



# Empirical Analysis of Value at Risk (*VaR*) of Stock Portfolio Based on Python

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## Abstract

This paper mainly introduces the financial sector is widely acclaimed VaR risk quantitative analysis method, VaR method (Value at Risk), known as value-at-risk model, is often used in the risk management of financial institutions. We select and download the data of four stocks (APPL, BAC, AXP, KO) which the company Berkshire Hathaway had heavy position in. Then according to the shareholding ratio released in its quarterly statement, we build the corresponding weights of portfolio. In this paper, we estimate Value at Risk (VaR) by the method of the simple variance - covariance VaR method. On the assumption that the sample data obey the normal distribution, the simple VAR method is used to get the reliable value of risk, and the prediction made by the value of risk can better estimate the risk which has an important practical guiding significance for the risk management of securities investment.

**Keywords**-Python; Jupyter Notebook; Berkshire Hathaway; Value at Risk (*VaR*); Variance - covariance VaR method; Portfolio risk

## 1. INTRODUCTION

The calculation of value at risk (VaR) is the mainstream method of risk management and financial supervision in the world financial market. Financial institutions' trading portfolios often depend on hundreds or thousands of market variables, and traders' daily analysis can involve a lot of calculations. While risk indicators such as Delta, Gamma and Vega are important to traders which are calculated for exposures to specific market variables (such as stock indexes, interest rates and commodity prices), they may not provide executives and regulators of banks, securities companies, fund companies and other financial institutions with a comprehensive and visual picture of overall risk.

Both indicators Value at Risk (VaR) and Expected Shortfall (ES) attempt to measure the overall risk of the financial institutions' portfolio by just one figure. Among them, the concept of VaR was first proposed by JPMorgan and has been widely adopted by the corporate asset department, fund managers and financial institutions. VaR is also a traditional tool used by many regulators to calculate bank capital setting, which includes market risk, credit risk and operational risk capital [1].

However, as VaR was proposed by Jorion P [2], conditional value at risk (CVaR) is also widely accepted and used as one of the measures to measure tail risk as well. Moreover, as a conditional expectation of losses exceeding VaR, it is a coherent risk measurement standard [4]. In addition, related studies on CVaR have included the linear programming model of risk conditional value and portfolio optimization by Mansini et al.[5], the application of worst-case conditional value at risk in robust portfolio management [6]. At the same time, due to the limitations of VaR and CVaR in measurement, expectile-based VaR (EVaR) based on the expected value at risk has been used as a risk measurement in recent years, which has also attracted a lot of attention in the field of risk management. Jakobsons and Vanduffel, Bellini and Di Bernardino and other scholars conducted in-depth studies on the derivation of lower and upper limits of EVaR under dependent uncertainty and the calculation of EVaR using econometric models [7-8]. Moreover, the empirical analysis of VaR portfolio optimization based on cardinality constraint expectation [9] and the optimal investment problem with financial institutions under VaR constraint and portfolio insurance constraint [10] are also extended applications of VaR in securities investment.

The research objective of the paper is to use Python to process the data of American stocks in a certain time

range, then calculate and test the value at risk of the constructed portfolio by variance - covariance method. The purpose is to systematically and comprehensively introducing the meaning and measurement methods of value at risk (VaR) for scholars, practitioners and investors in the field of finance and risk management. The paper also provides a simple processing of the financial data onto THE US stock and a specific method and steps to calculate the value at risk of the portfolio by using the variation-covariance method of the Jupyter Notebook. In China, most of the similar empirical analysis using the variance-covariance method choose the data of A-share in Shanghai and Shenzhen or Hong Kong stocks, and few choose the data of American stocks. Therefore, this is also the other purpose of this study which can fill the relevant research gap.

The remainder of the paper is organized as follows: Section 2 introduces the background and causes of this empirical research. The research motivation is derived from the simulation portfolio of Berkshire Hathaway's four heavy positions with corresponding weights based on the fact that some stocks held for a long time have maintained considerable return on investment; Section 3 describes the development of VaR correlation, concept, format framework, parameter selection, and the variance-covariance value-at-risk measurement method mainly used in this paper. After data processing of the downloaded historical data of 4 American stocks from 2011 to 2021, the value at risk (VaR) of the portfolio will be estimated according to the five steps in Section 4. After the VaR value is obtained, Section 5 shows the visualization test of returns and normal distribution carried out for each stock selected. Finally, in Section 6 the empirical analysis of this paper is summarized and the future research is prospected.

