Research on Macaulay Duration's Role of Immunization to Interest Rate Risk

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Abstract
This article explored basic features of the Macaulay duration in the context of its study. It also examines how Macaulay durations are immune to interest rate fluctuations using comparative examples and the theme of combining financial products with maturities. Combining the abstract Macaulay duration with practical examples aims to provide a better understanding of the specific uses of Macaulay duration in reducing risk and can better inform the reader of the logic of financial products containing duration.

Keywords: *Macaulay duration, limitation of Macaulay durations, Immunity*

1.INTRODUCTION

In 1983, Macaulay combined the duration effect with coupons to describe the fluctuation of bond prices by dividing the present value of each bond cash flow by the bond price to obtain the weight of each cash payment, and multiplying the timing of each cash flow by the corresponding weight to sum up the duration of the entire bond. Paul Samuelson, John Skees and Reddington subsequently used duration as a measure of interest rate sensitivity of assets/liabilities, according to which when the duration of a counterparty's bond portfolio is equal to the holding period of the debt, the counterparty is "immune" in the short run, its total wealth is not affected by interest rate fluctuations in the short run. Nowadays, it is mainly used for risk management and portfolio sensitivity indicators, and this paper will investigate this property of duration immunity to interest rate fluctuations. Zeyuan Zhang's (2022) [1] study provides investors with a detailed introduction to Macaulay duration, affirming its usefulness, but also reminding them of its limitations and providing solutions. The study by Zeyuan Zhang examines the immunity of duration to interest rate fluctuations, which is now used as a risk management and portfolio sensitivity indicator. Zeyuan Zhang's study provides investors with a detailed introduction to Macaulay duration, affirming its usefulness, but also reminding them of its limitations and providing solutions. The study by Zeyuan Zhang points out the shortcomings of the Macaulay duration and modified duration concepts, but does not indicate how duration comes into play in financial products. However, duration is an academic concept that needs to be understood by purchasers of financial products because it incorporates the concept of duration. This paper will introduce the basic nature of Macaulay duration, explain how Macaulay duration is calculated, the relationship between bonds and duration, the application of duration and the limits of Macaulay duration, and examples in simple financial products in order to give the reader a more complete and comprehensive understanding of Macaulay duration. The purpose of this paper is to provide the reader with a more concrete and detailed understanding of duration as an abstract academic concept.

2.MACAULAY DURATION AND ITS APPLICATION

2.1. Concept of duration

F. R. Macaulay (1938) [2] developed the duration concept based on the maturity structure of bonds to measure the interest rate risk of bonds. Macaulay duration is calculated as a weighted average of the average maturity of a bond. It is a weighted average of the time, it will take for the bond to generate cash flows in the future, weighted by the present value of each period as a proportion of the bond price. According to H., Shirvani & B., Wilbratte (2002) [3] one manifestation of the concept's acceptance as a valuable tool by the investment community is found in the fact that investment companies now regularly compute and report the durations of bond

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mutual funds, especially of Treasury securities. Z. Leszek (2017) [4] pointed out that duration is not just a time concept, its true value is its ability to represent the sensitivity of bond prices to interest rate movements, and it is an essential tool to measure interest rate risk. The concept of Macaulay duration is a good approximate value for the sensitivity of the bond's price.

2.2. Calculation of Macaulay duration

Z. Bodie. (2012) [5] indicated that Macaulay duration is equal to the weighted average of the timing of each coupon or bond principal payment on the bond. G. O.Bierwag. (1978) [6] calculate Macaulay duration's measurement and Hicks' "average maturity" of a cash flow, in some occasion is used to make comparison of different assets which is an index figure.

Assume a T-year bond with a cash payment of

$CFt\ (1 \le t \le T)$ at time t, y equals to a yield to maturity and P represent a bond price. The weight is related to the cash flow occurring at CF_t (time t) and is expressed as (1):

$$wt = (CFt/[1+y]t)/P \quad (1)$$

The present value of the cash flows occurring at time t is represented by the right-hand numerator of the equat ion. The right-hand denominator represents the cash flo w's present value at time t. The denominator is equal to t he total value of all bond payments. Since the total cash flows discounted at the yield to maturity equal the bond price, these weights sum to 1.0. When these values are u sed to calculate a weighted average of all bond payments over time, the Macaulay duration formula is expressed a s follows (2):

$$\sum_{t=1}^{T} t * wt \quad (2)$$

According to Bierwag, G., Kaufman, G., & Khang, C (1978) [7], duration measures have been developed to: acquire a more meaningful indicator of the time characteristic of a particular payment stream, such as a bond, and to relate changes in interest rates to changes in the capital value of particular payment streams, such as bonds, so as to better understand the underlying mathematical relationships, and to develop portfolio strategies that can mitigate the risks associated with unexpected changes in interest rates.

