Research on the Applications of Duration and Immune Theory and Their Implications

Naiyu Zhang

Liaoning University, Anshan, China
18402466@masu.edu.cn

Abstract

With the rapid development of the bond market and the financial field, more and more people realize the importance of the interest rate risk management for an enterprise to survive in the bond market, so they pay more attention to the interest rate risk management. Since the emergence of Macaulay Duration model in the last century, people began to pay attention to the application of duration theory and immune theory in interest rate risk analysis. This paper focuses on duration theory, immune theory and their applications in financial and debt markets. In this paper, a tabular analysis method is used to clearly show the development of the duration of immunity theory and the research conclusions in each period. Besides, the comparative analysis method is also used to show the advantages and disadvantages of the present duration theory and the model. Finally, we find that the immune duration theory has a positive effect on the interest rate risk management in the commercial bank and bond market, which is also an important application and advantage of the duration theory. However, there are still some deficiencies in the theory of duration, for example, the calculation of the theory of duration is still complex to analyze.

Keywords: *duration theory, immunization theory, immune duration theory, interest rate risk management, interest rate liberalization, investment portfolio.*

1.INTRODUCTION

Since the middle of last century, the commercial banks around the world have gradually started the reform of interest rate liberalization. At the same time, the commercial banks have gradually relaxed and eliminated the regulation of interest rates and followed the law of market value, and the market determines the level of interest rates. The interest rate liberalization can improve the external environment for commercial banks to operate and promote commercial banks to continuously operate in the market, thus speeding up the pace of financial creation, but the interest rate liberalization will also bring huge challenge for the commercial bank. With the gradual deepening of interest rate liberalization, the frequent fluctuation of interest rate also has an impact on the earnings of commercial banks, making interest rate risk become one of the main risks of commercial banks and bond market. Therefore, it is very important for commercial banks and bond markets to strengthen interest rate management, so this paper focuses on the application of duration theory in interest rate risk management. The paper consists of three major parts, namely duration theory, immune theory and the final conclusion. In addition, this paper discusses each theory, introduces their development process, application fields and development conclusions in detail, and finally emphasizes the application in bond market and commercial bank. In the conclusion part, the article summarizes the advantages and disadvantages of duration theory, and based on the current development of immune duration theory, the prospects for its development are also expected.

2.DURATION THEORY

2.1. Duration theory research trends

On the research trend of the duration theory, this paper makes a comprehensive analysis from two angles, time and country. In 1938, F. R. Macaulay proposed the concept of duration for the first time in western financial market, and also introduced Macaulay duration model. However, after this model was put forward, it was not taken seriously and applied for a long time. However, in the 1970s, the western financial market, especially the bond market and the stock market, underwent drastic

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changes and it also faced huge interest rate risks. Only then did the industry begin to appreciate the significance of the duration model in practice. So from the 1970s, the idea of duration really began to enter the world of financial market research and application fields. But it should be mentioned that since the idea of duration was put forward, it was most widely applied to the financial markets of some developed western countries to measure interest rate risk. Until the 1990s, the duration model is recognized as one of the most accurate and reliable criteria for interest rate risk management in international commercial banks. In 1992, the United States began to use the duration model to control interest rate risk on the recommendation of the Federal Reserve Board. To date, duration theory and practice have been accepted and widely used by commercial banks and bond markets in most developed countries. However, in some developing countries, such as China, the study of duration concept and duration model only began at the end of last century, and most of the research is still on the general introduction of the model. It can be seen that the global trend of duration theory is still more in favor of the financial markets of western developed countries, and the time of total research began in the 1970s. Although the duration theory has been a new research idea in the financial market for nearly half a century, its appearance is instructive to the analysis and control of interest rate risk.

2.2. Duration theory research results obtained and relevant analysis

2.2.1. The research history of duration theory

People started duration theory firstly since 1940s and based on this simple duration theory model many researchers keep on expanding its applications and understandings. Since 1970s, plenty of new duration models and theories have been pushed out. As the theory developing, duration's application has already expanded to many fields, such as bond market, financial market, interest rate risk analysis and so on. The following table will briefly introduce the development course of duration theory and draw important conclusions for explanation.