## 2. BACKGROUND AND RESEARCH CAUSATION

The reason for this study is that Berkshire Hathaway has held some stocks for a long time and has maintained a considerable return on investment, so we are interested in the investment portfolio of its stocks held. According to the proportion of its major holdings of stocks in its second quarter report of 2021, we construct the corresponding stock weight portfolio in this paper. Then, the return rate and value at risk of stock portfolio are analyzed and calculated by using Python.

As the equity securities investment projects of company in the 2020 second quarter report of the company showing in figure 2, the paper selects the positions of four stocks (Apple, Bank of America, American express, Coca-Cola), building simulation portfolio, matched with the corresponding investment weight, and do the empirical analysis of the simulated portfolio's value at risk.

| Stock                              | % Port |
|------------------------------------|--------|
| AAPL Apple Inc                     | 41.47% |
| BAC Bank of America Corp. (Nort... | 14.21% |
| AXP American Express Co            | 8.55%  |
| KO Coca Cola Co                    | 7.39%  |

Figure 1 Major Positions in Stocks

## 3. RESEARCH METHODS

### 3.1. Introduction of Value at Risk (VaR)

#### 1) The generation and development of VaR

In order to address all kinds of problems that cannot be solved by traditional risk measurement methods, VaR is generated as a comprehensive measure of market risk in complex financial asset portfolios. The VaR method was first proposed by JP.Morgan company. The risk managers of JP.Morgan developed a risk measurement method that could measure the market risks of different transactions and business departments and integrate these risks into a numerical value—VaR, which was promoted and widely used since 1993. In the same year, Bank for International Settlements for International Settlements (BIS) accepted the VaR as an analysis tool, which was reflected in the Basel Capital Accord [11].ight portfolio in this paper. Then, the return rate and value at risk of stock portfolio are analyzed and calculated by using Python.

The following year, in October 1994, J.P. Morgan Bank developed a risk management information system called RiskMetrics, a simplified model for measuring internal corporate risk. At the same time, J.P. Morgan made daily fluctuations and correlations of various market risk factors available for free on the Internet, and VaR became popular around the world. After more than ten years of development and improvement, the VaR measurement model has become a mainstream market risk measurement tool in Europe and the United States, and widely used in banking, securities, futures, financial supervision, financial derivatives and other aspects [12].

#### 2) Definition of VaR

Value at Risk (VaR) is the maximum possible loss of a financial asset or portfolio under normal market fluctuations. More precisely, it refers to the maximum possible loss of the value of a financial asset or portfolio in a specific future period at a certain probability level (confidence) in a specific holding period [14]. Its mathematical formula is as follows:

$$\text{Prob}(\Delta P \Delta t \leq VaR) = c \quad (1)$$

- **Prob**—the probability that an asset value loss is less than the maximum possible loss

- $\Delta P$ —the loss of value of a financial asset over a given holding period  $\Delta t$
- $VaR$ —the value at risk at the given confidence level  $c$ , which is also called the maximum possible loss
- $c$ —the given confidence level

### 3) Parameter selection of VaR model

In the definition of VaR, there are two important quantitative factors: holding period ( $\Delta t$ ) and confidence level ( $c$ ). In essence, the calculation formula of VaR can be regarded as a function of these two parameters  $\Delta t$  and  $c$ . The longer  $\Delta t$  is, the larger  $c$  is, and the larger VaR is calculated and vice versa [15]. When other factors remain the same, the VaR value completely depends on the two parameters  $\Delta t$  and  $c$ . Therefore, to obtain the VaR definite value of the portfolio, the values of the two parameters holding period ( $\Delta t$ ) and confidence level ( $c$ ) must be determined.

### 4) The format framework for VaR

In the sense of statistics, VaR is a number which refers to “the value at risk” in the face of “normal” market fluctuations. In other words, it is the expected maximum loss (can be absolute value or relative value) within a given confidence level and a certain holding period[4].

