# Retailers' Optimal Advance Selling Pricing and Inventory Decision with Virtual Showrooms 

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#### Abstract

With the development of e-commerce, the advance selling strategy with showrooms is widely adopted by many retailers in order to solve the uncertainty of consumer valuation and reduce inventory risk. In order to make the optimal pricing and inventory decision when the retailers share information with consumers in the pre-sale period, this paper studies the advance selling decision with virtual showrooms. The virtual showroom allows consumers get an imperfect signal of their valuation online, which may lead to returns in regular selling period. The pre-sale strategy with virtual showrooms hasn't been studied. This paper establishes a model and derives the optimal advance selling price and ordering quantity for the regular season. Next, this paper analyzes how different parameters affect retail's optimal decision in the numerical examples. The results show that the retailer's optimal price varies along with the retailer's return cost, the consumer's hassle cost and the informatization degree of the virtual showroom.


Keywords-advance selling; showroom; strategic consumer; product valuation uncertainty

## 1. Introduction

Advance selling is a marketing method in which the seller provides the buyer with purchase opportunities before its release date ${ }^{[1][3]}$. It can benefit retailers for several reasons. For instance, retailers can reduce inventory risk by updating demand forecasts ${ }^{[45]}$.During advance selling period, consumers' uncertainty about the value of products may lead to refusal of purchase or return, which has strong influences on their behavior ${ }^{[6]}$. Thus, taking return behavior into consideration is essential in this research. To address this problem, some retailers have employed the advance selling strategy with virtual showrooms.

With the development of virtual reality technology, showrooms have made the transition from physical to virtual. Shoppers can try new products online through virtual showrooms ${ }^{[7]}$. On the website of glasses retailer BonLook, consumers can know how well the glasses fit them through the lens. Taobao has also launched similar activities. For example, shoppers can inspect the fit of a pair of shoes through virtual showrooms. More and more suppliers provide many advanced technologies. For example, 3D scanning technology ${ }^{[8]}$. Some studies believe that the virtual exhibition hall may become a potential remedy for the return of products from online channels ${ }^{[911]}$.

As an alternative way to alleviate strategic waiting behavior, physical showrooms and virtual showrooms provide consumers with offline and online places to experience products before purchase. This is usually used in omni-channel retail as two channels for retailers to provide information to consumers ${ }^{[1216]}$. Reference [17] made a distinction between single channel and selling with virtual showrooms. Reference [18] established a model to study consumers inter product display behavior and information provision. Physical showrooms and virtual showrooms can be regarded as two ways for retailers to share product information with consumers.

Although both physical showroom and virtual showroom are designed to solve the problem of uncertain product valuation of consumers and induce consumers to buy, in addition to the different setting costs, the two mechanisms still have differences in the following aspects: on the one hand, the physical showroom allows consumers to check goods in the store. Through personal trial, the uncertainty of consumers' valuation of products is basically solved. The virtual showroom imitates the physical existence, so that online shoppers can evaluate products and reduce the uncertainty of product value. The uncertainty, however, still exists. Because technology is never perfect. On the other hand, trying products in physical showrooms will incur hassle costs for consumers (e.g., going to offline shop or searching the shelves for the
product). The hassle costs arise before customers purchase products. However, the hassle costs through virtual showrooms arise after customers purchase products online, that is, the possible return cost due to inaccurate valuation. The research most closely related to the studies in [8] and [22]. But there are several differences between this research and theirs Reference [8] studied how retailers can effectively provide information to consumers, including physical and virtual showrooms, but do not consider advance selling. Reference [22] discussed the optimal advance selling strategy when retailers provide physical showrooms, but this paper considers the optimal advance selling strategy when retailers provide virtual showrooms. Second, this paper studies the pricing and inventory decision of advance selling, while [22] only studied the pricing decision.

Thus, it is crucial to consider the optimal pricing and inventory decision of advance selling with virtual showrooms. Specifically, this paper attempts to answer the following questions: (1) What are the optimal pricing and inventory decisions with virtual showrooms? (2) How do the costs of retailers and consumers (for example, hassle cost of consumers, and return cost of retailers) affect the optimal decision?

