Water Allocation Plan Based on Linear Programming

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Abstract

Water allocation in the Colorado River directly affects greatly the water availability in the U.S. states of Arizona, California, Wyoming, New Mexico, and Colorado. In order to mitigate the influence of water shortage in these regions, this study develops a series of multi-objective linear programming models to propose a water allocation plan. The whole study is based on data from 2010 to 2030. First, we utilize the contour volume method to calculate reservoir capacity and introduce the satisfaction function to measure the rationality of water allocation in the Glen Canyon and Hoover dams. Then we introduce the multi-objective linear programming model and conclude that 40.165 km^3 and 50.978 km^3 volumes of water should be drawn from Lake Powell and Lake Mead, respectively. Then, to tackle the problem of competing interests in water availability, we propose multi-objective Ant Colony Optimization to measure the amounts of water needed for general usage and hydroelectricity generation. We find that at least ($10.660k_1+21.010k_2$) km^3 volume of water is needed (k_1 and k_2 refer to the efficiency of water for these two usages, respectively). Finally, to further optimize the model, we re-run the frequency of the model. The results of this study conclude that to get a satisfactory water allocation plan, we must take several influence factors into consideration, such as the ecological environment, technologies, and reuse of water and electricity.

Keywords: *Water Allocation, Hydroelectric Power, Reservoir Capacity Curve, Multi-objective Linear Programming, Gaussian Curve.*

1.INTRODUCTION

Reservoirs, artificial lakes formed by building barrages at the narrow mouth of ravines or rivers, can serve as flood control, water storage and irrigation, water supply, power generation, fish farming, recreation, etc. In recent years, the role of reservoirs to serve as hydroelectric power stations have become increasingly prominent as more and more dams are built to store water globally.

Hydroelectric power (hydropower) is a kind of electricity produced when water's kinetic energy is converted into electricity by turbines. So, compared to most other electricity generation technologies, hydropower is renewable, reliable, clean, and largely carbon-free, and represents a flexible peak-load technology [1]. However, climate change, especially in terms of changes in temperature and patterns of precipitation, has resulted in decreasing volume of water [7]. Consequently, hydropower suffers greatly. Therefore, some actions should be taken immediately to address this problem.

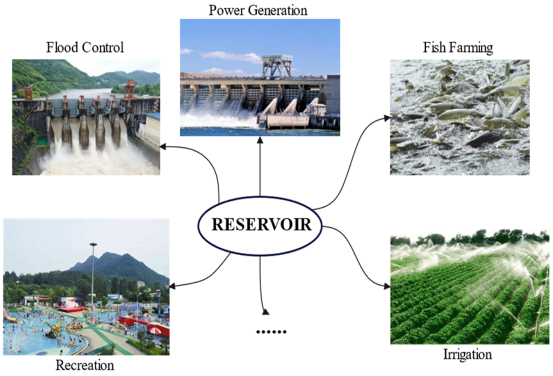

Figure 1: Functions of reservoirs.

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In the U.S. states of Arizona (AZ), California (CA), Wyoming (WY), New Mexico (NM), and Colorado (CO), recent rainfall shortages and hotter temperatures will continue to passively affect water availability and electricity requirements. We are to develop a water allocation plan about how to best allocate the water resources of Glen Canyon dam and Hoover dam. Our specific tasks include the following:

- Build a mathematical model to inform dam operations in a fixed set of water supply and demand conditions.
- Use our model to propose the best approaches to balance between benefits of water availability for general usage and hydropower production.
- Use our model to provide solutions when water supplement fails to meet all demands.
- Reallocate water resources when certain conditions change.

2.METHODOLOGY AND ASSUMPTIONS

2.1. Our Work

We mainly use the multi-objective linear programming to solve the water allocation problem. We divide the water in the dams into two parts. One is used to meet the general (agricultural, industrial, residential) demand of the five states of AZ, CA, WY, NM, and CO. The other is used to generate hydroelectric power to meet the electricity demand. Based on the law of conservation of mass and the law of conservation of energy, we finally get the results of the supply of these two parts of water, respectively. When water is sufficient, we solve the problem mainly based on the principles of satisfaction maximization and benefits maximization. When water is insufficient, we solve the problem mainly based on the principle of insufficiency minimization.

For better understanding of the overall model, a flow chart is provided to describe our sequence of modeling.