In table 1, there will be 2 kinds of bonds' calculation (coupon bond and zero coupon bond):

Table 1 2 kinds of bonds' calculation

2 KINDS OF BONDS' CALCULATION	Time			Present value
	Year1	Year2	Year3	
COUPON BOUND	10	10	1010	-1000
ZERO COUPON BOND	0	0	1030	-1000
EXPLAIN	FACE VALUE BONDS PURCHASED BY PAYING $1000	INTEREST AT $10	INTEREST AT $10	RECEIVED A TOTAL OF $1,030 IN INTEREST AND PRINCIPAL

Macaulay duration for coupon bond:

$$time\ period * (interest\ in\ time\ preiod1 / (sum\ of\ total\ interest\ and\ principal)) +$$

$$time\ period2 * (interest\ in\ time\ preiod2 / (sum\ of\ total\ interest\ and\ principal)) +$$

$$time\ period3 * (interest\ in\ time\ preiod3 / (sum\ of\ total\ interest\ and\ principal))(3)$$

2.3. The connection between bond and duration

Duration is the weighted sum of the time. Duration takes for an investor to buy a bond and get all the principal and interest back.

According to G.O. Bierwag, George G. Kaufman & Alden Toevs (1978) [8], The duration of bonds is used to determine their cash flow characteristics and to approximate their price sensitivity. Additionally, it can develop bond portfolio strategies, particularly ones that try to immunize the interest risk.

The concept of duration in bond analysis has evolved beyond the concept of time and is now used by investors to quantify the sensitivity of bond price movements to interest rate changes more frequently, with some adjustment to allow for accurate quantification of the influence of interest rate changes on bond prices. The greater the corrected duration, the more susceptible the bond price is to yield changes. The duration's concept has been widely adopted, not only for individual bonds, but also for bond portfolios. A combination of a long-dated bond and a short-dated bond can result in a portfolio of medium-dated bonds, whereas investing more in a particular bond category can tilt the portfolio toward its duration. With a defined duration and flexible adjustment of the weighting of each bond class, the desired result can basically be achieved.

If we buy a bond with an annual coupon rate of 5%, a yield to maturity YTM of 10%, a maturity of three years and a maturity payment of $100 face value of the bond, what is the purchase price of the bond today? Coupon rate of the bond is 5%, so the coupon received each year is: coupon=100X5%=$5 This $5 is also equivalent to the interest that the bond will pay to the investor each year; but the bond has a yield to maturity of 10%, which you can interpret as the market rate of interest, so the cash flow from discounting the annual coupon back to today (time zero) is discounted first year coupon

$$5/(1+10\%) = \$4.5 \tag{4}$$

discounted second year coupon:

$$5/[(1+10\%)(1+10\%)] = \$4.13. \tag{5}$$

discounted third year coupon:

$$5/[(1+10\%)(1+10\%)(1+10\%)] = \$3.76 \tag{6}$$

At the end of the third year another $100 of the face value of the bond will be received, also discounted: discounted third year face value:

$$\frac{100}{[(1+10\%)(1+10\%)(1+10\%)]} = \$75.13 \tag{7}$$

So the total discounted amount for the third year is:

$$\$3.76 + 75.13 = \$78.89 \tag{8}$$

According to the formula for the bond price:

$$Present\ value = \frac{C}{1+Y} + \frac{C}{[(1+Y)(1+Y)]} + \frac{C+100}{[(1+Y)(1+Y)(1+Y)]}; \tag{9}$$

$$P = \$4.54 + 4.13 + 78.89 = \$87.56 \tag{10}$$

Therefore, the purchase price of the bond today is $87.56. At time zero, the purchase price of the bond and the issue price are the same price, so the issue price of the bond is also $87.56. Macaulay calculates the average maturity of a bond using a weighted average method, which is often referred to as the Macaulay duration. Macaulay derives the weighted average time to maturity by assigning different weights to different times of the bond, the weight being the ratio of discounted cash flow to the price of the bond. In the example above, the bond has a purchase price of $87.56 today and we can find the weight of the discounted cash flows to the purchase price for each year: weight of the first year cash flows

$$W1 = 4.54/87.56 = 5.19\% \tag{11}$$

weight of the second year cash flows

$$W1 = 4.13/87.56 = 4.72\% \tag{12}$$

weight of the third year cash flows

$$W1 = 78.89/\ 87.56 = 90.10\% \tag{13}$$

The bond has a total of three years to maturity, so we assign weights of 5.19%, 4.72% and 90.10% to the first, second and third years respectively, and then sum up:

$$Duration = 1*W1 + 2*W2 + 3*W3 = 1*5.19\% + 2*4.72\% + 3*90.10\% = 2.876\text{years} \tag{14}$$

Thus, the average time for the bond investor to recover all of the principal and interest is 2.8763 years. The average time to recover all of the principal and interest is 2.8763 years, indicating that the bond has a 2.8763-year duration.

