Approximate Analytical Method of Maximum Likelihood
Estimation of Weibull Distribution

TIAN Ye*, HE Heping and ZHANG Yu

College of Economics and Management, China Jiliang University, Hangzhou 310018, China
**tianye9802272021@163.com, 15597396680@163.com*
**Corresponding author*
**Email address of correspondence author*

Abstract
Weibull distribution is commonly applied in survival analysis and reliability analysis, whose shape and scale parameters are most commonly estimated by maximum likelihood estimation and related numerical methods. In this paper, by introducing an artificial parameter and then using the perturbative method, for the first time in the statistics literature, we conveniently obtain approximate analytical formulas of maximum likelihood estimates of the parameters in Weibull distributions, which are important complements to those tedious and unreliable numerical methods. Monte-Carlo simulations show that the approximate analytical method proposed in this paper is fairly feasible and accurate. Using the similar method, we can also obtain the approximate analytical formulas of maximum likelihood estimates in many other statistical problems.

Keywords: *Approximate Analytical Method; Perturbative Method; Artificial Parameter; Maximum Likelihood Estimation*

1.INTRODUCTION

Weibull distribution can well fit all kinds of life data, and describe the failure process and life characteristics of the research object more truly; hence, it plays a very important role in the survival and reliability analysis. The most prominent feature of Weibull distribution is that its shape parameter can take different values, which can describe different failure mechanisms of the research object. The scale parameter in the Weibull distribution can stretch the density function after determining its density function form, so that the failure rate of the research object can be described by coupling. With the development of the times, the Weibull distribution has gradually evolved into various improved forms, such as the Weibull mixed model, the Weibull piecewise model, and the Weibull regression model. However, as the most basic form, the two-parameters Weibull distribution still plays a central role in the statistical modelling of survival analysis and reliability analysis.

Due to the importance and great practicability of Weibull distribution, there is much literature at home and abroad to study. [13] gave the fitting test method of two-parameters Weibull distribution under the condition of small sample fixed number censoring. [12] estimated the parameters of two-parameters Weibull distribution by two-logarithmic transformation and the least square method. Under the condition of a complete sample, [5] summarized various estimation methods of two-parameters Weibull distribution. [14] studied the parameter evaluation of timing censoring test under two-parameters Weibull distribution. [1] introduced a graphic method to solve the maximum likelihood estimation of two-parameters Weibull distribution, and then used this method to prove the existence and uniqueness of the maximum likelihood estimation. In the case of random censoring, [11] proposed an EM algorithm method to estimate the parameters of two-parameters Weibull distribution. Based on the algorithm obtained from the allowable value analysis of new composite material, [7] proposed a simple and effective iterative method for the maximum likelihood estimator of parameters in two-parameters Weibull distribution. In the case of zero failure test data, [8] gave the modified maximum likelihood estimation method of Weibull

G. Vilas Bhau et al. (Eds.): MSEA 2022, ACSR 101, pp. 674–680, 2023.
https://doi.org/10.2991/978-94-6463-042-8_97

distribution shape parameter and its solution equation. [3] gave the approximate maximum likelihood estimation of parameters of two-parameters Weibull distribution products based on grouping data. [4] studied the parameter approximate maximum likelihood estimation method of two-parameters Weibull distribution under the condition of increasing the type-II censored life test step by step, and considered the two expansion methods of the first-order Taylor expansion of the function involved in this method. Based on parameter estimation of the least square method, [6] proposed the maximum likelihood optimization method for the distribution parameter point estimation and the distribution parameter interval estimation of Weibull distribution life-span products, so as to solve the problem of fast searching the transcendental equation for solving the maximum likelihood estimation of parameters in the real number range. [2] found a small parameter when proving the existence and uniqueness of the maximum likelihood estimator of two-parameters Weibull distribution, and then obtained the approximate analytical solution of the maximum likelihood estimator by using the small parameter perturbation method. In this paper, only a little literature on the large literature of Weibull distribution is listed.