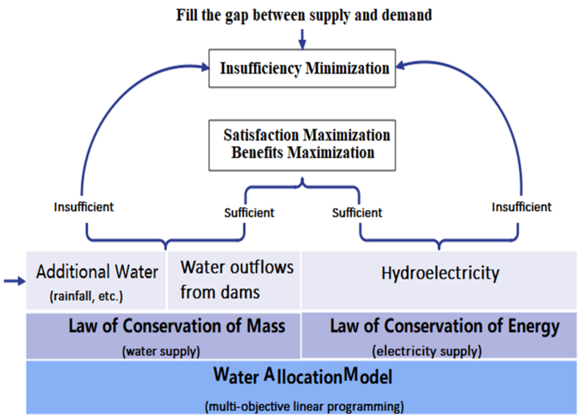

Figure 2: Flow chart of our work.

2.2. Assumptions

To simplify the problem, we make the following basic assumptions, each of which is properly justified.

- Assumption 1: Ignoring the evaporation from water surfaces in a short period of time. Our proposed water allocation plan is mainly for the short-term period in the future. In the short term, the evaporation of water is negligibly small, so it's reasonable to ignore the evaporation from water surfaces during this period [4, 6].
- Assumption 2: Water quality in different basins in the Colorado River is the same. We mainly focus on the rational allocation of water resources to address the diverse needs of people, so we ignore the differences in water quality in different watersheds.
- Assumption 3: The water in the dams will not be reused. If water resources are reusable, then the accessible water will be much more than the actual situation, so we assume water to be disposable.
- Assumption 4: Ignoring generators when considering hydroelectric power generation. Since we focus on the allocation of water resources, we suppose that the electricity generated by hydroelectric power is converted from the potential energy of water, solely. We do not consider generators in the process of electricity generation.
- Assumption 5: Supposing there are 365 days in a year. To simplify the problem, we assume that there are 365 days in each year from 2010 to 2030.

2.3. Notations

The primary notations used in this paper are listed in Table 1.

Table 1: Notations.

Symbol	Definition
a_i	Daily water requirements from reservoirs
b_i	Daily electricity requirements
c_i	Water outflows from the Glen Canyon dam
d_i	Water outflows from the Hoover dam
W	Hydropower generated by Lake Powell and Lake Mead
$\hbar_1$	The lower limit of water depth at Glen Canyon dam

h_2	The upper limit of water depth at Glen Canyon dam
h_3	The lower limit of water depth at Hoover dam
h_4	The upper limit of water depth at Hoover dam
Z_1	Satisfaction towards water allocation when additional water is available
Z_2	Satisfaction towards water allocation when additional water is not available

3 DAM OPERATIONS

3.1 *Model Overview and Data Processing*

Multi-objective linear programming turns a complex problem into a series of structurally similar optimal subproblems. Each subproblem has a smaller number of variables and a relatively simpler set of constraints than the original problem. Therefore, we create a multi-objective linear program to solve the water allocation problem. We use the satisfaction function to measure the rationality of water allocation in dams. We make constraints on the demand and supply of water and electricity based on the law of conservation of energy and the law of conservation of mass. Finally, we can get the best water allocation plan.

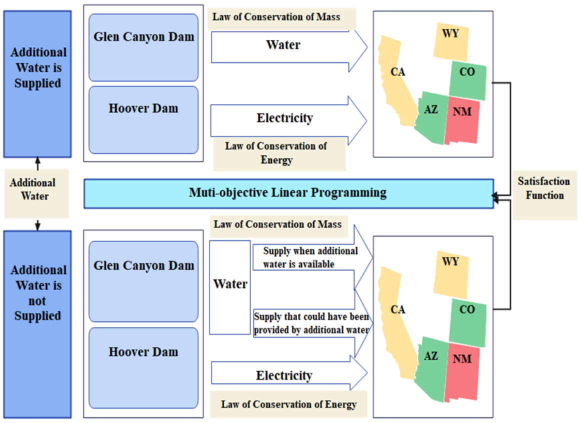

Figure 3: Flow chart of task 1.

We use a1, a2, a3, a4 and a5 to refer to the daily water requirements from reservoirs of the five states of AZ, CA, WY, NM, and CO, respectively. Similarly, we use b1, b2, b3, b4 and b5 to refer to the daily electricity requirements of the five states, respectively.