Estimating portfolio risk is important for long-term capital growth and risk management, especially within large firms or institutions. For example, the confidence of the stocks portfolio held by an investment company in the next 24 hours is 95% and under the normal fluctuation of the securities market, the VaR value is 250,000 dollars. It means that the securities portfolio of the company is in a day (24 hours), the probability of a maximum loss of more than 250,000 dollars due to market price changes is 5%, which occurs only once every 20 trading days on average. In other words, the investment company has a 95% confidence that its portfolio will not lose more than 250,000 dollars in the next trading day. The probability of 5% reflects the risk aversion degree of financial asset managers, which can be determined according to the risk appetite degree and tolerance of different investors.

## 3.2. The measurement of VaR

There are three main methods to calculate the VaR:

1. Historical simulation method
2. Variance - covariance method
3. Monte Carlo simulation method

In this paper, we will focus on the method (2) variance - covariance method. The variance-covariance method, also known as model building method, based on

the historical price movement (standard deviation, average price) of a given stock or portfolio of stocks over a given backdate period, using probability theory to measure the maximum loss within a specified confidence interval. We will use Python to step through the calculations in the data processing and empirical analysis in section 4 below.

The core of VaR measurement of portfolio is to solve the estimation of covariance matrix  $\Sigma$  [12].  $\Sigma$  can be estimated by using the historical data of the return rate of each security, and the formula is as follows:

$$\sigma_{ij} = \frac{1}{(T-1)} \sum (x_{t,i} - \mu_i)(x_{t,j} - \mu_j) \quad (2)$$

- $T$ —the length of time
- $x_{t,i}$ —the yield of the  $i$  bond at the end of time  $t$

Before the calculation, the following conditions are assumed for the standard VaR measurement:

1. Normal distribution of returns—VaR assumes that portfolio returns are normally distributed. This is certainly unrealistic for most assets, but it allows us to use simpler calculations for benchmarking in this paper. (It can modify VaR to illustrate different distributions, but here we focus on standard VaR calculations)

2. Standard market conditions—Like many financial instruments, VaR is best used to account for losses in standard markets and is not suitable for extreme or unusual events.

Finally, the following steps will be performed in this paper in order to calculate the VaR of the portfolio:

1. Calculate the periodic returns on the stocks in the portfolio;
2. Create the covariance matrix based on the return rates;
3. Calculate portfolio mean and standard deviation (weighted according to the level of investment in each stock in the portfolio);
4. Calculate the inverse function of the normal cumulative distribution by using the specified confidence interval, standard deviation and mean;
5. Estimate the VaR of the portfolio by subtracting the calculation in step (4) from the initial investment.

## 4. DATA PROCESSING AND EMPIRICAL ANALYSIS

In this paper, variance - covariance calculation method is used to calculate the VAR value. In order to facilitate the calculation of VAR value, it requires special assumptions and data samples. In this case, API is used to download and select the 2516 trading days' daily closing price pre-weighted data of four U.S. stock which

are Apple (AAPL), Bank of America (BAC), American Express (AXP), Coca-Cola (KO) from December 21, 2011 to December 20, 2021, and saved in CSV format.

#### 4.1. Calculate the periodic returns of the stocks in the portfolio

According to the requirements of the research method, process the original data and the daily return rate of each stock was calculated separately. The return on a stock is calculated using the closing price:

$$\text{daily return rate} = (\text{today's closing price} - \text{yesterday's closing price}) \div \text{yesterday's closing price} \times 100\% \quad (3)$$

Write code and calculate the daily return rate of the 4 stocks in Jupyter Notebook, the code is as follows:

```
# import pandas as pd
# from datetime import datetime
# import matplotlib.pyplot as plt
# code = ['AAPL','BAC','AXP','KO']
# data = pd.DataFrame()
# for c in code:
#     temp = pd.read_csv(c+'.csv',encoding='gbk')
#     temp = temp.set_index(['date'])
#     data =
pd.concat([data,temp['change(%)']],axis=1)
# data.columns = code
# data.index = [datetime.strptime(x,'%Y-%m-%d')
for x in data.index]
# n = list(data.index).index(datetime(2011,12,21))
# data = data[n:]
# data = data/100
# data
```