From the above literature on Weibull distribution and other related literature, it can be seen that the most important method of parameter estimation in Weibull distribution is the maximum likelihood method, and various cumbersome and unreliable numerical methods must be used in the process of solving. Although [2] gives the approximate analytical solution of the maximum likelihood estimator using the small parameter perturbation method, it uses a very special small parameter, so it is difficult to apply to other statistical problems. The perturbation method is an approximate analytical method originating from mechanics, and has been successfully applied in mechanics and some engineering fields. However, the authors found that only [2] has used the perturbation method in statistics so far. For the perturbation method and the artificial parameter perturbation method to be used in this paper, the specific content is referred to [10] and [9].

In this paper, by introducing an artificial parameter and using the perturbation method, the approximate analytical expressions of maximum likelihood estimations of parameters in two-parameters Weibull distribution are found, which provide an important supplement and guarantee for those cumbersome and unreliable numerical methods. At the same time, since the obtained approximate analytical expressions directly indicate the relationship between the approximate maximum likelihood estimators and the data, it has a certain theoretical significance. In this paper, a large number of Monte-Carlo simulations are carried out. These simulation results show that the approximate analytical method proposed in this paper has high accuracy. More importantly, the method proposed in this paper for solving the approximate analytical solution of the maximum likelihood estimation is generality. Similar to the method in this paper, the approximate analytical expressions of maximum likelihood estimates in many other statistical problems can also be obtained.

2.THE CONSTRUCTION OF WEIBULL DISTRIBUTION LIKELIHOOD EQUATION

Assuming that the independent and identically distributed lifetime data $t_i, i = 1, 2, \cdots n$ obey the Weibull distribution $W(\alpha, \beta)$ with two parameters, the probability density function $f(t;\alpha,\beta)$ is :

$$f(t;\alpha,\beta) = \frac{\beta t^{\beta-1}}{\alpha^{\beta}} \exp[-(\frac{t}{\alpha})^{\beta}] \tag{1}$$

where α and β are scale parameter and shape parameter, respectively.

We can get the following likelihood function:

$$L(\alpha,\beta) = (\frac{\beta}{\alpha^{\beta}})^{n} [\prod_{i=1}^{n} t_i^{\beta-1}] \exp[-\sum_{i=1}^{n} (t_i / \alpha)^{\beta} \tag{2}$$

The logarithmic likelihood function is :

$$l(\alpha,\beta) = n \ln \beta - n\beta \ln \alpha + \beta \sum_{i=1}^{n} \ln t_i - \sum_{i=1}^{n} \ln t_i - \frac{\sum_{i=1}^{n} t_i^{\beta}}{\alpha^{\beta}} \tag{3}$$

According to the maximum likelihood method, the partial derivatives of $l(\alpha,\beta)$ about α and β are obtained respectively, and the likelihood equations are obtained by making them equal to 0. According to $\partial l(\alpha,\beta)/\partial\alpha = 0$, we can get the maximum likelihood estimates of scale and shape parameters satisfying the following relation :

$$\alpha = \alpha(\beta) = (\frac{1}{n} \sum_{i=1}^{n} t_i^{\beta})^{1/\beta} \tag{4}$$

Substituting Equation (4) into $\partial l(\alpha,\beta)/\partial\beta = 0$, we obtain the maximum likelihood estimation of shape parameter β after simplification and consolidation, which satisfies Equation (5) :

$$\frac{\sum_{i=1}^{n} t_i^{\beta} \ln t_i}{\sum_{i=1}^{n} t_i^{\beta}} - \frac{1}{n} \sum_{i=1}^{n} \ln t_i - \frac{1}{\beta} = 0 \tag{5}$$

Since Equation (5) is a transcendental equation, it is impossible to obtain the specific expression of the solution of the equation. Usually, it can only be solved by various tedious and unreliable numerical methods. The main purpose of this paper is to construct a general perturbation equation by introducing an artificial parameter, obtain the approximate analytical expression of the maximum likelihood estimation of the shape parameter β, and then obtain the approximate analytical expression of the maximum likelihood estimation of α.

