Simulation of French CAC40 Index for the First 20 Years in the 21st Century with Random Walk Model

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Abstract
The French CAC40 index represents the most important index for the French stock market, and is composed of 40 most significant stocks in the Euronext Paris. Thus, the behavior of the CAC40 index is the subject of numerous studies, of which the efficient market hypothesis (EMH) is a hot topic. One way to prove EMH is to conduct statistical tests on the given stock index to determine whether or not a stock index meets the random walk. Apart from statistical tests, we prefer to use the random walk model to simulate a stock index because such results are more visible and convincing. In this study, we conduct two types of random simulations, (i) to use the classical random walk, which is in the 1/–1 form, to simulate the CAC40 in a simplified form for 2020, and (ii) to use the random walk in decimal form to simulate the real-life CAC40 for the first 20 years in the 20th century stratified into five segments with increment of five years for each. The results show that the simulations in theory have a chance to perfectly simulate the CAC40, but this theoretical perfect simulation is limited by the ability of the Monte Carlo algorithm in generation of random numbers.

Keywords*-CAC40 index; random walk; simulation; finance and trade; stock market*

1. INTRODUCTION

French CAC40 index represents the most important index for the French stock market, and is composed of 40 most significant stocks in the Euronext Paris. Thus, the behavior of the CAC40 index is the subject of numerous studies, of which the efficient market hypothesis (EMH) and adaptative market hypothesis (AME) are hot topics [1, 2].

A way to verify EMH is the random walk hypothesis [3-5]. Usually, the verification goes through various statistical tests, for instance, variance ratio test, unit root test, autocorrelation test, and run test [5-8].

We employ a different approach to test the random walk hypothesis by means of directly simulation of stock indices with random walk [9-15] because the computational power increased significantly over recent years. Thus, the direct fitting becomes possible in this regard.

However, the results not only from our studies but also from the studies using statistical tests are inconclusive, even contradict one another. Hence, we necessarily conduct more studies on this topic using diverse approaches to different datasets in order to understand the underlined mechanisms from different angles and aspects.

2. MATERISAL AND METHODS

2.1. CAC40 Data

The daily French CAC40 index for the first 20years in the 21st century was obtained from Yahoo Finance. These data from 2001 to 2020 include 5111 daily open, high, low, close, adjusted close, and volume, of which we choose the daily close as the target for random walk simulation. The simulations were designed to follow five segments with increment of five years for each, 2020 includes 227 trading days, 2016-2020 includes 1279 trading days, 2011-2020 includes 2554 trading days, 2006-2020 includes 3832 trading days, and 2001-2020 includes 5111 trading days.

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2.2. Random Walk Model

We use the random walk model, which is a path generated by tossing a fair coin continuously while two sides of coin are defined as 1 and –1 [14]. The continuous tossing generates a series of 1 and –1 over each tossing. The addition of these 1 and –1 generates a random walk over time. In computation, the random walk is generated using the random numbers produced by the Monte Carlo algorithm, and we round each generated random number to its integer.

2.3. CAC40 Index in Simplified Form

Because the classical random walk in the 1/–1 form, we can also change the CAC40 index into such a form, which practically means if the CAC40 index in a given day is higher or lower than that in the previous day, then we mark these changes in the 1/–1 form. However, we have no intention to simulate this type of simplified data for all the time segments in our study, but just focus on the simulation from January 2, 2001 to December 30, 2020.

2.4. Random Walk in Decimal Form

To advance our simulation, we also design a random walk in the decimal form, not only because the target of our simulation is in decimal form but also more importantly because the random walk in essence is the addition of a series of random numbers together. So it does not matter to limit us in the 1/–1 form random walk.

2.5. Simulation

With the random walk in both 1/–1 and decimal forms, we use software, SigmaPlot [15], with the Monte Carlo algorithm to random numbers with command, number of generated random numbers, seed, upper and lower ranges. The random walk generated by addition of random numbers is compared with the CAC40 index in either 1/–1 or decimal forms. The minimal squared difference between random walk simulation and the CAC40 is found from 100,000 seeds ranging from 0 to 1 with the difference of 0.00001 between seeds.