Table 1: Some relevant conclusions

Years	Researchers	Research area/Research aspect	Research conclusion/Research value
1938	Frederick.R.Macaulay	Average repayment period of railroad bond	Firstly presented the concept of duration
1971	Fisher&Weil	Academic papers on immune risk	Firstly presented F-W duration
1988	George.Kaufmann	Bond market(bond's price and yield rate)	Firstly proved that the actual relation between bond price and yield rate is nonlinear convexity
1977&1985&1992	Vasicek Cox&Ingersoll and Ross&Health, Jarrow and Mortorn	Financial field (rate of return and interest rate risk aspect)	Firstly introduced random duration model
1988	Priman&Shores	The disadvantages of random duration area	Multivariate parameter duration model is presented.
1990	Reitano	Direction vector area	Directional duration and partial duration are presented
1992	Ho	Rate of interest area	Firstly introduced key-rate duration
1992	Klaffky etc.	The disadvantages of random duration area	Firstly introduced reshaping duration
1993	Frank Fabozzi	The disadvantages of Macaulay duration area	Firstly presented effective duration and effective convexity model
1996	Barber&Copper	The disadvantages of random duration area	Firstly introduced Principal component duration
1997	Nawalkha&Chambers	The disadvantages of random duration area	M-vector model is presented
2000	Wu, Xueping	A New Stochastic Duration Based on the Vasicek and CIR theories.	The new duration becomes larger, increasing with bond maturity,than the original duration.
2005	EC Johnson	Duration's role in interest rate risk analysis.	It illustrates duration's value as a tool of interest-rate risk management, and introduces

			the "duration ratio" as a measure of the term-volatility of interest rates.
2008	Tom Arnold& David S. North	Duration measures for corporate project valuation.	A duration-type measure is generated that provides a single number for the assessment of project cash flows relative to changes in the discount rate [1] .
2011	Yin, Ying & Liang-rong Song	Optimization of commercial bank's value based on random duration.	The random duration method is applied to manage bank's interest rate risks [9].
2018	Beccacece&Francesca&Roberto Tasca&Luisa Tibiletti	Macaulay duration's important role in risk-adjustment in fair value.	It shows that because of different Macaulay durations, projects which at the same redemption date and endowed with the same EPV and/or the same total inflow may differ considerably in risk-adjustments [2].
2022	Zhang, Zeyuan	Basic theories of Macaulay duration and its limitations.	This paper affirms Macaulay duration's role but also reminds investors of Macaulay duration's limitations and offers modified duration and effective duration to make up for its shortages [11].

2.2.2. Modern duration model and its comparative research

Since Macaulay duration theory was put forward, dozens of duration models have been developed after decades of development. After decades of research, people gradually found out the limitations and deficiencies of Macaulay duration model. Compared it with F-W model, random duration model, directional duration model and effective duration model, this research is going to show their different strengths and weaknesses.

Table 2: Comparison between duration models

Model	Advantage	Disadvantage	Scope of application
Macaulay Duration	Reflect the sensitivity of bond prices to changes in interest rates	It is impossible to measure the interest rate risk under the interest rate fluctuation, non-flat ,non-parallel and implicit option.	Under the condition of small fluctuation of interest rate and parallel, flat and non-implicit option
F-W Duration	Interest rate maturities are allowed in a variety of shapes	The parallel movement of yield curve is hidden.	Non-flat and non-implicit option
Random Duration model	The interest rate risk of financial instruments can be measured when the yield curve is not moving in parallel	Need excellent modeling skills and the calculation cost is large.	Non-implicit option
Directional Duration	It can measure the risk of non-flat and non-parallel financial instruments	Difficult to calculate and can not measure assets with no maturity date	Non-implicit option
Effective Duration	It can measure the interest rate risk of implied option financial instrument	Very difficult to calculate and prepayment models and simulation techniques are needed.	Under the condition of implied option

2.3. Duration theory's application fields

Before the discovery and deep research of duration theory, commercial banks lack the methods and consciousness of interest rate risk management. In the 1970s, people began to study the duration theory, and extended F-W duration, random duration model and other applications, these theories and models play an active role in the interest rate risk management of the bond industry and commercial banks. Based on a large number of related articles, this paper found that the application of duration theory is very wide, but most commonly use in the interest rate risk management.