3.THE APPROXIMATE ANALYTICAL EXPRESSION OF THE MAXIMUM LIKELIHOOD ESTIMATION OF SHAPE PARAMETER

First of all, using the idea in the literature of [2], we carry out Taylor expansion of each item in the likelihood equation (6) at $\beta=0$; secondly, the general perturbation equation (16) is constructed by introducing the artificial parameter into the expanded likelihood equation (15); thirdly, the solution of Equation (16) is expanded into the power series form of the artificial parameter, and the expansion is substituted into the perturbation equation (16); then, by setting the coefficient of each power front of the artificial parameter to zero, we can obtain a system of equations; then, by solving the equations, we can obtain the power series expansion of the solution of Equation (16) with respect to the artificial parameter; finally, by taking the value of the artificial parameter as 1 and intercepting the first three terms of the power series expansion, we can obtain the approximate analytical expression of the maximum likelihood estimation of the shape parameter β, thus obtaining the approximate analytical solution of Equation (5).

Equation (5) can be rewritten as follows :

$$\beta\sum_{i=1}^{n}\left(\frac{t_i^{\beta}}{\sum_{i=1}^{n}t_i^{\beta}}-\frac{1}{n}\right)\ln t_i-1=0 \tag{6}$$

The left side of Equation (6) needs to use the relevant knowledge of Taylor series expansion. Firstly, t_i^{β} is expanded at $\beta=0$, and we can give the following expansions of t_i^{β} and $\sum_{i=1}^{n}t_i^{\beta}$ in turn :

$$t_i^{\beta}=\sum_{k=0}^{\infty}\frac{(\beta\ln t_i)^k}{k!} \tag{7}$$

$$\sum_{i=1}^{n}t_i^{\beta}=n+\sum_{j=1}^{\infty}\frac{\beta^j}{j!}\sum_{i=1}^{n}(\ln t_i)^j \tag{8}$$

Then, according to the Taylor expansion formula of $1/(n+x)$ at $x=0$, we can expand $1/\sum_{i=1}^{n}t_i^{\beta}$ into the following form :

$$\begin{aligned}\frac{1}{\sum_{i=1}^{n}t_i^{\beta}}=&\frac{1}{n}-\frac{1}{n^2}[\beta\sum_{i=1}^{n}\ln t_i+\frac{1}{2}\beta^2\sum_{i=1}^{n}(\ln t_i)^2\\&+\frac{1}{6}\beta^3\sum_{i=1}^{n}(\ln t_i)^3+\cdots]+\frac{1}{n^3}[\beta\sum_{i=1}^{n}\ln\\&+\frac{1}{2}\beta^2\sum_{i=1}^{n}(\ln t_i)^2+\frac{1}{6}\beta^3\sum_{i=1}^{n}(\ln t_i)^3\\&+\cdots]^2-\frac{1}{n^4}[\beta\sum_{i=1}^{n}\ln t_i\\&+\frac{1}{2}\beta^2\sum_{i=1}^{n}(\ln t_i)^2+\frac{1}{6}\beta^3\sum_{i=1}^{n}(\ln t_i)^3\\&+\cdots]^3+\cdots\end{aligned} \tag{9}$$

By multiplying formula (7) with formula (9), the following expansion is obtained:

$$\begin{aligned}\frac{t_i^{\beta}}{\sum_{i=1}^{n}t_i^{\beta}}=&\frac{1}{n}-\frac{1}{n^2}\beta\sum_{i=1}^{n}\ln t_i+\frac{1}{n}\beta\ln t_i\\&-\frac{1}{2n^2}\beta^2\sum_{i=1}^{n}(\ln t_i)^2\\&+\frac{1}{n^3}\beta^2(\sum_{i=1}^{n}\ln t_i)^2\\&-\frac{1}{n^2}\beta^2\ln t_i\sum_{i=1}^{n}\ln t_i\\&+\frac{1}{2n}\beta^2(\ln t_i)^2\\&-\frac{1}{6n^2}\beta^3\sum_{i=1}^{n}(\ln t_i)^3\\&+\frac{1}{n^3}\beta^3\sum_{i=1}^{n}\ln t_i\sum_{i=1}^{n}(\ln t_i)^2\\&-\frac{1}{n^4}\beta^3(\sum_{i=1}^{n}\ln t_i)^3\\&-\frac{1}{2n^2}\beta^3\ln t_i\sum_{i=1}^{n}(\ln t_i)^2\\&+\frac{1}{n^3}\beta^3\ln t_i(\sum_{i=1}^{n}\ln t_i)^2\\&-\frac{1}{2n^2}\beta^3(\ln t_i)^2\sum_{i=1}^{n}\ln t_i\\&+\frac{1}{6n}\beta^3(\ln t_i)^3+O(\beta^4)\end{aligned} \tag{10}$$