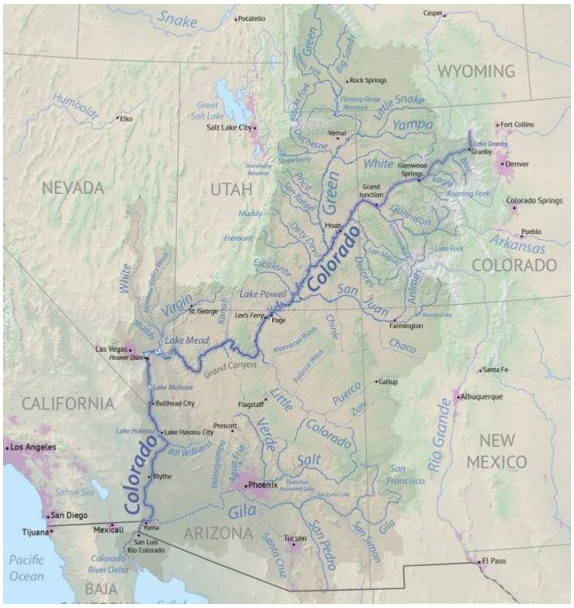

Figure 4: Location of the five states and the Colorado River.

There are three types of uses for the water in Glen Canyon dam: generating hydroelectric power, supplying the agriculture, industry, and residences of the five states and supplying Hoover dam. (Water outflows from the Glen Canyon dam supply part of the water input to the Hoover dam.) There are two types of uses for the water in Hoover dam, generating hydroelectric power and supplying the agriculture, industry, and residences of the five states. The corresponding mathematical symbols are shown in Figure 5. All of these variables are non-negative.

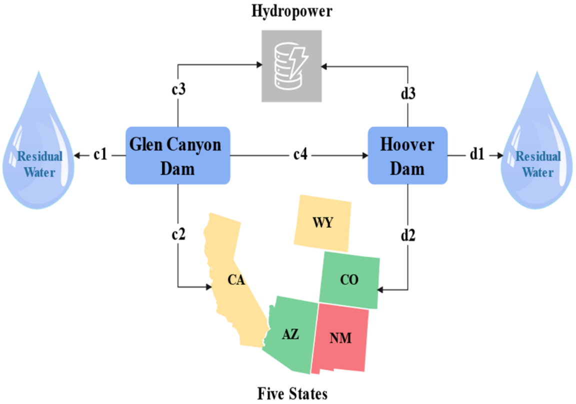

Figure 5: Allocation of water in Glen Canyon Dam and Hoover Dam.

We obtain the data of the total average annual precipitation and average annual electricity consumption of the five states from 2010 to 2020. We use Gaussian curve to predict the values of precipitation and electricity consumption from 2021 to 2030. The basic function of Gaussian curve is:

$$f(x) = A_1 \exp\left(-\left(\frac{x-B_1}{C_1}\right)^2\right) \qquad (1)$$

Where A_1, B_1 and C_1 are parameters. The physical meanings of A_1, B_1 and C_1 are the peak height, peak position and half-width information of the curve, respectively. We get the ultimate Gaussian curves, which are shown in equations (2) and (3):

$$S_p = f(t) = 907.8 \exp\left(-\left(\frac{t-2024}{83.43}\right)^2\right) \tag{2}$$

$$S_w = f(t) = 9616 \exp\left(-\left(\frac{t-2014}{79.22}\right)^2\right) \tag{3}$$

Where S_p and S_w refer to the total amounts of the average annual precipitation and electricity consumption of the states of AZ, CA, WY, NM, and CO and t donates year. The specific results are shown in Figure 6. Subsequently, we will use the data from 2010 to 2030 to get the optimal water allocation plan.

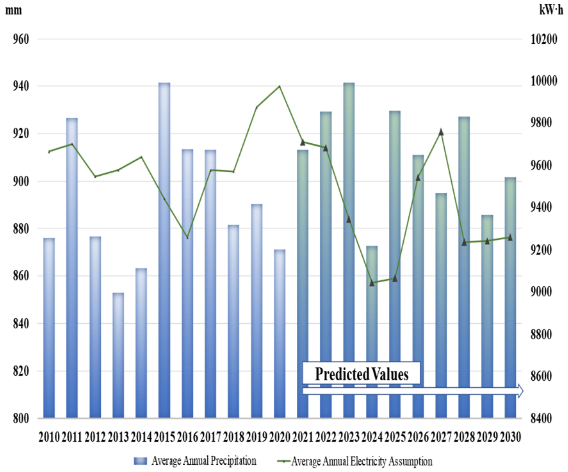

Figure 6: 2010-2030 average annual precipitation and electricity assumption of the five states.