Firstly, the research introduces the background of the application of duration theory in interest rate risk management. Commercial banks in many countries have all implemented asset-liability ratio management and

risk management. However, because the interest rate control system weakens the role of interest rate as a compensation mechanism for loan risk, so many commercial banks of the commercial operation of the initiative is not high. Under the controlled interest rate policy, when deciding whether to adjust the interest rate or not and the level of the interest rate, the governments put the coordination of interests in the first place and put the supply and demand of funds in the market in the second place, thus make the interest rate adjustment become the main task consciously. And in the process of interest rate liberalization, the interest rate spread and investment income of commercial banks will be increasingly affected by the interest rate fluctuation, so the interest rate risk management will become increasingly important in the risk management of commercial banks. In commercial banks' interest rate risk management, risk measurement is the most important, which involves the formulation of the final risk management strategy.

Secondly, this research introduces the application of duration theory in risk management. The core idea is that duration is used to assess and manage a bond portfolio's interest rate risk. It is a weighted average of the time when future cash flows are generated. The larger it is, the longer the weighted maturity of future payments, and the more sensitive it is to market interest rates, the greater the interest rate risk. In the interest rate risk management, this paper will introduce a new term called duration gap. Duration Gap (GAP) is the difference between the duration of assets and the duration of liabilities. It has an impact on commercial banks' net asset changes and may be used to assess their interest rate risk exposure.

Here are some relevant formulas for us to analyze.

$$\mathrm{GAP} = \mathrm{D_A} - \frac{\mathrm{L}}{\mathrm{A}}\mathrm{D_L} \tag{1}$$

$\mathrm{D_A}$ represents modified duration of assets, $\mathrm{D_L}$ represents correction period of liability and L represents total liabilities, A represents total assets.

The duration of assets and liabilities may be estimated using a weighted average of commercial bank assets and liabilities and the weight is the proportion of the assets or liabilities to the total assets or liabilities, so:

$$\mathrm{D_A} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{D_i^A} \times \frac{\mathrm{A_i}}{\mathrm{A}} \tag{2}$$

$$\mathrm{D_L} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{D_i^L} \times \frac{\mathrm{L_i}}{\mathrm{L}} \tag{3}$$

D_i^A represents duration of asset I, A_i represents asset I, D_i^L represents duration of the liability in item I and L_i represents Liability I.

Changes in net assets of commercial banks may be expressed as changes in assets less changes in liabilities, so:

$$\Delta\mathrm{E} = \Delta\mathrm{A} - \Delta\mathrm{L} \tag{4}$$

ΔArepresents changes in assets and ΔL represents changes in liabilities.

$$\Delta\mathrm{A} = -\mathrm{D_A} \times \frac{\Delta\mathrm{r}}{(1+\mathrm{r})} \times \mathrm{A} \tag{5}$$

$$\Delta\mathrm{L} = -\mathrm{D_L} \times \frac{\Delta\mathrm{r}}{(1+\mathrm{r})} \times \mathrm{L} \tag{6}$$

$$\Delta\mathrm{E} = -\mathrm{A}\left(\mathrm{D_A} - \frac{\mathrm{L}}{\mathrm{A}} \times \mathrm{D_L}\right)\frac{\Delta\mathrm{r}}{(1+\mathrm{r})} = -\mathrm{A} \times \mathrm{GAP} \times \frac{\Delta\mathrm{r}}{(1+\mathrm{r})} = -\mathrm{A} \times \mathrm{GAP}^* \times \Delta\mathrm{r} \tag{7}$$

ΔE represents changes in net assets.