TABLE 1. PROCEDURE TO CONSTRUCT A RANDOM WALK SIMULATION IN 1/–1 AND DECIMAL FORMS

Date	CAC40 index Close	Compare Preceding Close	Random Walk in 1 or –1 Form	Generated Random Number	Compare Preceding Random Number	Random Walk in 1 or –1 Form	Generated Random Number	Random Walk in Decimal Form
Jan 2, 2020	6041.5			0.96164			–6.20127	
Jan 3, 2020	6044.16	1	1	0.31973	–1	–1	88.73797	6130.24
Jan 6, 2020	6013.59	–1	0	–0.44631	–1	–2	–67.23357	6063.00
Jan 7, 2020	6012.35	–1	–1	–0.23487	1	–1	–20.90443	6042.10
Jan 8, 2020	6031	1	0	0.75029	1	0	19.75531	6061.86
Jan 9, 2020	6042.55	1	1	0.61871	–1	–1	–37.53048	6024.32
Jan 10, 2020	6037.11	–1	0	–0.86685	–1	–2	–85.01246	5939.31
Jan 13, 2020	6036.14	–1	–1	–0.00005	1	–1	–34.67969	5904.63
Jan 14, 2020	6040.89	1	0	–0.31101	–1	–2	–67.68956	5836.94
Jan 15, 2020	6032.61	–1	–1	0.47518	1	–1	–14.21329	5822.73

3. RESULTS AND DISCUSSION

Table 1 shows the procedure of how to perform the random walk simulation in both 1/–1 and decimal forms. Columns 1and 2 are the date and its corresponding CAC40 index close for the beginning of 2020. Column 3 is the comparison between two CAC40 index closes in sequential days with 1 or –1 for larger or smaller than its preceding day, for example, 6044.15, the CAC40 index close on January 3, 2020 is larger than 6041.5, the CAC40 index close on January 2, 2020, so 1 was given to cell 2 in column 3. Column 4 is the addition of each cell in column 3, and builds a curve of CAC40 index in the 1/–1 form. Column 5 is a series of random numbers generated by the Monte Carlo algorithm. Column 6 is the comparison between two random numbers in sequential cells in Column 5, which is as the same as what is done in Column 3. Column 7 is the addition of each cell in Column 6, and builds a random walk simulation for comparison with Column 4. The last two columns show the procedure of how to build a random walk simulation in the decimal form. Column 8 is a series of random numbers generated by the Monte Carlo algorithm with the standard deviation of the CAC40 index close in 2020 as upper and lower ranges. Column 9 is the addition of each cell in Column 8 to the corresponding CAC40 index close value in Column 2, i.e. the random walk simulation in decimal form. Finally, comparison can be made between Columns 2 and 9.

Figure 1 shows that the random walk simulation in the 1/–1 form can very approximately fit the CAC40 index in the simplified form although the chance for a simulation without a single mistake is very small, $(½)^{227}$.

Figure 2 shows that the random walk simulation in decimal form cannot deal with the sudden fall due to Covid-19 pandemic in March 2020. This could be the failure of random walk. However, to the best of our knowledge, no other models can handle the unexpected fall in stock due to Covid-19, too.

The difference between Figures 1 and 2 is the difference between 1/–1 and decimal forms, but they reveal the same mechanism, that is, the random walk can simulate the CAC40 index. However, the random walk in the 1/–1 form has the advantage that is absent from the random walk in decimal form, i.e., it does not care the upper and lower ranges in command because we round all random numbers generated by Monte Carlo algorithm into integer. In contrast, the random walk in decimal form suffers the uncertainty in upper and lower ranges.

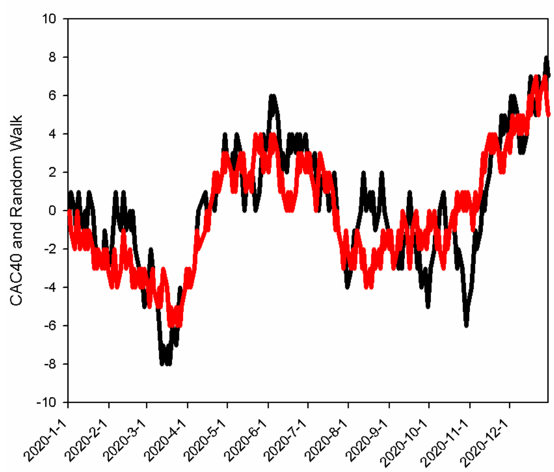

Figure 1. The CAC40 inxed in 2020 in 1/–1 form (black line) and its random walk simulation (red line) in 1/–1 form using the seed of 2.14840.

