Finding the Best Plant Location Given the Costs

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Abstract. This paper is trying to find the optimal location of the production plant out of the few possibilities that have different costs associated with building the plant there. This paper first shows that this problem can be solved if the costs are rational numbers, then proves that it is impossible to have a computer program that can always solve this problem if the costs are constructive real numbers.

Keywords: Constructive Mathematical Economics, Rational number, Constructive Real Number

1 Introduction

The concept of the rational number and constructive number are introduced by giving their definitions.

1.1 Mathematical Economics

Mathematical economics refers to the discipline that systematically uses advanced mathematical methods to express, study and demonstrate economic theories. It differs from general economics expressed in words mainly in the research methods. When mathematical economics studies theoretical problems, it generally starts from assumed conditions, and then uses mathematical methods to deduce theoretical conclusions from these assumptions. The content of western mathematical economics research mainly includes: production theory, utility theory, price theory, equilibrium theory, decision theory, preference theory, group behavior theory, social choice theory, as well as static economic models, dynamic economic models, multi-sector input-output models, etc. Mathematical economics, as an independent discipline of Western economics, was formed in the early 20th century [1-4].

The application of mathematics in Western economic theory has been developing continuously for more than half a century. On the one hand, the theoretical field of research using mathematical methods is still expanding; Mathematical methods are discussed in more depth. The former is such as: various macroeconomic models of British J.M. Keynes and various schools of Keynesianism; how individual preferences are ag-

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gregated into social choices and their relationship with social welfare functions; optimal growth theory, etc. The latter, such as the general equilibrium system pioneered by Walras, has become the focus of continued theoretical research, because he only takes the equality of equations and unknowns as a condition for obtaining an equilibrium solution, while ignoring how the equilibrium is achieved and whether it is stable [5-9].

Since the 1930s, British J.R. Hicks (1904-1989.5.20) and American P. Samuelson have carried out precise mathematical analysis and solutions, but still use calculus as the main tool, subject to continuous functions. Therefore, J. von Neumann (1903~1957), K.J. Arrow (1921~), G. Debreu (1921~2004.12.31), etc. successively used set theory and linear model to develop new exploration. After the 1960s, mathematical economics was combined with calculus, set theory, and linear models. At the same time, the application of mathematical methods spread to almost every field of bourgeois economics. After the Second World War, the needs of economic life and the invention of electronic computers led to the rapid development of econometrics related to mathematical economics, which in turn promoted mathematical economics to continue to advance [10-12].

Although mathematical economics has made some achievements in analyzing the quantitative relationship of economic things, it ignores the qualitative aspects of economic things to a certain extent, especially the study of production relations. This research method has great limitations, especially the research that reveals the law and essence of social and economic relations has little application value [13].

The application of mathematics in modern economic theory is more and more extensive. On the one hand, the theoretical field of research using mathematical methods is still expanding; After the 1960s, mathematical economics was combined with calculus, set theory, and linear models, and mathematical methods were used in almost every field of economics. Continuing to use mathematical methods to study economic problems is conducive to discovering the essence of economic problems and indicating the development and changing trends of economic problems. Mathematical analysis is an indispensable aspect of studying economic problems these days, and any analysis of economic problems without mathematics will be considered unreliable. With the deepening of people's understanding of economic activities, mathematical economics is also constantly developing and improving [14-15].

1.2 Rational number

A rational number is a number that can be obtained by p/q where p and q are integers, and q cannot be 0. Rational numbers are the general term for integers (positive integers, 0, negative integers) and fractions. Positive integers and fractions are collectively called positive rational numbers, and negative integers and negative fractions are collectively called negative rational numbers. Therefore, the numbers of the rational number set can be divided into positive rational numbers, negative rational numbers and zero. Since any integer or fraction can be converted into a decimal recurring decimal, and conversely, every decimal recurring decimal can also be converted into an integer or fraction, so rational numbers can also be defined as decimal recurring decimals.

The set of rational numbers is an extension of the set of integers. In the set of rational numbers, the four operations of addition, subtraction, multiplication, and division (the divisor are not zero) are unimpeded.

An important difference between the set of rational numbers and the set of integers is that the set of rational numbers is dense while the set of integers is dense. After arranging the rational numbers in order of magnitude, there must be other rational numbers between any two rational numbers, which is the density. Integer sets do not have this property, and there are no other integers between two adjacent integers.

Rational numbers are a close subset of real numbers: every real number has an arbitrarily close rational number. A related property is that only rational numbers can be reduced to finite continued fractions. According to their sequence, rational numbers have an ordered topology. A rational number is a (dense) subset of the real numbers, so it also has a subspace topology.