Then, import the data of 'AAPL', 'BAC', 'AXP' and 'KO' in CSV format and obtain the table of 2516 rows × 4 columns by the above calculation method. The running result of the Jupyter Notebook is as follows:

|            | AAPL    | BAC     | AXP     | KO      |
|------------|---------|---------|---------|---------|
| 2011-12-21 | 0.0014  | 0.0116  | -0.0090 | 0.0173  |
| 2011-12-22 | 0.0053  | 0.0459  | 0.0029  | -0.0055 |
| 2011-12-23 | 0.0119  | 0.0238  | 0.0067  | 0.0107  |
| 2011-12-27 | 0.0080  | -0.0214 | -0.0060 | -0.0009 |
| 2011-12-28 | -0.0096 | -0.0347 | -0.0145 | -0.0052 |
| ...        | ...     | ...     | ...     | ...     |
| 2021-12-14 | -0.0080 | 0.0126  | -0.0028 | 0.0007  |
| 2021-12-15 | 0.0285  | -0.0043 | -0.0037 | 0.0045  |
| 2021-12-16 | -0.0393 | 0.0241  | 0.0047  | 0.0102  |
| 2021-12-17 | -0.0065 | -0.0249 | -0.0183 | -0.0157 |
| 2021-12-20 | -0.0081 | -0.0164 | -0.0257 | -0.0033 |

2516 rows × 4 columns

Figure 2 Daily Return Rate of The 4 Stocks

#### 4.2. Create the covariance matrix based on the return rates

After getting the daily closing price changepercent of each stock in the portfolio, the next step is to create a portfolio of the four U.S stocks—Apple, Bank of America, American Express and Coca-Cola—selected according to the major heavy positions of Berkshire Hathaway listed in the 2021 Q2 report. The code is as follows:

```
# import pandas as pd
# from pandas_datareader import data as pd
# import numpy as np
# import datetime as dt
# tickers = ['AAPL','BAC', 'AXP', 'KO']
```

We have constructed a portfolio of four American stocks, and the weight of the portfolio is 1:1 to that is on the report. In this case, the initial investment amount of the simulated constructed portfolio is set as \$1,000,000:

```
# weights = np.array([.4147, .1442, .0855, .0739])
# initial_investment = 1000000
```

Then combined with the daily return rate of the stock obtained in the previous step, we establish the variance - covariance (Var - Cov) matrix, and the code is as follows:

```
# cov_matrix = data.cov()
# cov_matrix
```

After running, the variance-covariance (Var - Cov) matrix is shown below, which will enable us to calculate the standard deviation and mean return of the entire portfolio in the next step.

```
# Outcome:
```

|      | AAPL     | BAC      | AXP      | KO       |
|------|----------|----------|----------|----------|
| AAPL | 0.000319 | 0.000132 | 0.000117 | 0.000064 |
| BAC  | 0.000132 | 0.000397 | 0.000244 | 0.000089 |
| AXP  | 0.000117 | 0.000244 | 0.000320 | 0.000097 |
| KO   | 0.000064 | 0.000089 | 0.000097 | 0.000122 |

**Figure 3** Var - Cov matrix of The 4 Stocks

### 4.3. Calculate portfolio mean and standard deviation (weighted according to the level of investment in each stock in the portfolio)

Before calculating the average value and standard deviation of the portfolio, we firstly calculate the daily average return of each stock of the four stocks in the portfolio, and write the code as follows:

```
# avg_rets = data.mean()
# avg_rets
# Outcome:
      AAPL    0.001203
      BAC    0.001100
      AXP    0.000679
      KO     0.000371
      dtype: float64
```

**Figure 4** The Daily Average Return of The 4 Stock

Calculate the average return of the entire portfolio and normalize the investment weight:

```
# port_mean = avg_rets.dot(weights)
# port_mean
# Outcome: 0.0007429043925035794
Calculate portfolio standard deviation:
# port_stdev =
np.sqrt(weights.T.dot(cov_matrix).dot(weights))
# port_stdev
# Outcome: 0.010154772959830884
Calculate the average value of investment:
# mean_investment = (1+port_mean) *
initial_investment
# mean_investment
# Outcome: 1000742.9043925036
Calculate the standard deviation of investment:
# stdev_investment = initial_investment * port_stdev
# stdev_investment
# Outcome: 10154.772959830885
```

Thus, we can get the standard deviation and average value of the portfolio:

**Table 1** the standard deviation and average value of the portfolio

|                            |                    |
|----------------------------|--------------------|
| Standard deviation (stdev) | 10154.772959830885 |
| average value (mean)       | 1000742.9043925036 |

### 4.4. Calculate the inverse of the normal distribution cumulative function with the specified confidence interval, standard deviation, and mean

We input the standard deviation and mean variables got in step C into the Percent Point Function (PPF). The Percent Point Function (PPF) is the inverse of the cumulative distribution function. For this reason, the Percent Point Function is also often called the inverse distribution function. That is, in the cumulative distribution function, for a given  $x$ , we calculate the probability that the variable is less than or equal to  $x$ . For the the Percent Point Function, we start with a given probability and compute the corresponding  $X$  for the cumulative distribution.