3.2. Hydroelectric power generation

According to the law of the conservation of energy, energy can neither be created nor destroyed; it can only be transformed from one form to another. And hydroelectric power was developed based on this principle. Precisely, the principle of hydroelectric power generation is to build a dam on a large river that has a large drop in elevation. The dam stores lots of water in the reservoir. Gravity causes water to fall through the penstock inside the dam. At the end of the penstock, a turbine propellor is turned by the moving water. The shaft from the turbine goes up into the generator, which then produces the hydroelectric power.

Therefore, based on the principle mentioned above, the water in the reservoir passing through the dam converts potential energy into electrical energy, and then we can obtain the following equations:

$$W = P\rho_w g c_3 \eta_1 + M\rho_w g d_3 \eta_2 \tag{4}$$

Where W refers to hydropower generated by Lake Powell and Lake Mead, ρ_w refers to the density of water, g refers to gravity, d3 and c3 refer to the amount of water used for hydropower generation in Lake Powell and Lake Mead respectively, P and M refer to the water level in the two lakes respectively and η_1 and η_2 refer to the power generation efficiency of the Glen Canyon dam and Hoover dam, respectively.

3.3. Reservoir Capacity

The cross-sectional method is a common method for reservoir capacity calculation, which is both simple and convenient. However, due to the lack of basic research work and the calculation restraints, all the methods can hardly reach desirable results. Therefore, we introduce the contour volume method for reservoir capacity calculation. The contour volume method is used to calculate the volume of an irregular column by transforming it into a standard truncated cone of equal volume. Usually, the volume is divided into several layers by equally spaced horizontal surfaces. The volume of each layer is calculated by the truncated cone formula and then summed up to obtain the volume between adjacent sections [3][5]. The method is highly accurate considering the irregularity of the internal shape of the reservoirs. We can define the basic model of the reservoir capacity curve according to the law of conservation of mass:

$$\Delta V = \frac{1}{3}\left(A_n + \sqrt{A_n A_m} + A_m\right)\Delta Z \tag{4}$$

Where ΔV refers to the reservoir capacity, ΔZ refers to water level difference between two adjacent contours (water levels), An and Am refer to the area of the water surface surrounded by each of the two adjacent contour lines. By substituting the actual parameters of the Glen Canyon dam and Hoover dam, the model can be rewritten as:

$$\sum_{i=1}^{4} c_i = \frac{1}{3}\left(A_1 + \sqrt{A_1 A_2} + A_3\right)P \tag{6}$$

$$\sum_{i=1}^{3} d_i = \frac{1}{3}\left(A_3 + \sqrt{A_3 A_4} + A_4\right)M + c_1 \tag{7}$$

Where A1 and A2 refer to the area of the water surface of Lake Powell surrounded by each of the two adjacent contour lines, and A3 and A4 refer to the area of the water surface of Lake Mead surrounded by each of the two adjacent contour lines.

3.4. Multi-objective linear Programming

The amount of water stored in the dams should be controlled within a tolerable range. If it is too high, it may cause flooding, and if it is too low, it will be difficult to support hydroelectric power generation. Let h_1 and h_2 denote the lower and upper limits of water depth at Glen Canyon dam, respectively. Similarly, let h_3 and h_4 denote the lower and upper limits of the water depth at Hoover dam, respectively. We can obtain two constraints of $h_1 < c_1 < h_2$ and $h_3 < d_1 < h_4$. Plus, the demand for water and electricity in the five states must be no more than the supply. Combining all known conditions, we can obtain the following set of inequalities:

1) When additional water is supplied.

$$\begin{cases} h_1 < c_1 < h_2 \\ h_3 < d_1 < h_4 \\ \sum_{i=1}^{5} a_i < W \\ \sum_{i=1}^{5} b_i < c_2 + d_2 \\ \sum_{i=1}^{4} c_i = \frac{1}{3}\left(A_1 + \sqrt{A_1 A_2} + A_3\right)P \\ \sum_{i=1}^{3} d_i = \frac{1}{3}\left(A_3 + \sqrt{A_3 A_4} + A_4\right)M + c_1 \\ c_1,\ c_2,\ c_3,\ c_4,\ d_1,\ d_2,\ d_3 \geq 0 \end{cases} \tag{8}$$

We establish the following objective function based on the satisfaction function:

$$\max\ Z_1 = \frac{W - \sum_{i=1}^{5} a_i}{\sum_{i=1}^{5} a_i} + \frac{(c_2 + d_2) - \sum_{i=1}^{5} b_i}{\sum_{i=1}^{5} b_i} \tag{9}$$

Where W and $(c_2 + d_2)$ are actual values, and $\sum_{i=1}^{5} a_i$ and $\sum_{i=1}^{5} b_i$ are expected values.