where $O(\beta^4)$ denotes a quantity of the same order as β^4. From formula (10), we can get the following formula (11) :

$$\sum_{i=1}^{n}\left(\frac{t_i^{\beta}}{\sum_{i=1}^{n}t_i^{\beta}}-\frac{1}{n}\right)\ln t_i=-\frac{1}{n^2}\beta\left(\sum_{i=1}^{n}\ln t_i\right)^2+\frac{1}{n}\beta\sum_{i=1}^{n}(\ln t_i)^2-\frac{1}{2n^2}\beta^2\sum_{i=1}^{n}(\ln t_i)^2\sum_{i=1}^{n}\ln t_i+\frac{1}{n^3}\beta^2\left(\sum_{i=1}^{n}\ln t_i\right)^3-\frac{1}{n^2}\beta^2\sum_{i=1}^{n}\ln t_i\sum_{i=1}^{n}(\ln t_i)^2+\frac{1}{2n}\beta^2\left(\sum_{i=1}^{n}\ln t_i\right)^3-\frac{1}{6n^2}\beta^3\sum_{i=1}^{n}\ln t_i\sum_{i=1}^{n}(\ln t_i)^3+\frac{1}{n^3}\beta^3\sum_{i=1}^{n}(\ln t_i)^2\left(\sum_{i=1}^{n}\ln t_i\right)^2-\frac{1}{n^4}\beta^3\left(\sum_{i=1}^{n}\ln t_i\right)^4-\frac{1}{2n^2}\beta^3\left(\sum_{i=1}^{n}(\ln t_i)^2\right)^2+\frac{1}{n^3}\beta^3\left(\sum_{i=1}^{n}\ln t_i\right)^2\sum_{i=1}^{n}(\ln t_i)^2-\frac{1}{2n^2}\beta^3\sum_{i=1}^{n}\ln t_i\sum_{i=1}^{n}(\ln t_i)^3+\frac{1}{6n}\beta^3\sum_{i=1}^{n}(\ln t_i)^4+O(\beta^4) \quad (11)$$

Let

$$u_0\equiv-\frac{1}{n^2}\left(\sum_{i=1}^{n}\ln t_i\right)^2+\frac{1}{n}\sum_{i=1}^{n}(\ln t_i)^2 \quad (12)$$

$$u_1\equiv-\frac{1}{2n^2}\sum_{i=1}^{n}(\ln t_i)^2\sum_{i=1}^{n}\ln t_i+\frac{1}{n^3}\left(\sum_{i=1}^{n}\ln t_i\right)^3-\frac{1}{n^2}\sum_{i=1}^{n}\ln t_i\sum_{i=1}^{n}(\ln t_i)^2+\frac{1}{2n}\sum_{i=1}^{n}(\ln t_i)^3 \quad (13)$$

$$u_2\equiv-\frac{2}{3n^2}\sum_{i=1}^{n}(\ln t_i)^3\sum_{i=1}^{n}\ln t_i+\frac{2}{n^3}\left(\sum_{i=1}^{n}\ln t_i\right)^2\sum_{i=1}^{n}(\ln t_i)^2-\frac{1}{n^4}\left(\sum_{i=1}^{n}\ln t_i\right)^4+\frac{1}{6n}\sum_{i=1}^{n}(\ln t_i)^4-\frac{1}{2n^2}\left(\sum_{i=1}^{n}(\ln t_i)^2\right)^2 \quad (14)$$

Equation (6) can be rewritten as follows :

$$-1+\beta^2u_0+\beta^3u_1+\beta^4u_2+\cdots=0 \quad (15)$$

Considering the relationship between the Weibull distribution and the extreme value distribution, the artificial parameter ε is introduced in Equation (15) in the following way, we construct the following perturbation equation:

$$-1+\beta^2u_0+\beta^3u_1\varepsilon+\beta^4u_2\varepsilon^2+\cdots=0 \quad (16)$$