## 2. The Model with Virtual Showrooms

Retailers sell products in a pre-sale period and a regular selling period. Consumers are divided into informed and uninformed consumers. Informed consumers make purchase decisions in the pre-sale period. Uninformed consumers are ignorant of advance selling. Thus, they will decide whether to buy the product in the regular selling period. Let $N_{i}$ and $N_{u}$ denote the size of informed and uninformed consumers, respectively. Suppose $N_{j}, j \in\{i, u\}$, are bivariate normal distributions with means $\mu_{j}$, standard deviations $\sigma_{j}$, and correlation coefficient $\rho \in(-1,1)$. The valuation of consumers, denoted by $V$, is uncertain in the pre-sale period. It will not be resolved until the product arrives. This paper assumes that $V$ follows a uniform distribution between $l$ and $h$, where $l<h$.

The process of events is described as follows. (1) During advance selling period, retailers announce the advance selling price $X$ and the regular selling price $p^{[19]}$. (2) The informed consumers arrive and decide whether to examine the new product in the virtual showroom before they purchase in advance. (3) Before the beginning of the regular period, retailers decide the quantity of products to procure (i.e., $Q+n$ ) at cost $c$ per unit. And $n$ denotes the advance demand, and $Q$ is expected demand in regular period. The paper assumes that pre-sale products will not be out of stock, which means each consumer who participates in pre-sale can certainly receive his product. (4) During regular selling period, uninformed consumers decide whether to buy. Previously purchased customers will receive the product. Depending on whether they like
the product, consumers decide whether to keep or return it. If they keep it, their surplus will be $V-X$. This research assumes that each return will bring a net loss $r>0$ for retailers and the same troublesome cost $h_{r}>0$ for consumers. (5) After the end of the regular period, unsold items are salvaged at $s$ per unit.

Let $\eta$ denotes the risk of stock-out. Let $U_{W}, U_{S}$ be the expected utility of consumers who don't purchase in advance and purchase in advance, and the expected utility of consumers who solve the value uncertainty and purchase in advance, respectively. Hence consumers' expected utility can be expressed as

$$
\begin{gather*}
U_{S}=\int_{X}^{h}(v-X) f(v) d v,  \tag{1}\\
U_{W}=(1-\eta) \int_{p}^{h}(v-p) f(v) d v . \tag{2}
\end{gather*}
$$

When retails offer virtual showrooms during pre-sale, consumers can try on products virtually, even though it's not perfect. That means that consumers can realize their valuation through virtual showrooms. As stated in [8], virtual showrooms only screen out part of low-type customers. Low-type consumers refer to those who will eventually choose to return products. And high-type customers will eventually choose to buy products. We assume that the customers who are screened out include a fraction $\alpha \in(0,1]$ of the low-type customers. Note that if $\alpha=1$, the virtual showrooms give perfect show that consumers realize their valuation. However, if $\alpha<1$, virtual showrooms screen out a fraction $\alpha$ of the low types. Therefore, this paper interprets $\alpha$ as the informatization degree of virtual showrooms.

Bayesian updating is adopted by this research to the consumers' proportion who will eventually choose to buy the product. Let $F^{\prime}\left(X-h_{r}\right)$ denotes the posterior probability of a remaining customer being of low type. Then, we have

$$
\begin{gather*}
\bar{F}^{\prime}\left(X-h_{r}\right)=\frac{\bar{F}\left(X-h_{r}\right)}{\left(1-\alpha F\left(X-h_{r}\right)\right)}>0,  \tag{3}\\
F^{\prime}\left(X-h_{r}\right)=1-\bar{F}^{\prime}\left(X-h_{r}\right)=(1-\alpha) F(X- \\
\left.h_{r}\right) /\left(1-\alpha F\left(X-h_{r}\right) .\right. \tag{4}
\end{gather*}
$$

As a result, the total demand size in advance selling period is

$$
\begin{equation*}
N_{i}^{\prime}=\left[1-\alpha F\left(X-h_{r}\right) N_{i} .\right. \tag{5}
\end{equation*}
$$

We consider that consumers have only two purchase options: inspect before purchase or waiting for the next period. Compared with purchase in advance directly, informed consumers prefer to inspect before buying, and the utility will not be reduced.

Under this condition, informed consumers inspect new products through virtual showrooms before purchase if and only if $U_{S} \geq U_{W}$; uninformed consumers make
purchase decisions based on utility during the regular period.