Firstly, 95% confidence is selected in this case:

```
# conf_level1 = 0.05
```

Since the inverse cumulative distribution function is still normally distributed, the mean value and standard deviation of the portfolio obtained in the above steps are inserted with the following codes and results:

```
# from scipy.stats import norm
# cutoff1 = norm.ppf(conf_level1, mean_investment,
stdev_investment)
# cutoff1
# Outcome: 984039.789258657
```

### 4.5. Estimate the VaR of the portfolio by subtracting the calculation in step D from the initial investment.

Using the following code, we can estimate the portfolio's value at risk (VaR) with a 95% confidence interval by subtracting the value returned by the inverse cumulative distribution function obtained in Step C from the initial investment:

```
# var_1d1 = initial_investment - cutoff1
# var_1d1
# Outcome: 15960.210741342977
```

So we get the value at risk of the portfolio based on the proportion of Berkshire Hathaway's major stock holdings: VaR = 15960.210741342977. Here, we have a level of 95% confidence to say that our \$1 million



portfolio will not lose more than \$16,000 in a single trading day.

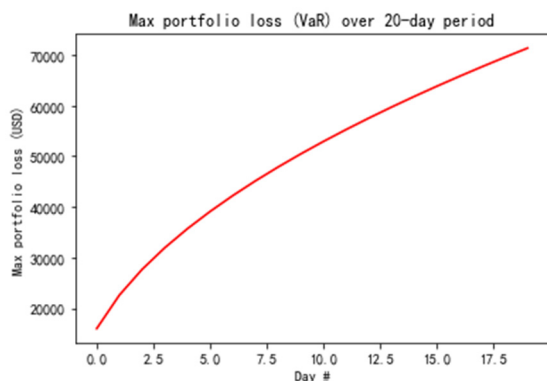
When it comes to a larger time horizon like the VaR over  $n$  days, it can simply take the 1-day VaR and multiply it by the square root of the time period (this is because the standard deviation of stock returns will increase with the square root of time).

For example, the VaR of the portfolio at risk in this case can be calculated and visualized in Python for the VaR of the 20-day period based on the given VaR of 1 day. The code, results and visualization are shown as follows:

```
# var_array = []
# num_days = int(20)
# for x in range(1, num_days+1):
#     var_array.append(np.round(var_1d1 *
#                               np.sqrt(x),2))
# print('VaR of 40days',var_array)
# print(str(x) + " day VaR @ 95% confidence: " +
# str(np.round(var_1d1 * np.sqrt(x),2)))
# plt.xlabel("Day #")
# plt.ylabel("Max portfolio loss (USD)")
# plt.title("Max portfolio loss (VaR) over 20-day
# period")
# plt.plot(var_array, "r")
# plt.show()
# import matplotlib.mlab as mlab
# import matplotlib.pyplot as plt
# import scipyNote

# Outcome: VaR of 20days [15960.21, 22571.15,
27643.9, 31920.42, 35688.12, 39094.37, 42226.75,
45142.29, 47880.63, 50470.62, 52934.03, 55287.79,
57545.36, 59717.64, 61813.63, 63840.84, 65805.63,
67713.44, 69568.95, 71376.23]
```

# 20 day VaR @ 95% confidence: 71376.23

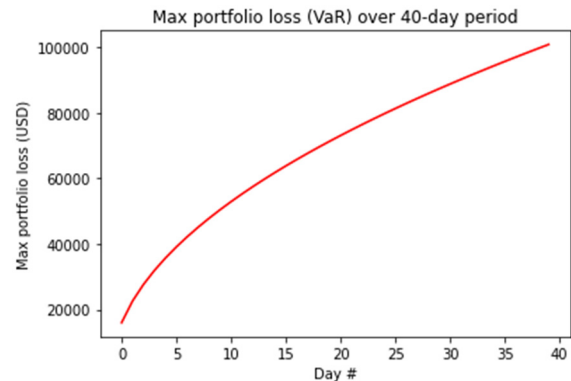


**Figure 5** Max Portfolio Loss (VaR) over 20-day Period

Here, we have 95% confidence to say that our 1 million dollars portfolio will not lose more than \$71,376 within 20 days.