Using the idea of perturbation method, the solution β of Equation (16) is expressed as the following power series form of parameter ε :

$$\beta=a_0+a_1\varepsilon+a_2\varepsilon^2+a_3\varepsilon^3+\cdots \quad (17)$$

Replace the formula (17) with the equation (16) and retain it to term ε^2, we can get the following result :

$$-1+a_0^2u_0+(2a_0a_1u_0+a_0^3u_1)\varepsilon+(2a_0a_2u_0+a_1^2u_0+3a_0^2a_1u_1+a_0^4u_2)\varepsilon^2=0 \quad (18)$$

According to the perturbation method, the coefficient of each power front of the artificial parameter ε on the left side of Equation (18) is set to 0, and the following equations are obtained :

$$\begin{cases}-1+a_0^2u_0=0\\ 2a_0a_1u_0+a_0^3u_1=0\\ 2a_0a_2u_0+a_1^2u_0+3a_0^2a_1u_1+a_0^4u_2=0\end{cases} \quad (19)$$

By solving the equations (19), the following solution can be obtained :

$$\begin{cases}a_0=1\big/\sqrt{u_0}\\ a_1=-a_0^4u_1\big/2\\ a_2=\left(\frac{5}{4}u_1^2a_0^7-a_0^5u_2\right)\Big/2\end{cases} \quad (20)$$

So we get the following power series expansion of the solution β of Equation (16) with respect to ε :

$$\beta=\frac{1}{\sqrt{u_0}}-\frac{a_0^4u_1}{2}\varepsilon+\frac{\frac{5}{4}u_1^2a_0^7-a_0^5u_2}{2}\varepsilon^2+O(\quad (21)$$

Let $\varepsilon=1$, the approximate analytical expression $\tilde{\beta}$ of the maximum likelihood estimation of β can be obtained by taking the first three terms of expression (21) as follows :

$$\tilde{\beta}\equiv\frac{1}{\sqrt{u_0}}-\frac{a_0^4u_1}{2}+\frac{\frac{5}{4}u_1^2a_0^7-a_0^5u_2}{2} \quad (22)$$

Then, by substituting formula (22) into formula (4), the approximate analytical expression $\tilde{\alpha}$ of the maximum likelihood estimation of the scale parameter α can be obtained.

As for the accuracy of the approximate analytical expression (22) of maximum likelihood estimation, we can make the following simple theoretical analysis. If the random variable T obeys two-parameters Weibull distribution $W(\alpha,\beta)$, where α and β are scale and shape parameters, then the random variable $X \equiv \ln T$ obeys extreme value distribution $E_v(\gamma,\eta)$, where $\gamma = \ln\alpha$, $\eta = 1/\beta$. It is easy to know that the variance of the random variable X is $Var(X) = \pi^2\eta^2/6 = \pi^2/(6\beta^2)$, so the shape parameter β can be expressed as $\beta = \pi/\sqrt{6Var(X)} = \pi/(\sqrt{6}\sqrt{Var(X)})$ by the variance of the corresponding extreme value distribution. According to the large number theorem, it is easy to know that a_0 is a good estimator of $1/\sqrt{Var(X)}$, so the value of a_0 is not far from the real value of the shape parameter β, and then through the adjustment of a_1 and a_2, the approximate analytical expression $\tilde{\beta}$ of the maximum likelihood estimation of the shape parameter β can be quite accurate.

4.NUMERICAL SIMULATION

In this section, we use the software RStudio to do the Monte-Carlo simulation to verify the reliability and accuracy of the approximate analytical expression of maximum likelihood estimation. Firstly, we use the software RStudio to generate the random numbers of Weibull distribution, which obey the preset parameter values. Then, the generated random numbers are substituted into the approximate analytical expressions of maximum likelihood estimates to obtain the approximate maximum likelihood estimators of the shape parameter and the scale parameter. Because the random numbers generated in the simulation are pseudo-random numbers, we simulate 5000 times for different sample sizes of each set of real parameter values. Approximate maximum likelihood estimators of the shape parameter and the scale parameter are calculated for each simulation. Finally, the average values of 5000 approximate maximum likelihood estimators are calculated respectively. By comparing these average values with the corresponding real parameter values, we can verify the reliability and applicability of the approximate analytical expressions of maximum likelihood estimates. Since Equation (5) in this paper is a strong nonlinear problem, the true value of maximum likelihood estimation of shape parameter β cannot be found. However, we can use the standard function optim in R language to obtain the maximum likelihood estimation $\hat{\beta}_{mle}$ and $\hat{\alpha}_{mle}$ of the parameters of the corresponding Weibull distribution as the standard values and compare them with the approximate analytical solutions.