We use advance demand information to improve the prediction of regular demand $N_{2}{ }^{[20]}$. After updating, demand $N_{2}$ has a new mean $\mu_{2}{ }^{\prime}$ and standard deviation $\sigma_{2}{ }^{\prime}$, which is expressed as follows:

$$
\begin{align*}
& \mu_{2}^{\prime}=\left[\mu_{u}+\rho\left(n-\mu_{i}\right) \frac{\sigma_{u}}{\sigma_{i}} \bar{F}(p),\right.  \tag{6}\\
& \sigma_{2}^{\prime}=\sigma_{2} \sqrt{1-\left[\operatorname{corr}\left(N_{1}, N_{2}\right)\right]^{2}}=\sigma_{u} \bar{F}(p) \sqrt{1-\rho^{2}} . \tag{7}
\end{align*}
$$

The retailer decides the optimal order quantity $Q_{l}$ to maximize the expected profit $\prod_{1}$. According to [21], the optimal order quantity and corresponding total expected profit can be expressed as follows:

$$
\begin{gather*}
Q_{1}=\mu_{2}^{\prime}+k \sigma_{2}^{\prime}-F^{\prime}\left(X-h_{r}\right) N_{1}=\mu_{2}^{\prime}+k \sigma_{2}^{\prime}- \\
(1-\alpha) F\left(X-h_{r}\right) N_{i}, \\
\Pi_{1}=\left[(X-c) \bar{F}^{\prime}\left(X-h_{r}\right)-r F^{\prime}\left(X-h_{r}\right)\right][1- \\
\left.\alpha F\left(X-h_{r}\right)\right] \mu_{i}+(p-c) \bar{F}(p) \mu_{u}-(p- \\
s) \varphi(k) \sigma_{u} \bar{F}(p) \sqrt{1-\rho^{2}} . \tag{9}
\end{gather*}
$$

where $k=\varphi^{-1}\left(\frac{p-c}{p-s}\right), \varphi(\cdot)$ represents the density distribution functions of the standard normal distribution. The first term indicates the profit during pre-sale period. Moreover, $r(1-\alpha) F\left(X-h_{r}\right) \mu_{i}$ gives retailer's loss of consumers' return. The remaining items give retailer's profit in the regular period.

## 3. The Model Analysis and Numerical Simulation

In this section, we give the retailer's optimal pricing decision, as shown in Proposition 1 below.

Proposition 1 When the retailer adopts advance selling with virtual showrooms, he will set an optimal advance selling price

$$
\begin{equation*}
X^{*}=\min \left\{X_{1}^{*}, X_{2}^{*}\right\} \tag{10}
\end{equation*}
$$

where

$$
\begin{gather*}
X^{*}=X_{1}^{*}=\frac{h+h_{r}+c+r(\alpha-1)}{2},  \tag{11}\\
X^{*}=X_{2}^{*}=h+h_{r}-\sqrt{(1-\eta)(h-p)^{2}+2(h-l) h_{r}} \tag{12}
\end{gather*}
$$

Then all informed consumers choose to inspect through virtual showrooms before purchase. The corresponding total profits are:

$$
\begin{align*}
& \quad \Pi_{11}^{*}= \\
& {\left[\frac{\left(h+h_{r}-c\right)^{2}-r^{2}(1-\alpha)^{2}-2(1-\alpha) r\left[h+c-r(1-\alpha)-h_{r}-2 l\right.}{4(h-l)}\right] \mu_{i}+(p-} \\
& \text { c) } \bar{F}(p) \mu_{u}-(p-s) \varphi(k) \sigma_{u} \bar{F}(p) \sqrt{1-\rho^{2}}, \tag{13}
\end{align*}
$$

$$
\begin{gather*}
\Pi_{12}^{*}=\left[\frac{\left.\left(h+h_{r}+r-r \alpha-c\right) \sqrt{(1-\eta)(h-p)^{2}+2(h-l) h_{r}}\right)}{h-l}-\right. \\
\left.\frac{(1-\eta)(h-p)^{2}+2 h_{r}(h-l)+r(1-\alpha)(h}{h-l}\right] \mu_{i}+(p-c) \bar{F}(p) \mu_{u}- \\
(p-s) \varphi(k) \sigma_{u} \bar{F}(p) \sqrt{1-\rho^{2}} . \tag{14}
\end{gather*}
$$

Proposition 1 indicates that the optimal pre-sale price is separated into two situations according to the return cost. Then, Lemma 1 presents some analytical properties of the optimal price $X^{*}$.