Figure 3 shows that the random walk in the decimal can relatively well simulate the CAC40 index before the impact of Covid-19 pandemic. In order to follow the big fall due to Covid-19, the simulation curve has to end its uptrend to the peak before Covid-19. Thereafter, the simulation has a somewhat similar tendency to follow the CAC40 index approaching to the end of 2220.

Figure 4 shows the random walk simulation on CAC40 index for ten years. As seen, the simulation looks better than those in Figures 2 and 3, because the CAC40 index' sharp decline in March 2020 appears less visible. This is actually attributed to the increased range in the CAC40 index. It goes up from 2781.68 on September 22, 2011 up to 6111.24 on February 19, 2020 (Figure 4) whereas the CAC40 index ranges from 3754.84 on March 18, 2020 in Figure 2 and Figure 3. Therefore, the enlarged range compensates the shape decline in March 2020.

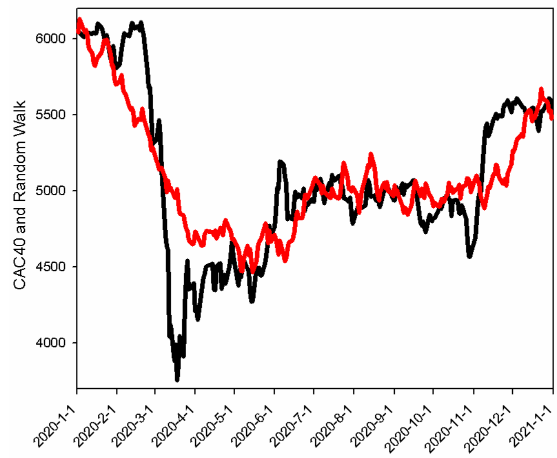

Figure 2. The CAC40 index in 2020 (black line) and its random walk simulation (red line) in decimal form using the s eed of 0.82081.

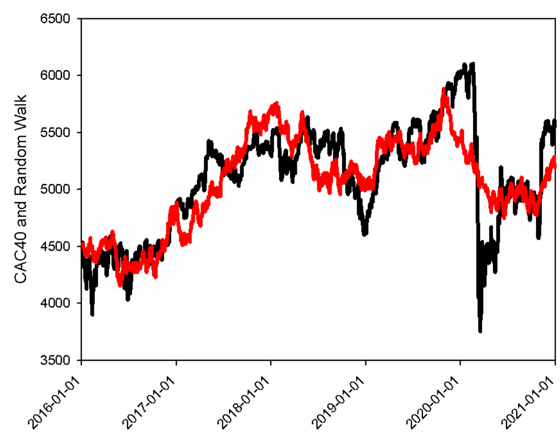

Figure 3. The five-year CAC40 index from 2016 to 2020 (black line) and its simulation (red line) generated by random walk in decimal form using the seed of 2.36571.

Figure 5 shows the random walk simulation on CAC40 index for fifteen years. The random walk simulation in this segment of time faces two significant peaks and several valleys. Although the simulation cannot rise up rapidly for the first peak, 6168.15 on June 1, 2007, and dives fast enough to three big valleys, it rises for the second peak in 2020. In the sense of general trend, the random walk simulation follows the CAC40 index along its upward and downward trends. This could be potentially useful to forecast the general trend in CAC40 index for a certain period of time in future.

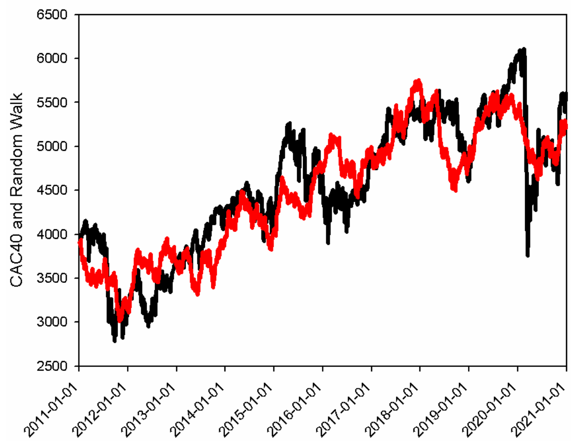

Figure 4. The ten-year CAC40 index from 2011 to 2020 (black line) and its simulation (red line) generated by random walk in decimal form using any of four seeds from 5.29542 to 5.29546 with increament of 0.00001.