Considering the Weibull distributions of the following five sets of real parameter values:
$\beta_{true} = 1, \alpha_{true} = 2; \beta_{true} = 2, \alpha_{true} = 2;$
$\beta_{true} = 6, \alpha_{true} = 3; \beta_{true} = 0.1, \alpha_{true} = 0.8;$
$\beta_{true} = 0.5, \alpha_{true} = 1,$

the Monte-Carlo simulation results are as follows:

Table 1: $(\beta_{true},\alpha_{true}) = (1,2)$: The simulation results of approximate analytical method and optim function.

n	100	1000	10000
$\tilde{\beta}$	1.036285	1.00887	1.004516
$\tilde{\alpha}$	1.954278	1.943475	1.971819
$\hat{\beta}_{mle}$	1.031884	0.9830727	0.9947199
$\hat{\alpha}_{mle}$	2.210849	1.9190384	2.0044835

Table 2: $(\beta_{true},\alpha_{true}) = (2,2)$: The simulation results of approximate analytical method and optim function.

n	100	1000	10000
$\tilde{\beta}$	2.073536	2.018431	2.008921
$\tilde{\alpha}$	1.87147	1.945517	1.997938
$\hat{\beta}_{mle}$	2.387537	1.958556	1.994001
$\hat{\alpha}_{mle}$	2.044412	1.990811	1.984543

Table 3: $(\beta_{true},\alpha_{true}) = (6,3)$: The simulation results of approximate analytical method and optim function.

n	100	1000	10000
$\tilde{\beta}$	6.210159	6.055795	6.026922
$\tilde{\alpha}$	3.003619	3.001516	3.001014
$\hat{\beta}_{mle}$	6.415885	5.832113	6.022557
$\hat{\alpha}_{mle}$	3.097322	2.964182	2.997604

Table 4: $(\beta_{true},\alpha_{true}) = (0.1,0.8)$: The simulation results of approximate analytical method and optim function.

n	100	1000	10000
$\tilde{\beta}$	0.1037109	0.1008289	0.1004669
$\tilde{\alpha}$	1.4647502	0.8735634	0.8200441
$\hat{\beta}_{mle}$	0.09670285	0.1005416	0.09973063
$\hat{\alpha}_{mle}$	0.30341086	0.7347735	0.80063109

Table 5: $(\beta_{true},\alpha_{true}) = (0.5,1)$: The simulation results of approximate analytical method and optim function.

n	100	1000	10000
$\tilde{\beta}$	0.5177883	0.5044067	0.5021977
$\tilde{\alpha}$	1.0372842	1.0089008	1.0035362
$\hat{\beta}_{mle}$	0.5083463	0.4951559	0.5016259
$\hat{\alpha}_{mle}$	0.8263514	0.9357381	1.0116090

As can be seen from the above tables, with the increase of sample size n, the approximate analytical solutions of parameter maximum likelihood estimates are closer to the real values; when the sample size reaches 10000, the results obtained from the approximate analytical expressions are very close to the real values of the parameters and the maximum likelihood estimates obtained from the optim function, which indicates the feasibility and accuracy of the proposed method. At the same time, it is also found that sometimes the results obtained by the approximate analytical expressions are closer to the real values of the parameters, and sometimes the maximum likelihood estimates obtained from the optim function are closer to the real values of the parameters. In addition, when the real shape parameter takes different values, the accuracy of the approximate analytical expression of the maximum likelihood estimation is different. This is mainly because the value of the real shape parameter determines the shape of the Weibull distribution density function. The size of the real shape parameter is one of the most important factors affecting the accuracy of the parameter estimation method of the Weibull distribution. Overall, the results in the above tables clearly show that the method proposed in this paper for solving the approximate analytical solutions of the maximum likelihood estimates is quite accurate.

