF_0$  [6].

### 2.2.1 Advantages of IRR

Different to NPV, which was mentioned before, IRR is calculated to find the point at which the rate of return that make the two projects in comparison indifferent. While comparing to NPV, according to Satyasai, to calculate the profitability of certain projects, the IRR model is a more preferred approach for investors due to its simplicity and also to be compared with [5].

Firstly, according to Mackevičius and Tomaševičthe, the IRR model provides a clearer extent of profitability of each individual project, as a comparison between the hurdle rate (the opportunity cost of capital) and the required rate of return that is calculated would directly suggest the difference in the rates of return, rather than the discounted cash flows, which the discounted cash flows may provide a more misleading result when encountering interest rate risks, which means that the calculation of IRR avoids the problem induced by the interest rate, and it can be concluded that the IRR model truly

indicates the minimal rate of return that a project should offer in order to remain profitable for the investors [7]. In addition, as Arshad and Asma had suggested, only the expected future cash flows are required for the calculation of IRR, which less input is required to be found comparing to NPV which both the future cash flows and the cost of capital are required as inputs [11].

### 2.2.2 Disadvantages of IRR

Unfortunately, the IRR model does have some of its drawbacks considering its practical applications. For example, IRR is based on the assumption that all of its future cash flows are reinvested back to the project so that to attain the final calculated required rate of return, which can be impractical in real life situations as firms or individuals would be spending the cash inflows acquired for other purposes, but continuing the projects simultaneously [12]. Moreover, the duration of the projects is ignored in the calculation of the internal rate of return, in other words, the life span of the projects is not taken into consideration, so that the IRR model is not able to distinguish between a long-term project and a short-term project which might disrupt the plans of a firm or individual investment [12].

Despite those problems that the use of IRR model would generate with all kinds of investment projects, there are also additional issues that the IRR model would encounter when dealing with a non-conventional cash flow. The issue that might be encountered while dealing with non-conventional cash flows is that multiple results may be collected, making investment decisions vaguer to be made. This is a consequence of the multiple sign changes in the non-conventional cash flows [13, 14]. In addition, rather than using the IRR alone, firms may also calculate NPV for the same projects to ensure that the project chosen is the most profitable. Nonetheless, NPV and IRR can occasionally contradict with each other. This is also shown in the studies conducted by Arshad and Asma, when comparing two mutually exclusive projects, the NPV and IRR may sometimes suggest the same results, but only due to the cost of capital, if the cost of capital was to change, the final result provided by the two models may suggest different answers to whether invest in 'project A' or 'project B' [11].

To sum up, the internal rate of return model is a more favored approach to some investment decision makers due to its simplicity to compare, and several advantages over the net present value. However, there are considerable confinements to its applications in certain real life situations which the use of IRR model would obstruct the investors to make the correct decision.

## 2.3 The Applications of NPV and IRR Methods

The ranking might be inconsistent when comparing the financial projects using NPV and IRR methods [15]. It is essential for a company to make an optimal investment to be competitive in the market with fierce competition. Different countries in the world have different situation, the preferred capital budgeting method can be different among developed countries and developing countries. From 1966 to 2016, some companies in developed countries, for instance, the USA and UK, preferred NPV and IRR methods to other capital budgeting methods when making the financial decisions, as for some firms

in developing countries, taking India and South Africa as examples, there is an increasing use of NPV and IRR, especially NPV [16].

## 2.4 Modified versions of NPV

### 2.4.1 Max-NPV

It is known that a project does not always perform well. Potential factors such as climate change, problems with machines, and lack of raw materials can lead to the postponement of the delivery schedule. However, NPV cannot cope with these uncertainties. In 1970 max-NPV was developed by Russell [17], net present value of the project is maximized by advancing cash-inflowing activities as quickly as possible while delaying cash-outflowing activities as late as possible [18]. However, Max-NPV assumes that investors have perfect information about the project [18]. These include the sequence of operation steps and resource allocation.

Max-NPV project scheduling problems can be categorized into two types: resource-unconstrained project scheduling problems and resource-constrained project scheduling problems [19]. The resource-unconstrained project scheduling problem means that the project has no restrictions on the use of resources. An effective method of resolving the unconstrained max-NPV problem is to use nodes to show each activity and connect them by arrows, take every cash flow in the project as events [20], while heuristic algorithms are used for solving resource-constrained problems [21].

Markovian projects were tried to be solved by using max-NPV by Buss and Rosenblatt [22]. In addition, max-NPV can be used to improve the project profit in an uncertain environment according to the research by Zheng et al. [23]. The reasonable outcomes of several risky projects were computed and found out that max-NPV is more stable in the reaction of disruptions.

### 2.4.2 Decouple NPV

DNPV's approach to evaluating projects begins with identifying project risks and then accounting for those risks as project costs [24]. Furthermore, it depends on how much the investor rebels at making loss [25]. This method is related to prospect theory, which supposes that different investors have different attitudes toward loss depending on their situations. It is beneficial for investors as it is a more subjective approach.