2) When additional water is not supplied.

$$\begin{cases} h_1 < c_1 < h_2 \\ h_3 < d_1 < h_4 \\ \sum_{i=1}^{5} a_i < W \\ \sum_{i=1}^{5} b_i + p_0 < c_2 + d_2 \\ \sum_{i=1}^{4} c_i = \frac{1}{3}\left(A_1 + \sqrt{A_1 A_2} + A_3\right)P \\ \sum_{i=1}^{3} d_i = \frac{1}{3}\left(A_3 + \sqrt{A_3 A_4} + A_4\right)M + c \\ c_1,\ c_2,\ c_3,\ c_4,\ d_1,\ d_2,\ d_3 \geq 0 \end{cases} \tag{10}$$

Where p0 refers to the total amount of daily precipitation of the five states. According to equation (2), $p_0 = 907.8 \exp\left(-\left(\frac{t-2024}{83.43}\right)^2\right)$, $t = year + \frac{day}{365}$. In the absence of additional water supplies, reservoirs need to supply more water to meet the fixed set of water supply and demand conditions. We establish the following objective function based on the satisfaction function:

$$\max\ Z_2 = \frac{W - \sum_{i=1}^{5} a_i}{\sum_{i=1}^{5} a_i} + \frac{(c_2 + d_2) - (\sum_{i=1}^{5} b_i + p_0)}{\sum_{i=1}^{5} b_i + p_0} \tag{11}$$

Assuming that the drainage rates of Glen Canyon dam and Hoover dam are v1 and v2, respectively, the time required to meet the fixed water demand is:

$$T = \max\left(\frac{c_2}{v_1}, \frac{d_2}{v_2}\right) \tag{12}$$

If $T \leq 1day$, then the dams can meet all the water demand of that day. If $T > 1day$, then the dams can not meet the water demand of the day, and additional water is needed. The amount of water to be replenished on that day is:

$$\Delta Q = c_2 + d_2 - \max\left(\sum_{i=1}^{4} c_i, \sum_{i=1}^{3} d_i\right) \tag{13}$$

The total amount of water to be replenished is:

$$S_Q = \left[c_2 + d_2 - \max\left(\sum_{i=1}^{4} c_i, \sum_{i=1}^{3} d_i\right)\right]T \tag{14}$$

3.5. Results

We get the following results.

1) When additional water is supplied.

By solving the objective function (9) in section 4.4, we conclude that in order to meet stated water and electricity demands, a total amount of 59 millimeters of precipitation are needed to meet the water demand. To be precise, 33 millimeters of precipitation are needed from Lake Powell and 26 millimeters of precipitation are needed from Lake Mead. The five states of AZ, CA, WY, NM, and CO each covers an area of 295000 km², 411013 km², 253596 km², 315194 km² and 269997 km². The total floor area is 1544800 km². By multiplying the total area by the precipitation and then we can get that 40.164800 km3 amount of water should be drawn from Lake Powell, and 50.978400 km3 amount of water should be drawn from Lake Mead.

2) When additional water is not supplied.

By solving equation (12) in section 4.4, it will take 1.167 days to meet the total water demand. Because 1.167 days > 1 day, the Glen Canyon dam and Hoover dam can not meet the water demand of the day.

And by solving the objective function (11) and equation (14) in section 4.4, we conclude that if no additional water, such as rainfall, is supplied, 86 millimeters of precipitation is needed to ensure that these fixed demands are met. We multiply the total area by the precipitation and then we can get that 132.852800 km3 amount of water in total are to be replenished over time to meet the water demand.

4.BALANCE BETWEEN COMPETING INTERESTS

4.1. Model Overview

We divide the water in Lake Mead and Lake Powell into two parts, one is used directly for electricity generation and the other is used to meet basic usages, such as agricultural, industrial and residential needs. These two parts of the water will produce different benefits. Therefore, while satisfying people's water demand as much as possible, we try to maximize the total benefits of water.

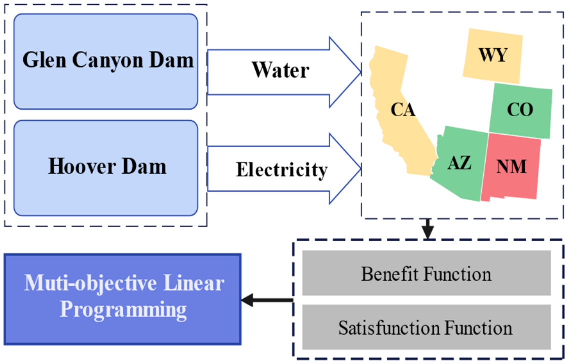

Figure 7: Flow chart of task 2.