2.4. Application of duration

Using the duration approximation to calculate bond price movements, there will be an effect on bond price due to the change of interest rates. Bond prices are more sensitive to changes in interest rates as the duration of the bond increases. It is possible to quantify the effect of interest rate changes on bond prices by using duration.

Calculation:

$$\frac{\Delta P}{P} = -\frac{D}{1+r} * \Delta r = -Dm * \Delta r \tag{15}$$

where Dm is the modified duration and $\triangle r$ is the amount of change in yield to maturity, which is equal to the new yield after the change minus the original yield and reflects the absolute magnitude of the changing in yield to maturity.

$\triangle P/P$ is ratio of changing in bond prices, which is a relative number. This formula demonstrates that the percent change in the price of a bond is equal to the product of the modified duration and the absolute value of the change in yield to the formula shows that the percentage change in bond prices is equal to the product of the absolute value of the change in yield to maturity. The larger modified duration is, the greater the impact of interest rate movements on bond prices. Therefore, modified duration is no doubt a composite reflection of the bond's exposure to interest rate movements. The negative sign in the formula indicates that bond price changes in the opposite direction to interest rate movements.

There are two methods described in International Fin ancial Reporting Standard (IFRS) 13 for adjusting the ex pected present value (EPV) for risk, under one of the two methods the expected cash inflows are discounted at the risk-free rate, plus a risk premium. EPV Method 2 (IFR S 13, B26) requires that the expected cash flow should b e discounted at the risk-free rate plus an addition for the risk premium. The risk adjustment in this method illumi nates the important role played by the Macaulay maturit y as opposed to the key role played by the project life.

3. Limitation of Macaulay duration

According to J.Nie, Z.Wu, S.Wang & Y.Chen (2021) [9]there is still some shortcomings in Macaulay duration, which will limits the accuracy of the tool.

Macaulay duration has its limitation. T. Robert Daigler & M., Smyser (1987) [10]by analyzing the effects of additive rate shocks to the term structure it is possible to determine the theoretical limitations of the Macaulay. To measure the interest rate risk, the model of Macaulay Duration is an important tool, but it is so simple and has its own assumptions so they limit its accuracy and it can not be used under all circumstance. W.Zuo. (2010) [11] pointed out the duration model has two assumptions, namely, that the interest rate levels of banks' assets and liabilities are equal and that the interest rates on commercial banks' assets and liabilities fluctuate at the same rate, which limit the usefulness and accuracy of the duration model

W. Zuo (2009) [12] analyze the four assumptions implicit in the Macaulay duration model have significant limitations in its practical application.

3.1. Price-to-yield curve

The price-to-yield curve is linear. It is considered as a linear relationship between price changes and yields. In reality, the true relationship is not the same as Macaulay duration's assumption. It is called convexity.

This non-linear feature is known as the convexity of the yield curve. Therefore, the linear relationship represented by the Macaulay duration can only be approximated when there is a small change in yield; if the change in yield is large, then Macaulay duration can not be used to estimate the change in price.

3.2. Term structure

The term structure of interest rates is flat. The same discount rate is used in the cash flow of Macaulay duration, it assumes that the term structure is flat. In reality, a flat term structure is very rare, as interest rate maturity structures are generally not flat and tend to have an upward non-linear shape.

3.3. Future cash flows

Macaulay duration assume the future cash flow does not change in response to changes in interest rate. However, for financial instruments with implied options, the future cash flows will change in response to interest rate fluctuations and their prices will change. The price of the instrument will change accordingly.

3.4. Yield curve

The yield curve is moving in parallel. The Macaulay duration only considers the case of parallel shifts in yields. In practice, due to the time factor, yields at different maturities respond differently to market influences. In practice, due to time factors, yields on bonds of different maturities respond differently to market influences, i.e. the magnitude of changes in yields varies between maturities, resulting in a variety of yield movements, such as butterfly and twist shapes.