Lemma 1 (i) The optimal pre-sale price of retailers increases with the troublesome cost of return if $0<h_{r}<$ $\max \{0, G\}$ or $h_{r}>H$, but is convex in the troublesome cost if $\max \{0, G\} \leq h_{r}<H$, where
$G=c+3 h-4 l-r(1-\alpha)-$
$2 \sqrt{(1-\eta)(h-p)^{2}+2 c(h-l)+2 h^{2}-6 h l-2 h r(1-\alpha)+2 \operatorname{lr}(1-\alpha)+4 l^{2}}$,
$H=c+3 h-4 l-r(1-\alpha)+$
$2 \sqrt{(1-\eta)(h-p)^{2}+2 c(h-l)+2 h^{2}-6 h l-2 h r(1-\alpha)+2 l r(1-\alpha)+4 l^{2}}$.
(ii) The optimal pre-sale price of retailers decreases with the increase of return cost if

$$
\begin{equation*}
r>\frac{c-h-h_{r}+2 \sqrt{(1-\eta)(h-p)^{2}+2(h-l) h_{r}}}{1-\alpha} \tag{17}
\end{equation*}
$$

Otherwise, the optimal pre-sale price does not change with the return cost.
(iii) The optimal pre-sale price of retailers increases with the increase of the informatization degree of the virtual showroom if

$$
\begin{equation*}
\alpha<\frac{h+h_{r}-r-c-2 \sqrt{(1-\eta)(h-p)^{2}+2(h-l) h_{r}}}{r} . \tag{18}
\end{equation*}
$$

Otherwise, the optimal pre-sale price does not change with the degree of informativeness.

Lemma 1 (i) indicates that the optimal pre-sale price increases first, decreases afterwards, and at last increases with the troublesome cost of consumers. It means that if the troublesome cost is relatively low, shoppers' decisions are rarely affected by rising pre-sale price. Shoppers decide whether to keep or return the product according to whether they like it or not. It will bring about more advance needs. If the troublesome cost is moderate, retailers need to lower their price. Because in this case, consumers will hesitate to buy in advance. Nevertheless, if the troublesome cost becomes very high, retailers' cost will increase accordingly. For this reason, retailers should raise the pre-sale price. Lemma 1 (ii) indicates that there is a threshold for retailers' return cost. When the return cost is higher than the threshold, retailers need to lower their price to reduce returns. Lemma 1 (iii) means that there is a threshold for the informatization degree of the virtual showroom for retailers. When the degree is lower than the threshold, the optimal pre-sale price of retailers increases with the informatization degree of virtual showrooms. In order to figure out these properties, this
paper varies $h_{r}, r$ and $\alpha$ with other parameters constant (i.e., $p=100, h=120, l=60, c=70$ and $s=50$ ) and graphs the optimal pre-sale prices as shown in Figure 1, Figure 2, and Figure 3, respectively.

## 4. Conclusions

This paper considers the advance selling strategy with virtual showrooms through modeling and numerical simulation. During the pre-sale period, consumers always face uncertain product valuation. This will not be resolved until the product arrives in the regular selling period. The virtual showrooms allow consumers inspect before purchase.

We find that the optimal pricing and ordering decision of retailers depends on their return cost $r$, hassle cost $h_{r}$ and the informatization degree $\alpha$ of virtual showrooms. When the retailer adopts advance selling with virtual showrooms and informed consumers inspect the products before purchase, the optimal pre-sale price increases first, then decreases, and finally increases with the increase of consumers' hassle cost $h_{r}$. Moreover, it is non-increasing with return cost $r$, and it is non-decreasing with the informatization degree $\alpha$ of virtual showrooms. In addition, we find that retailers are more likely to order fewer products when using the advance selling strategy of virtual showrooms. The reason is that retailers can sell products returned in the regular period to consumers.

This paper assumes that the price of the regular period is exogenous, that is, we only decide the pre-sale price. The future research direction of this paper is to include the regular selling price into the research and consider a variety of pricing mechanisms, such as price commitment and dynamic pricing. In addition, the consumers' heterogeneity can be studied in the future. For example, some consumers may regret missing the purchase opportunity, which requires further research.


Figure 1. The optimal advance selling price with different hassle cost $h_{r}$


Figure 2. The optimal advance selling price with different return cost $r$


Figure 3. The optimal advance selling price with different informatization degree $\alpha$

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