4.2. Efficiency and Benefits of Water

Due to severe droughts, water evaporation is becoming more and more intense and precipitation is decreasing heavily. The Colorado River system has become very vulnerable. If climate change continues to adversely affect the Colorado River basin, the water volume at some point will be insufficient to meet the fundamental water and electricity needs of stakeholders. Therefore, in order to tackle the problem of balancing between competing interests of water availability, we firstly measure the benefits of water:

$$\theta_1 = k_1(c_2 + d_2) \tag{15}$$

$$\theta_2 = k_2 W \tag{16}$$

Where k1 and θ_1 refer to the efficiency and benefits of water available for general (agricultural, industrial, residential) usage, respectively. Similarly, k2 and θ_2 refer to the efficiency and benefits of water available for electricity production, respectively.

4.3. Multi-objective Ant Colony Optimization

Ants have inspired a number of methods and techniques, among which the most successful one is ant colony optimization (ACO). These ants deposit pheromone on the ground in order to mark some favorable path that should be followed by other members of the colony. ACO is a multi-agent system, which conducts solution search in multiple sections of the problem space independently. ACO not only improves the reliability of the algorithm but also makes it have a strong global search capability. At the same time, the results of ACO are not dependent on the initial route selection and do not require manual adjustment. Ant colony optimization exploits a similar mechanism for solving optimization problems [2]. Therefore, we choose multi-objective ACO to tackle the problem of competing interests in water availability.

The constraints are the same as (8) that we have mentioned in section 4.4. Considering that we have to meet the water requirements of the five states while maximizing the interests of the stakeholders, we establish two objective functions:

$$\begin{cases} \max\ Z_1 = \frac{W-\sum_{i=1}^{5} a_i}{\sum_{i=1}^{5} a_i} + \frac{(c_2+d_2)-\sum_{i=1}^{5} b_i}{\sum_{i=1}^{5} b_i} \\ max\, Z_3 = \theta_1 + \theta_2 \end{cases} \tag{17}$$

4.4 Results and Solutions

By solving objective functions (17) in section 5.3, (6.9*k*$_1$+13.6 *k*$_2$) millimeters of precipitation is needed to meet the general water usages. We multiply the total area by the precipitation and then we can get that (10.659672*k*$_1$+21.010368 *k*$_2$) km^3 amount of water in total is needed.

5.SOLUTIONS TO WATER SCARCITY

5.1. Maximize Satisfaction and Minimize Water Gap

As global warming becomes more and more severe, the Colorado River system is likely to suffer from water storage. Therefore, we reasonably take this scenario into account and incorporate the solution into our water allocation plan. When water is insufficient, our priority is to allocate available water to maximize people’s satisfaction level and minimize the impact of supply-demand asymmetries.

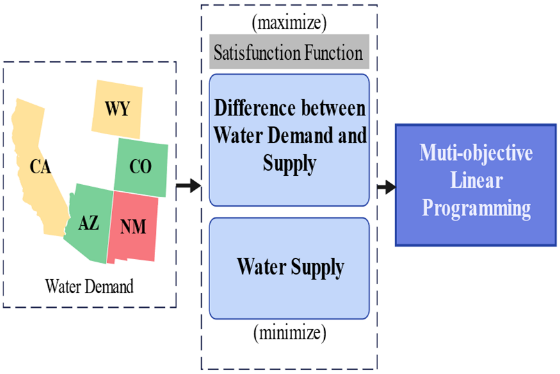

Figure 8: Flow chart of task 3.

The constraints are the same as (8) that we have mentioned in section 4.4.

$$\begin{cases} \max\ Z_1 = \frac{W-\sum_{i=1}^{5} a_i}{\sum_{i=1}^{5} a_i} + \frac{(c_2+d_2)-\sum_{i=1}^{5} b_i}{\sum_{i=1}^{5} b_i} \\ \min Z_4 = \sum_{i=1}^{5} b_i - (c_2+d_2) \\ \min Z_5 = \sum_{i=1}^{5} a_i - W \end{cases} \tag{18}$$

Where Z4 refers to the difference between the total supply and demand of water of the five states and Z5 refers to the difference between the total supply and demand of electricity of the five states when water is insufficient.