Repeat the similar procedure to get 40-days VaR above :

# 40 day VaR @ 95% confidence: 100941.24



**Figure 6** Max Portfolio Loss (VaR) over 40-day Period

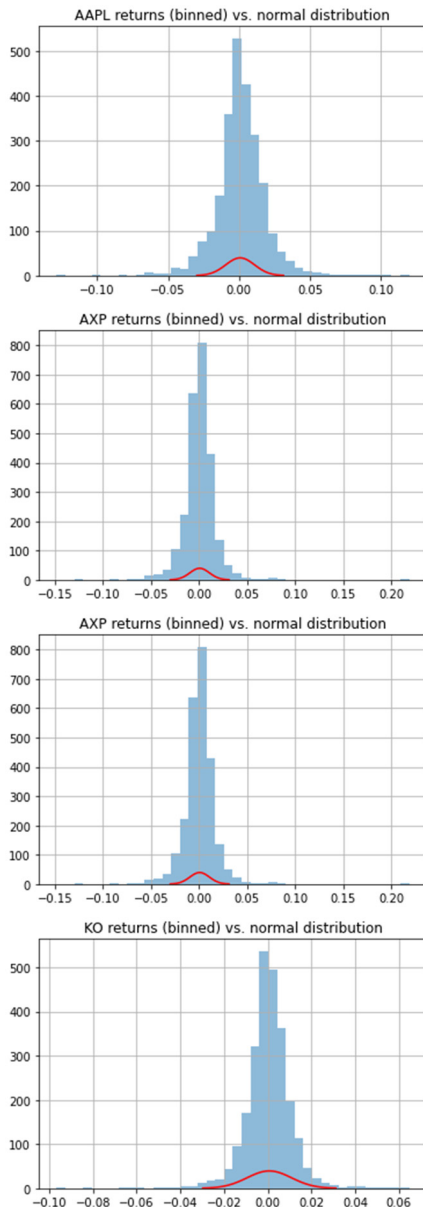
Here, we have 95% confidence to say that our 1 million dollars portfolio will not lose more than \$100,900 within 40 days.

## 5. VISUALIZATION TEST

The application of variance - covariance method is based on the assumption of normality [16]. Therefore, after the above data processing and calculation, we get the corresponding VaR, and then need to test the return rate of the closing prices of the four American stocks we selected by referring to the normal distribution. As mentioned in the calculation section when we use variance - covariance method to calculate VaR, the returns of stocks in the simulated portfolio constructed should be normally distributed. Admittedly, we can't predict the future of this distribution, but we can at least examine how the historical returns of all four stocks have been allocated to help us assess whether VaR is appropriate for our portfolio. For this reason, the following code is used to test the coincidence comparison between income and normal distribution:

```
# for c in code:
#     fig = plt.figure()
#     data[c].hist(bins=40,histtype="stepfilled",
alpha=0.5)
#     plt.show()
#     x = np.linspace(port_mean - 3*port_stdev,
port_mean+3*port_stdev,100)
#     plt.plot(x, scipy.stats.norm.pdf(x, port_mean,
port_stdev), "r")
```

```
# plt.title(c+" returns (binned) vs. normal
distribution")
# plt.show()
```