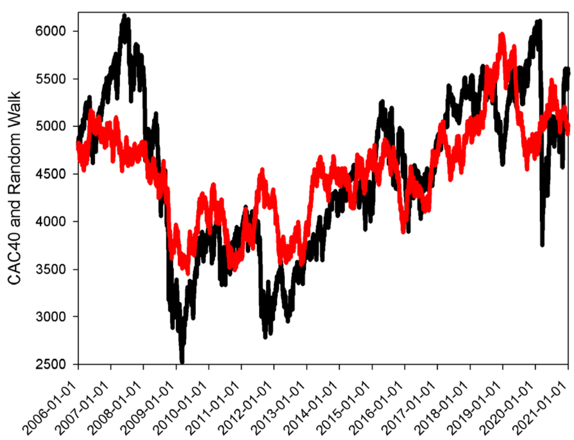

Figure 5. The fifteen-year CAC40 index from 2006 to 2020 (black line) and its simulation (red line) generated by random walk in decimal form using a seed of 1.26074.

Figure 6 shows the difficulty to use random walk simulation on CAC40 index for twenty years because the random walk simulation fails to follow the peaks and valleys. Clearly, this failure suggests the time-length dependency in the simulations. As said in Figure 1, the chance for a perfect simulation to CAC40 index in the simplified form is $(½)^{227}$. This is in terms of the 1/–1 form random walk. For the random walk simulation in the decimal form, the chance of perfect simulation will dramatically decrease. As the CAC40 index have six digitals in the first 20 years in the 21st century, it means a perfect simulation for 2020 will have a $(1/1000000)^{227}$ chance. This number is far, far beyond our searching scope, which is limited to 100,000 seeds for all our simulations.

For the other periods of time, the chance will decrease astonishingly, i.e. $(1/1000000)^{1279}$ for the period from 2016 to 2020, $(1/1000000)^{2554}$ for the period from 2011 to 2020, $(1/1000000)^{3832}$ for the period from 2006 to 2020, and $(1/1000000)^{5111}$ for the period from 2001 to 2020. This is the time-length dependency.

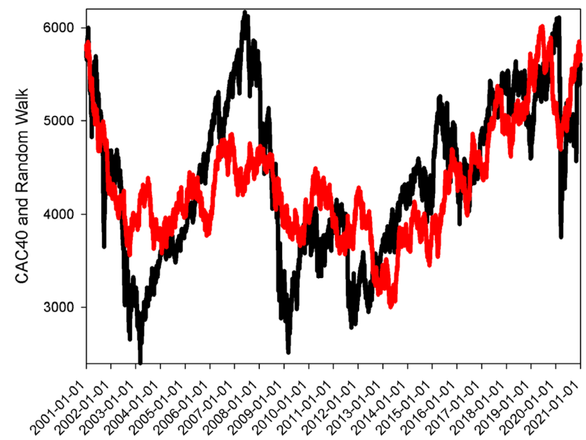

Figure 6. The twenty-year CAC40 index from 2001 to 2020 (black line) and its simulation (red line) generated by random walk in decimal form using any of seeds of 1.81538 and 2.85651.

Therefore, how to enlarge the searching scope of seeds in a problem to be solved regarding this time-length dependency because we used to search one million seeds ranged from 1 to 10, but we find that many seeds produce the same results. To reduce the computational time, we scale down the size of seeds from one million to one hundred thousands, but we can still find the case where several seeds produce the same results, for example, Figure 4 and Figure 6 in this study. In our previous studies, we also attempted to use the seeds outside range of 0 to 10, but these seeds did not produce encouraging and promising results.

At this point, the time-length dependency that we observed in our studies is different from the issues of whether the choice of time period for simulation affects the market efficiency in the weak-form and whether the market efficiency improves over time. In this context, we would be better to divide our samples into equal length periods in our future studies.

Practically, the random walk model is more explicit than neural network models, where one does not know the underlined mechanisms such as biases and weights, and simpler than any other deterministic and stochastic models, where one can find various parameters with different dimensions and meanings. However, the seed in random walk model has less clear meanings, which should be explored in our future.

In the past, our concern was whether the random walk model works in such simulation, but now the question goes to whether a totally random mechanism dominates the stock index to a tiny period of time. This requires more studies in future.

4. CONCLUSION

In this study, we employ the random walk in both 1/–1 and decimal forms to simulate the CAC40 index in five segments of time. The results show that the random walk can simulate the CAC40 index to different degrees, and has better simulations for short period of time than for long period of time. On the one hand, the results partially support the notion that the stock market behave randomly. On the other hand, the results also suggest the limitation in choosing seeds, lower/upper ranges for simulation.

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