5.2. Results and Solutions

The final results show that the amount of water that should be drawn from Glen Canyon dam and Hoover dam is 59 millimeters of precipitation. When water is sufficient, the difference between the total supply and demand of water of the five states is 36.71 millimeters of precipitation. When water is insufficient, the difference between the total supply and demand of electricity of the five states is 21.66 millimeters of precipitation. After changing precipitation in volume, we get that Z_1 equals 91.1432 km^3, Z_4 equals 56.709608 km^3, Z_5 equals 33.460368 km^3.

Therefore, we propose several solutions to water scarcity:

- Improve the ecological environment and increase the availability of water resources.
- Improve the ecological environment and increase the availability of water resources.
- Develop new wastewater disposal technologies to reduce the amount of wastewater.
- Reuse water and electricity as much as we can.
- Inter-basin water diversion and long-distance water transfer.

6.MODEL IMPLICATIONS UNDER CERTAIN CONDITIONS

Certain conditions, such as variety of demand for water, can lead to different model implications.

1)The demands for water and electricity in the communities of interest change over time.

When the total demand for water and electricity decreases in the five states, according to our model, the total amount of additional water that needs to be withdrawn from the two dams decreases. Plus, when there is no additional supply of water, such as rainwater, our model implicates that the time required for water resources to reach the stated demand is significantly reduced. Meanwhile, the additional water that needs to be taken from the reservoirs is reduced. Clearly, this result holds. A reduction in demand will inevitably lead to a reduction in supply. This can further justify that our model is realistic and highly adaptive.

2)The population, agricultural, and industrial demand for water grows or shrinks in the affected areas.

When the total water demand remains the same, but the population decreases and the agricultural and industrial water demand decreases while, the interests of stakeholders will surely change. We adjust the relevant variables, and our model shows the following results. A decrease in the general demand for water and an increase in the demand for hydropower generation can result in a downward trend in stakeholders' benefits. Similarly, when the general demand for water remains constant, but the population increases, the agricultural and industrial water demand increases, the demand for hydroelectric power generation increases. Our model also shows a decreasing trend in total benefits. This result can perfectly indicate that the amount of water used for general demand and electricity we obtained before can realize interest maximization.

3)The proportion of renewable energy technologies increases.

All of our analyses above are based on the assumption that water cannot be recycled. However, as recyclable resources continue to be developed and utilized, people will be exposed to renewable energy technologies more frequently in the future. Therefore, we now ignore this assumption and take renewable energy technologies into account. The use of this technology can be valued as an increase in the supply of water used to meet general usage. When this variable changes, we can infer from the results that the benefits of the stakeholders are greatly enhanced and the amount of water that should be drawn from the dams is greatly reduced.

4)Additional water and electricity conservation measures are implemented.

In the short term, it is difficult for mankind to effectively control climate change. Therefore, at this stage, saving water and electricity is one of the best ways to alleviate energy shortages. When people take measures to conserve water and electricity, the demand for both water and electricity decreases. We simultaneously reduce the values of these two variables of water and electricity demand in the five states, and the final result shows that the amount of water that should be drawn from the dams is greatly reduced.

7.MODEL OPTIMIZATION

It is assumed that the water demand in the five states remains constant over a certain period of time, that is, water demand is fixed. However, the actual precipitation will affect our water allocation plan. To further optimize the model, we re-run the frequency of the model. We have predicted the precipitation curves for the five states above, which can be denoted by C_{pre}. The actual precipitation curve can be denoted by C_{act}. We use ΔC to refer to the percentage error of the predicted value and actual value:

$$\Delta C = \frac{\int_0^{t_0} C_{pre}dt - \int_0^{t_0} C_{act}dt}{\int_0^{t_0} C_{pre}dt} \tag{19}$$

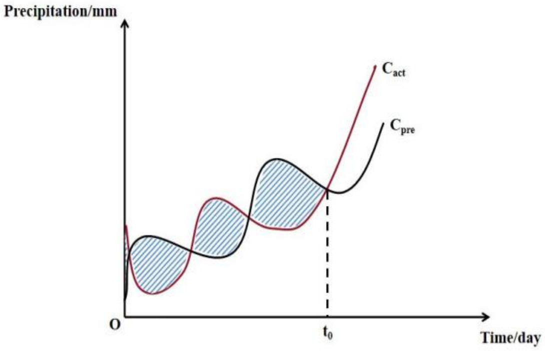

Figure 9: Model optimization.

If ΔC is greater than 40%, then we are to reuse the model for prediction.