4. Examples of financial product used duration

Duration research has expanded in many ways, Beccacece, F., Tasca, R. and Tibiletti, L. (2018) [13], two methods are used to correctly and quickly assess the risk adjustment of cash inflow items, highlighting the role of Macaulay durations. Adrian, T. and Shin, H.S. (2010) [14] analysis the duration's liquidity and leverage. Immunisation of duration is one of the most common financial products in our daily life.

Immunity means constructing a portfolio of inputs such that the effects of changes in interest rates within the portfolio on bond prices can cancel each other out so that the portfolio as a whole is not sensitive to interest rates. The basic method of constructing such a portfolio is to match duration so that interest-bearing bonds can approximate exactly zero-coupon bonds. Immunisation using duration is a passive input strategy, i.e. the portfolio manager does not seek excess returns through interest rate forecasts, but simply constructs the portfolio to achieve the stated return objective while avoiding crises of interest rate volatility.

In the design of the portfolio variety, apart from treasury bonds which can be selected into the portfolio, some higher yielding corporate bonds and financial bonds can also be added to the input portfolio, but with controlled matching maturities.

However, the immunisation strategy itself carries certain assumptions, such as the yield curve does not move very much and there is an equilibrium point between the level of yield to maturity and the change in market interest rates. not optimal for those inputs for which payment or termination dates are not certain.

The price risk of bonds is in the opposite direction to the reinvestment risk of coupons. The two can offset each other so that measures can be taken to fully offset the price risk of the bond and the reinvestment risk of the coupon payments. This is known as bond interest rate risk immunisation in bond portfolio management. The maturity immunisation strategy is the earliest and currently the most popular method of interest rate risk immunisation and is widely used in bond portfolio management by insurance companies, investment firms commercial banks, and others according to L. Linlin. (2006) [15]

For example, investor A has a debt payment of $100,000 after 3 years. He wants to choose one of two bonds, buy it and hold it until it matures, using the investment income to pay off the debt. Bonds A and B are known to have maturities of 3 and 4 years respectively, both have a face value of $100, both have a coupon rate of 15% and pay interest once a year. The current market interest rate is 10% and may fluctuate up or down after the bonds are purchased. Should you buy Bond A or Bond B?

First, let's calculate what the present value of the $100,000 debt will be in 3 years.

$$Present\ value = 100{,}000/(1+10\%)^{3} = 75{,}131.4 \tag{16}$$

This is the amount currently invested in bonds purchased by the investor.

Current selling price of Bond A

$$\sum_{t=1}^{3} 15/(1+10\%)^{t} + 100/(1+10\%)^{3} = 112.43$$

$$\frac{75131.4}{112.43} = 668.23\text{pieces} \tag{17}$$

Next, calculate the investment income that would be earned by holding Bond A for three years. This is derived from two components, the reinvestment of interest and the return of the principal at par at maturity. The amount of interest reinvested depends on the market interest rate after the purchase of the bond.

The market rate of interest is considered in three scenarios: it remains the same, it becomes larger and it becomes smaller.

If the market rate of interest remains constant at 10% after the purchase of Bond A

The interest income on the coupon was $15 and the interest reinvested income at the end of the first year was

$$15 * 1.1^{2} = 18.15 \tag{18}$$

Interest reinvested earnings at the end of second year were

$$15 * 1.1 = 16.5 \tag{19}$$

Interest reinvested earnings at the end of third year were 15

The sum of the interest reinvested earnings is $49.65 and the principal is returned in the third year at $100

Total investment per bond is $149.65

Total investment income after three years is

$$149.65 * 668.23 = 100000.6 \tag{20}$$

Can be exactly the right amount to pay off debts

If the market interest rate rises to 20% immediately after the purchase of Bond A and then stays the same

The interest income on the bond is $15 per year and the interest reinvested at the end of the first year is

$$\$15 * 1.2^{2} = \$21.6 \tag{21}$$

Interest reinvested at the end of second year is

$$\$15 * 1.2 = \$18 \tag{22}$$

Interest reinvested at the end of third year is $15

The sum of the interest reinvested earnings is $54.6 and the principal is returned in the third year at $100

Total investment per bond is $154.6

Total investment income after three years is

$$\$154.6 * 668.23 = \$103{,}308.4 \tag{23}$$

This will pay off the debt and give an additional $3,308.4

If the market interest rate drops to 5% after the purchase of Bond A and then stays the same

The interest income on the bond is $15 per year and the interest reinvested at the end of first year is

$$15 * 1.05^{2} = \$16.5375 \tag{24}$$

Interest reinvested at the end of second year is

$$15 * 1.05 = \$15.75 \tag{25}$$

Interest reinvested at the end of third year is $15

The sum of the interest reinvested earnings is $47.2875, with $100 of principal returned in the third year

Total investment per bond is $147.2875

Total investment income after three years is

$$\$147.2875 * 668.23 = \$98{,}421.93 \tag{26}$$

The income earned cannot afford the debt.