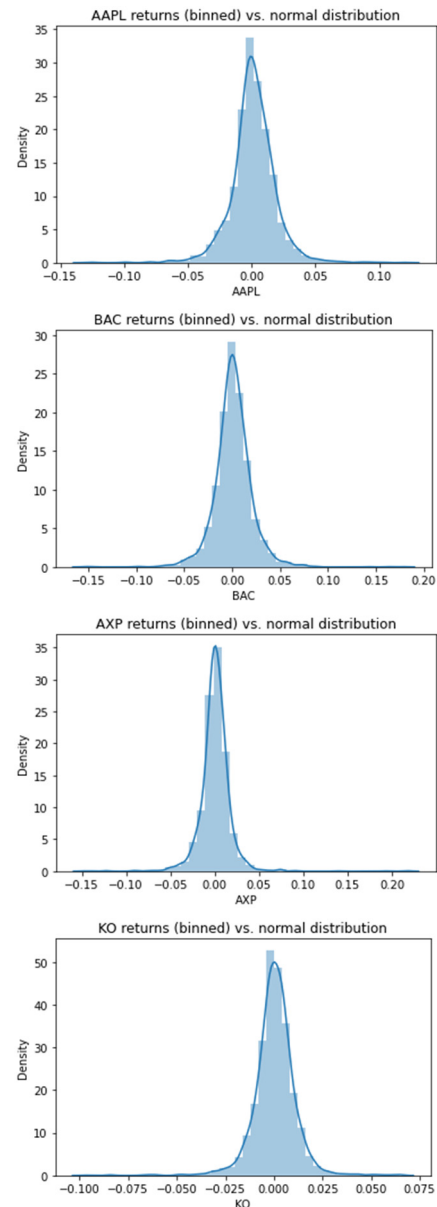


**Figure 7** Returns (Binned) & Normal Distribution (AAPL, BAC, AXP, KO)

By introducing seaborn library, visual optimization is carried out on the comparison graph of revenue and normal distribution:

```
# import seaborn as sns
# for c in code:
#     fig = plt.figure()
#     #x = np.linspace(port_mean - 3*port_stdev,
# port_mean+3*port_stdev,50)
#     #plt.plot(x, scipy.stats.norm.pdf(x, port_mean,
# port_stdev), "r")
```

```
# #data[c].hist(bins=40,histtype="stepfilled",
alpha=0.5)
# plt.title(c+" returns (binned) vs. normal
distribution")
# sns.distplot(data[c],bins=40)
# plt.show()
```



**Figure 8** Rest of Normal Distribution (AAPL, BAC, AXP, KO)

From the above visualization results, we can say that the returns of the four stocks selected by the simulated portfolio in this paper have been basically in line with normal distribution since 2011.

## 6. CONCLUSION

Based on the introduction of the concept, formula and measurement method of VaR above and the empirical

study of specific portfolio cases, we can know that it is very intuitive and simple to analyze the VaR based on the variance - covariance method of VaR model. In most cases, it is relatively accurate to predict the future price fluctuation of a given financial product or portfolio by using VAR model, and it is beneficial to financial investment to measure the value of risk of investment. The method is simple and feasible, and it is a good choice for analyzing VaR. In addition, risk measurement is based on probability theory and mathematical statistics, having a strong scientific nature. VaR changes the lack of unified measurement of risk expression in different financial markets, making different terms (such as base value, existing position, etc.) have a unified comparison standard, and making people in different industries have a common measurement when discussing their market risks. With unified standards, financial institutions can regularly measure and publish VaR values, which strengthen market transparency, help improve investors' understanding of the market, enhance investors' investment confidence and stabilize the financial market [16].

The variance - covariance method has the advantages of simple principle and easy calculation. However, it also has some disadvantages, mainly in three aspects: (1) Because of estimating the future based on historical data, it can not predict the risk of emergencies; (2) The normal assumption condition of variance - covariance method is questioned; (3) The variance - covariance method only reflects the first-order linear effect of risk factors on the whole portfolio, and cannot fully measure the risk of nonlinear financial instruments (e.g. option). Only when portfolio value changes are linearly related to daily percentage changes or actual changes in risk factors can results be fast produced and daily changes in risk factors are assumed to be multivariate normal distributions. Actually, the distribution of risk factors that vary from day to day is often different from the normal distribution. Users of the variance - covariance method hope for some form of statistical central limit theorem that will allow the daily gains and losses of a large portfolio to follow a normal distribution, even if the changes in the portfolio components themselves do not. At the same time, Monte Carlo Simulation Approach can also be used to analyze the nonlinear situation, which is also the direction of our future research on the VaR of portfolio.

Despite there are some disadvantages of the variance - covariance method in practical application, as a principle of simple and straightforward value at risk measure, it can still provide an effective risk management tools to the financial institutions and investors of the portfolios' VaR which can efficiently do the risk control and performance evaluation. It is of great practical significance for the construction of financial markets to prevent excessive speculation and enable the financial regulators to supervise financial investment effectively [17].

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