In contrast to NPV rules, DNPV not only takes time value of the project into account, but also estimates the performance and stability of the project, which is also an important component while making financial decisions.

However, over-evaluation and under-evaluation can still be a problem. In 1969, Stoll had proved it by using put-call parity. In 2020, Lopez-Marín et al. optimized the method by considering future protected value, and proved the accuracy of the method by the KuisebSun Gold Project [26].

In the model of NPV, the investor should only consider one parameter (i.e. the risk premium) [27]. However, because time and risk are two identical variables, only considering one factor would result in considerable valuation errors, which further causes investment failure. However, Decouple NPV allows investors to integrate interrogative techniques. That is an experience-based method.

## 2.5 Modified Versions of IRR

### 2.5.1 Modified Internal Rate of Return MIRR

Studies from Carry et al., states that the modified internal rate of return is an improved version of the original IRR model, which according to the theory of time value of money, the terminal value of a series of cash flows should be equivalent to the compounded present value of the same series of cash flows, with the discount rate remains unchanged [12]. One of the improvements that MIRR has is that as long as the largest cash outflow is counted in the calculation process for each individual investment projects, the ranking of MIRR and NPV would be consistent, suggesting that despite the non-conventional cash flows, both the MIRR model and the NPV model would derive to the same result, instead of inconsistent results attained from the original IRR and NPV, which is a significant pitfall for the original IRR model, providing a correct profitability of individual projects in order for the investor to choose the most profitable decision. This is fulfilled by MIRR since it discounts all the future cash outflows to the present and to combine with the initial cash outflow at the corresponding opportunity cost of capital, which eliminates the problem induced by the multiple sign changes in the original IRR model [12]. In addition, MIRR also allows another approach, which all the cash flows are compounded to the terminal value, so that the life span of the project is also taken into consideration, according to Carry et al. This can be shown by the example, when two investment projects differ in their sizes, and at the given cost of capital which is 10%, the original IRR and NPV show a conflicting result, however, a more appropriate and feasible solution is when the MIRR model is adopted along with the implied shadow investment.

Furthermore, in the study of Sarsour et al., they have suggested that the MIRR model is based on the assumption that a fixed amount of capital is invested on buying shares at the beginning of each year after acquiring the profit gained from the preceding year [28], which enhances its practicability in real life situations, as firms or individual investors tend to keep investing their capital in order to maximize the total profit attained.

MIRR is a return on the investment, assuming a particular return on the reinvestment of cash flows. As a more practical version of IRR, the real-life applications include: the analysis of investment in pigs fattening-evaluate the growth rate and the further economics revenue; and the analysis of watershed projects. According to the research conducted by Ivanovic et al. [29], they had a conclusion that this investment is economically benefit for the investor because the MIRR was higher than cost of capital. Moreover, the application of MIRR helps in the research of watershed evaluation. By the study of Satyasai [5] in 2009, who cited data from three evaluation study reports [30], a conclusion was made that MIRR could deal with the problem of the differences of scale and time span.

### 2.5.2 Average internal rate of return (AIRR)

Another modification of IRR developed by scholars is the AIRR model, so that the risk of a project is also taken into consideration, as AIRR may be applied to risky capital asset projects, where the use of IRR may be unreliable and induce additional problems. Utilizing the AIRR model, the figures derived would also refer the level of risk to different projects therefore to enable the investors to also distinguish the differences

between characteristics of the investment projects and to make a more profitable choice of investment with a relatively lower potential risk [31].

The AIRR is an average of one-period return rates derived from investment streams which are freely chosen by the analyst [32]. This AIRR notion works in association with the money flow, which in another word investment stream, because the consolidation of AIRR and investment stream determines the NPV of the project.

### 3 Conclusion

This paper reviews the original NPV method, IRR method and their advantages and disadvantages. Traditional NPV method does not clearly clarify the stability and risk of projects, and original IRR fails to cope with multiple signs and level of risk involved in projects. Four modified methods and their applications are summarized to draw the conclusion below:

Max-NPV method can be used to deal with the uncertainties in the production process by maximizing the Net Present Value of the project. Therefore, it will give a more stable reaction. Secondly, in DNPV, the risk of making a loss is directly involved in calculation. Therefore, two essential components while making financial decisions, time value and risk can be considered.

For traditional IRR, the situation that NPV and IRR derive different results may occur, using MIRR can provide a more reliable decision. MIRR will calculate the sum of outflows, which can solve the problem of multiple signs. In addition, AIRR will provide a profitable result with lower level of risk to investors.

In summary, this paper summarizes the original IPV and IRR methods and their modified versions. Furthermore, possible applications of those methods are also discussed. This paper could provide some useful references for those who are interested in investment valuation methods.

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