5.CONCLUSIONS

By introducing an artificial parameter and using the idea of perturbation method, this paper proposes a general method to obtain the approximate analytical formulas of maximum likelihood estimates of Weibull distribution parameters; these approximate analytical formulas are not only convenient for computer calculation, but also for exploring the rules in practical application problems. In this paper, some Monte-Carlo simulations are further carried out. These Monte-Carlo simulation results show that the approximate analytical method proposed in this paper has high reliability and accuracy for Weibull maximum likelihood estimation.

The authors have recently extended the approximate analytical method proposed in this paper to the maximum likelihood estimation of the Weibull Accelerated Failure Time Model. Some results have been obtained and will be published soon.

Since this paper presents a general method to obtain an approximate analytical expression of maximum likelihood estimation, the authors believe that this method is naturally applicable to parameter estimation of other distribution models of life data, such as log-normal distribution, gamma distribution, generalized gamma distribution, log-logistic distribution. The authors believe that the approximate analytical method is also applicable to the life data distribution model and the corresponding regression model in the case of censoring, such as the Weibull regression model. Moreover, the authors believe that the approximate analytical method should be applicable to the parameter estimation of the models such as the curve exponential family, the generalized linear model and the Cox proportional hazard model. The approximate analytical method proposed in this paper applied to the parameter estimation of these models mentioned above is not only an interesting exploration, but also has considerable practical and theoretical significance.

REFERENCES

[1] Balakrishnan, N. & M. Kateri (2008). On the maximum likelihood estimation of parameters of Weibull distribution based on complete and censored data. J. Statistics and Probability Letters. 78(17): 2971-2975.

[2] Coria, V. H. et al. (2016). Perturbative method for maximum likelihood estimation of the Weibull distribution parameters. J. SpringerPlus. 5(1) : 1-16.

[3] Gu Yiming. (2001). The approximate likelihood estimate of multicomponent data. J. Journal of Shanghai Normal University(Natural Sciences). 30(2).

[4] Gu Beiqing & Xu Xiaoling & Wang Ronghua. (2019). Approximate maximum likelihood estimations of two-parameter Lognormal distribution and Weibull distribution. J. Statistics & Decision. 35(03): 34-39.

[5] Jin Liangqiong. (2010). Two parameter estimation for Weibull distribution. D. Yunnan University.

[6] Jin Xing & Chen Jingpeng & Wen Ming. et al. (2003). Optimized maximum likelihood estimation method of parameters estimation for Weibull distributed item. J. Journal of Equipment Academy. (05): 46-48.

[7] Li Jian & Zhang Qunhui. (1994). An iterative method for the maximum likelihood estimate of the two parameters in the two-parameters Weibull distribution. J. Journal of Mechanical Strength. (03): 67-68.

[8] Li Baosheng & Tong Hongyu. (2017). The estimation of shape parameter m in Weibull distribution on zero failure products. J. Structure & Environment Engineering. 44(05): 47-51.

[9] Lyapunov, A. M., 1992. General problem of the stability of motion, Taylor & Francis. London, English translation.

[10] Nayfeh, A. H., 1981. Introduction to perturbation techniques, John Wiley & Sons. New York.

[11] Nandi, S. & I. Dewan (2010). An EM algorithm for estimating the parameters of bivariate Weibull distribution under random censoring. J.

Computational Statistics and Data Analysis. 54(6): 1559-1569.

[12] Wang Wenyue & Cui Jie. (2019). Weibull distribution analysis and its application in product life analysis. J. Electronic Product Reliability and Environmental Testing. 37(05): 16-26.

[13] Xu Xiaoling & Wang Ronghua & Gu Beiqing (2017). Fitting test of Weibull and Lognormal distribution for small sample under type-II censored situation. J. Journal of Applied Statistics and Management. 36(05): 810-820.

[14] Zeng Guohuan. (2018). Methods of parameter estimation of two-parameter Weibull distribution for type-I censoring test. D. School of Mechanical and Electrical Engineering.