From formula (7), it's easy to know that, when the duration gap(GAP) is positive, the change of net assets is inversely proportional to the change of interest rate; while when the duration gap is negative, the change of net assets is directly proportional to the change of interest rate. Moreover, the smaller the absolute value of the GPA, the smaller the change in net assets, that is, the smaller the risk that banks will face.

Table 3: Duration gap and interest rate change's influence on commercial bank's assets and liabilities and their net worth

Duration gap	Interest rate	Change in asset value	Change in liability value	Change in net asset value
Positive GAP	↑	↓	↓	↓
	↓	↑	↑	↑
Negative GAP	↑	↓	↓	↑
	↓	↑	↑	↓
Zero GAP	↑	↓	↓	→
	↓	↑	↑	→

The use of duration in commercial bank interest rate risk management is primarily seen in interest rate risk measurement and control. On the one hand, commercial bank managers calculate the GAP, the amount of assets and the change of rate to measure changes in net assets, or equity, and consequently the amount to which interest rate swings influence them. On the other hand, the risk controller can use the immune strategy, which is a conservative way to avoid interest rate risk. Its fundamental goal is to align the bank's assets and liabilities so that the long-term gap is zero, making the bank immune to interest rate changes.

3.IMMUNIZATION STRATEGY

3.1. What is immunization strategy?

Immunization strategy is widely used in the portfolio management of bond market and it is a kind of passive strategy that bond portfolio managers do not actively seek the possibility of trading in order to overcome the market. It is based on the assumption that the bond market is semi-efficient and that the current price of the bond accurately reflects all publicly available information. The immunization strategy can protect the bond portfolio from the loss caused by the change of interest rate risk.

3.2. The application of immunization strategy

3.2.1. Some current research results

This paper chooses six representative academic articles in the past 60 years (from 1970s to 2020s) and it find some useful conclusions to help us have a deeper understanding of the immunization strategy.

Table 4: Some relevant conclusions

Author	Academic field	Conclusion/Value
Bierwag, G. O. Khang, C. (1979)	Finance& accounting and mathematics	An immunized portfolio of coupon bonds stochastic-ally dominates a portfolio of pure discount bonds (zero coupon bonds) the return on which has a zero variance [4].
Prisman, E. Z. (1986)	Banking and finance	This paper is shown that if a world with taxes, the immunization strategy needs the portfolio to satisfy a constraint about the bond to be included, in addition to the duration constraint [5].
Prisman, E. Z., & Tian, Y. (1994)	Finance and quantitative analysis & tax revenue	The paper study the tax effects on immunization strategies and it shows that the mistake caused by ignoring tax is enormous [6].
Marida Bertocchi Rosella Giacometti Stavros A. Zenios(2005)	Bond market and risk analysis and portfolio immunity	Through developing a multi-factor model to analyze the factors which can influence the changes in the term structure of corporate bonds.
Verma, N. K., & Agarwal, D. (2011)	Bond immunization and investment and debt securities	Because there is a trade-off between immunization risk and diversification risk, so this paper presents a mix strategy using: M-absolute duration and fisher-Well duration measure which can be more effective to test corporate bond [7].
Zaremba, L. S., & Rządkowski, G. (2016)	Bond portfolio and investment	In this paper we identify those shifts (continuous functions) of the term structure of interest rates, against which a given bond portfolio (BP) is immunized [10].
Vukovic, D. B. Maiti, M. Kochetkov, D. Bystryakov, A(2020)	Portfolio investment and bond immune	The statistics of municipal bond market shows that both municipal general and revenue bonds had stable and goog level of fields to maturity in the past ten years [8].