8.SENSITIVITY ANALYSIS

8.1. Introduction of Grey Model

In order to prove that our model has a good internal relationship, especially can make a good response to electricity and consumption changes, we carry out the sensitivity analysis.

The Gray model is a method of prediction that has been widely used in recent years. Gray system is a type of system that contains both known and unknown information. By properly processing the original data, the intrinsic laws of the data can be discovered. Also, the Accumulated Generation Operation (AGO) allows us to weaken the interference of random factors. GM (1, 1) is a commonly used method when it turns to grey model. It performs a one-time accumulative generation of the raw data. The prediction can be achieved by a one-time cumulative reduction. The original time series data of annual water demand and electricity consumption in the five states are strongly random. However, the cumulative method can weaken the randomness, so it is reasonable to use the time series data after one cumulative reduction for prediction.

8.2. Sensitivity Analysis

To test the plausibility of the gray model, we perform posteriori error tests and calculate the posterior error ratio (C). Posterior error ratio is the ratio of the variance of the true error to the variance of the original data. If $C < 0.35$, then the prediction accuracy of the model is "good". If $0.35 < C \leq 0.5$, then the prediction accuracy of the model is "pass". If $0.5 < C \leq 0.65$, then the prediction accuracy of the model is "barely". If $C \geq 0.65$, then the prediction accuracy of the model is "fail".

1)Sensitivity Analysis of Electricity Consumption

The posterior error ratio of electricity consumption is 0.63625, between 0.5 and 0.65. Therefore, we conclude that the accuracy of the model is acceptable. 9950.4657 and 10179.4303 are two subsequent values fitted by the model.

2)Sensitivity Analysis of Water Consumption

The posterior error ratio of water consumption is 0.39681, between 0.35 and 0.5. Therefore, we conclude that the accuracy of the model is good. 874.5011 and 867.7148 are two subsequent values fitted by the model.

3)Sensitivity Analysis of Our Model

The posterior error ratio of our model is 0.27424, less than 0.35. Therefore, we conclude that the accuracy of the model is satisfactory.

9.STRENGTHS AND WEAKNESSES

Some strengths and weaknesses of our model are listed for better understanding and application.

9.1. Strengths

- Universality. Our model can provide rational, environmentally adapted water allocation strategies as water demand varies across the five states.
- Flexibility. When additional water resources are not available, our model is able to dynamically allocate the water resources in the reservoir over time.

- Thoughtfulness. In reality, the interests of allocators are also important. Therefore, we upgrade our model so that it can coordinate the interests between stakeholders and then maximize their benefits.
- Effectiveness. Nowadays, water is in scarce supply. In most cases, the water in the reservoirs is not adequate enough to meet the demand of the five states. Our model ensures that people can be provided as much water as possible from the reservoirs and their satisfaction can be maximized. We also take droughts and flood into considerations, so we try to keep the volume of water within a safe range.
- Practicability. Our model states that when actual precipitation is 40% less than predicted precipitation, we should restart the model based on actual conditions, so that the allocation plan can adapt to the changing environment.

9.2. Weaknesses

- Ignorance of generators. Because the role of generators in hydroelectric power generation is neglected, it is difficult to get the exact value of hydroelectric power generation from dams.
- Difficulty in data obtain. Explanation. When utilizing the contour volume method to get the reservoir volume, the area of the water surface surrounded by each of the adjacent contours is difficult to obtain.
- Inconsiderateness of Mexico's interests. Our model simply takes into account the amount of water that flows into Mexico, but fail to consider the benefits of Mexico in depth.

10.CONCLUSIONS

From the calculation results obtained, the following conclusions can be drawn:

- When additional water is supplied, 33 millimeters and 26 millimeters of precipitation are needed from Lake Powell and Lake Mead respectively to meet stated water and electricity demands. Also, 40.165 km3 and 50.979 km3 amount of water should be drawn from Lake Powell and Lake Mead, respectively.
- When additional water is not supplied, it will take 1.167 days to meet the total water demand. Also, 86 millimeters of precipitation is needed to meet the fixed demands and 132.853 km3 amount of water are to be replenished to meet the water demand.
- To maximize the interests of stakeholders, (6.9k1+13.6k2) millimeters of precipitation and (10.659672k1+21.010368 k2) km3 amount of water are needed to meet the general water usages.
- Water scarcity is affected by various factors. And there are several solutions to improve the situation. For example, we can improve the ecological environment and increase the availability of water resources, develop new wastewater disposal technologies, or reuse water and electricity.

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