If purchase bond B

$$\sum_{t=1}^{4} 15/(1+10\%)^{t} + 100/(1+10\%)^{4} = 115.85 \tag{27}$$

$$75131.4/115.85 = 648.5\text{pieces} \tag{28}$$

The investment income from Bond B comes from two parts: one is the reinvestment of interest and the other is the sale price of Bond B at the end of year 3 (because Bond B has a 4-year maturity, there will be no repayment of principal at par at the end of year 3 when the debt matures and Bond B should be sold at that point).

Investment income is still calculated on the basis of three scenarios: no change in interest rate, greater and less.

If the market interest rate stays the same at 10% after the purchase of Bond B

The interest income from the bond is $15 per year and the interest reinvested at the end of the first year is

$$\$15 * 1.1^2 = \$18.15 \tag{29}$$

At the end of second year the reinvested interest income is

$$\$15 * 1.1 = \$16.50 \tag{30}$$

Interest reinvested earnings at the end of third year is $15

The sum of the interest reinvested earnings is $49.65 and the third year B bond sells for $104.5

Total investment per bond is $154.2

$$154.2 * 648.52 = \$99{,}998.8 \tag{31}$$

(basically barely enough to pay off the debt)

If the market interest rate rises to 20% immediately after the purchase of Bond B and then remains unchanged

The interest income on the bond is $15 per year and the interest reinvested at the end of the first year is

$$\$15 * 1.2^2 = \$21.6 \tag{32}$$

Interest reinvested at the end of the second year is

$$\$15 * 1.2 = \$18 \tag{33}$$

Interest reinvested at the end of the third year is $15

The sum of the interest reinvested earnings is $54.6 and the third year B bond sells for $95.83

Total investment per bond is $150.43

$$150.43 * 648.52 = \$97{,}559.0 \tag{34}$$

(very little difference from the 10% market rate return)

If the market rate drops to 2% immediately after the purchase of Bond B and then remains unchanged

The interest income on the bond is $15 per year and the interest reinvested at the end of the first year is

$$\$15 * 1.02^2 = \$15.606 \tag{35}$$

Interest reinvested at the end of the second year is

$$15 * 1.02 = \$15. \tag{36}$$

Interest reinvested income at the end of the third year is $15

The sum of the interest reinvested earnings is $45.91 and the third year B bond sells for $112.75

Total investment per bond is 158.66,

$$158.66 * 648.52 = \$102{,}894.18 \tag{37}$$

(not much different from market rates of 10% and 20%)

This shows that Bond B has a small range of returns, regardless of market interest rate fluctuations, because the duration of Bond B is exactly equal to three years, which is exactly the same as the maturity of the debt, if the investment is in a portfolio of not one, but several bonds. In this case, it is necessary to make the duration of the portfolio equal to the maturity of the debt. The duration of a bond portfolio is the weighted average of the duration of each bond, with the weight being the proportion of capital invested in each bond.

The Immunity to interest rate fluctuation risk means that the income from bond investments can remain stable and unaffected by upward and downward fluctuations in interest rates. In addition to that, the rationale inherent in immunity is that reinvestment return risk and price risk due to changes in interest rates can offset each other. When interest rates rise, reinvestment income rises and bond prices fall; when interest rates fall, reinvestment income falls and bond prices rise. So there are two conditions for immunisation: firstly, the current investment amount equals the present value of future debt (investment target amount); secondly, a bond or bond portfolio with a duration equal to the debt service period (investment duration) is selected

5. Conclusion

This article describes the nature of Macaulay duration, its calculation method and its relationship with bonds. It illustrates the role played by Macaulay duration in financial products, but at the same time its specific nature imposes certain constraints on the accuracy of the results. In the examples in the last part of the paper, the duration shows very well its immunity to interest rates. The interest rate and duration of bonds are inextricably linked and the correct use of duration in certain financial products is necessary to maximize profitability. The limitations of this paper's research are based on the limitations of the Macaulay duration, which in practice, in most cases, affects the accuracy of the results when calculated directly using the Macaulay duration. The impact of the limitations of the Macaulay duration on the immunisation effect of the calculation of interest rates can be considered as a future research direction

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