3.2.2. Some trend analysis of immunization in the bond market

Through the six representative research it easy to see that the immunization strategy has been widely used in the bond market and enterprise's investment. In this part, the research will pay more attention to the research between 1970s-2020s. First, the immunization strategy is mostly used in the finance industry, bond market and commercial bank and it is hardly used by oneself. In many research examples, the immunization strategy is always combined with other theory such as duration theory and some mathematical theory and so on. Besides, the effectiveness of immunization is also an essential thing to talk about. After reading plenty relevant academic articles, there is a fact that immunization strategy is really valuable in risk management and confirming an investment portfolio.

3.2.3. Immunization theory's application in portfolio investment

Immunization strategy also impact a manager's decision when he chooses a portfolio. When managers choose their portfolios for analysis, the Macaulay duration equals to the maturity of their liabilities is often introduced as a criterion for choosing their portfolios (Arnold and North, 2005). The objective of establishing immune bond portfolio is to find a bond portfolio with Macaulay duration equal to the maturity of its liabilities.

The key problem is to determine the investment proportion of various bonds in the bond portfolio.

The reference formula:

$$D_p = \sum_{i=1}^{n} w_i D_i \quad (8)$$

And this is also an appropriate example to show that there is a combination use between immunization and duration theory.

【Example】A fund management company has set up an insurance fund whose liabilities are to pay the beneficiary 3 million yuan a year and never end. Fund managers plan to meet this requirement by creating a portfolio of bonds with a yield-to-maturity of 15 per cent on all bonds. If the bond portfolio consists of Bond A and bond B, Bond A has a coupon of 10% , a maturity of 5 years, and an annual interest payment; Bond B carries a coupon of 8% , has a maturity of 20 years and is payable annually. So what are the proportion of each bond in order to fully immunize against debt?

Table 5: Immunization strategy of bond portfolio management

The debt data		Basic bond data		
Annual payments(ten thousands)	300	Bond	A	B
		Coupon rate	10%	8%
Yield to maturity	15%	Years	5	20
		Number of interest payments	1	1

Table 6: Calculation process and results

Debt	Bond			
Present value:300/15%=2000		A	B	Portfolio
	Macaulay duration	4.08	7.85	7.67
Macaulay duration(year):(1+15%)/15%=7.67	Investment proportion	4.81%	95.19%	100%

Through analyzing two tables, the most suitable investment portfolio is 4.81% of bond A and 95.19% of bond B. Firstly, in table 5, Macaulay duration of debts are calculated which is about 5 years. And in table 6 Macaulay duration of A and B are also calculated which are 4.08 years and 7.85 years. The key point of investment portfolio is making Macaulay duration equal to the maturity of its liabilities. So such a bond portfolio would meet the demand for an indefinite annual payment of $3million to pension beneficiaries.

4.CONCLUSION

This paper mainly studies the development of duration theory and immune theory, their application fields, and their important role in the bond market. Generally speaking, in the aspect of theoretical research, this paper finds that duration theory and immune theory are inseparable, especially have a great advantage in the interest rate risk management and the determination of the optimal portfolio area [3]. So far, immune duration theory has been active in most financial fields, and it has been developed into a multi-dimensional duration model after a long period of research. In the comparative study, it selected five models and theories based on four important hypotheses of Macaulay duration, and found that they gradually weakened the limitation of Macaulay duration, and continuously improve the accuracy of calculation and broaden the scope of application. As far as the application of the two theories is concerned, the theory of protracted immunity is still only more widely applied in the western developed countries instead of the whole world. Because of the insufficient time for the study of Macaulay duration theory, at present, duration theory is only used as a financial tool to measure interest rate risk, but it has no further development in other financial fields. We hope that in the future through the development of duration and immune theory, the interest risk rate can be measured more precisely. Until now, the calculation cost of current duration models and theories is still really high, so this paper suggests future researchers to find out an accurate and efficient method to calculate. In the future, the research predicts that the theory of duration immunity will have a better application prospect, especially with the development of modern computer technology, the calculation of duration will be more convenient, and the results will be more accurate. Although the theory of duration immunity has some limitations and deficiencies, people cannot deny its significance in the interest rate risk management of commercial banks.

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