1.3 Constructive real numbers

A constructive real number is a real number constructible with respect to some collection of constructive methods. The formal definition is that we have an algorithmically generated Cauchy sequence of rational numbers satisfying the condition for all, such CRN are said to satisfy the condition of the standard regulator. The general regulator program, said β, is a program that tells how fast the Cauchy sequence converges: for . For the case of the standard regulator to be the identity function. Besides, a non-standard regulator can always be modified to a standard one by deleting some constructive terms.

A partially defined computer algorithm transforming positive integers to positive integers or to 0,1 is said to be unextendible if there is no computer algorithm that gives the same output when the first algorithm terminates and gives some output when the first algorithm does not ever terminate. As proven for example in A.Shen, N. K. Vereshchagin, Computable Functions, Theorem 12, this unextendible function always exists. A.Shen, N. K. Vereshchagin [1].

We define the non-extendable algorithm H, which prints 0 or 1 from positive integers inputs, and let be the constructive real number, defined as follow:

If H is still working at step k.

If H stops working at step m and it prints 0 then for all

If H stops working at step m and it prints 1 then for all

Since Cauchy sequences stabilizes after a certain moment, they all satisfy the standard regulator condition, and hence, they define constructive real numbers.

Constructive real numbers should appear in economics in the situation when the exact costs are not clear but can get better and better approximations by repeating measurements.

With those given definitions, we can further prove our topic's main idea.

1.4 Short introduction of two schools of constructive mathematics

Constructive real numbers were introduced first under the name of computable numbers by. Turning.

Bishop's constructive mathematics is often described as "obtained from classical mathematics by removing the law of excluded middle and the axiom of choice".

On the other hand, the Russian school of constructive mathematics, associated principally with Andrey Markov Jr and Nikolai Aleksandrovich Shanin, allows the principles of constructive choice, and sometimes allows to argue by contradiction.

2 Main Theorem

2.1 Rational number cost

If the costs are rational numbers, which can be expressed in the form of, where and are integers, it is relatively easy to make a decision by finding the town with the smallest numerator when the costs are brought to the common denominator. The town with the lowest numerator will be the optimal choice of building a factory, since its cost is the lowest. Thus, the problem is algorithmically solvable for any number of towns.

2.2 Constructive real number cost of two towns

The paper tries to prove that there does not exist a program that can always choose the optimal location for towns when the costs are constructive real numbers.

Based on the definition of constructive real numbers, the minimum value of the constructive real number is, which happens when H stops at step 1 and prints 0. The cost of the factory construction cannot be negative.

Suppose a factory needs to be built in one of the two towns. The two towns are town 0 and town 1 associated with different costs respectively. To ensure the cost is non-negative, the cost at town 0 is defined as the and the cost at town 1 is defined as 1.

Let's argue by contradiction and assume there is a program that always chooses the optimal location for the factory. This program should in particular work when there are only two towns.

Then, choice can be made based on the output of program H:

If H stops working and prints 0, town 0 will be chosen since is negative and the cost of town 0 factory placement is smaller. So, the program G which makes an optimal choice for the factory will place it in town 0.

If H stops working and prints 1, town 1 will be chosen since is positive and the cost of town 1 is smaller. So, the program G which makes an optimal choice for the factory will place it in town 1.

If H does not terminate ever, their costs are the same since for all k. The choice is supposed to be made by another program and the assumption is that it always does a correct choice which in this situation can be any of these two towns. In this situation program still makes a choice between two towns, and is clearly the extension of program H. However, H is not extendable, so there is a contradiction and hence the computer program could not exist.

Thus, this result successfully proved that it is impossible to have a program that can solve all such problems if the costs are constructive real numbers.

2.3 Constructive real number costs in the case of more than two towns

Suppose there are three towns, town 0, town 1, and town 2 associated with different costs respectively and a factory needs to be built in one of them.

When the costs are constructive real numbers, according to Vershchagin and Schen, there is an unextendible program which can be used to generate the valued costs and the program that chooses the optimal location should be defined for all the number of towns and all costs of placing a factory in this town. A.Shen, N. K. Vereshchagin[1]

The same as 1.2.2, we assume that the constructive real number which represents the cost of the factory in town 0 is and the constructive real number which represents the cost of the factory in town 1 is 1.

Further, lets assumed that the cost of town 2 to be 2022 or some other huge number that is not the median. Making the cost in the third town so huge means that will never choose that town.

Since town 1 always has less price than town 2, this process only needs to compare the cost of town 0 and 1.

By using the same method mentioned in 1.2.2, it can be proven that it is impossible to have a computer program G that can solve all such problems if the costs of three towns are constructive real numbers.

Similarly, it can also be proven that it is impossible to have a computer program G that can solve all such problems if there are more than three towns.

3 Conclusion

This paper successfully proved that this problem is algorithmically solvable for any number of towns if costs are rational numbers, and there is no program that can always choose the optimal location based if costs are constructive real numbers.

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