An Analysis of Optimized Asian Options

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ABSTRACT ÷

Nowadays, the option is an indispensable financial tool in the current financial market. Moreover, the Asian option, as one of the most famous and widely used exotic options, plays a significant role in it. This paper aims to propose a new type of Asian option, which is jointly designed by the authors and makes an improvement based on the regular Asian option better to meet the demands of the current financial market. This paper provides an explicit definition and comprehensive explanation of this new Asian option and explores its unique functions in the financial market compared with the normal Asian option. Furthermore, to verify the effectiveness of the proposed option, this paper selects representative real stock prices and makes some reasonable assumptions. After using Stata to process the data, the performance of the new Asian option is analyzed, and a numerical comparison between the proposed option and the normal Asian option is made. And the effectiveness of the proposed option in mitigating risk is validated. This paper expounds on the proposed option in detail so that readers can develop a deep understanding of it. It also provides some reference for the option issuers to issue new options.

Keywords: *A proposed option, The regular Asian option, Financial market, Payoff*

1. INTRODUCTION

1.1. Background

Option, which is a sort of an indispensable and popular financial derivative, has many important usages such as hedging, speculation, etc. Initially, the contract took the form of the most common European and American options, which gave the holder the right to buy or sell an asset at a fixed price on or before a specific date. Before obtaining this right, investors need to pay a certain option premium, which stimulated the emergence of various option pricing methods. In 1973, mathematician Fischer Black and Stanford University professor Myron Scholes put forward the Black-Scholes model, which laid a foundation for the rational pricing of derivative financial instruments in emerging derivative financial markets. The underlying asset of the derivative can be either stocks, bonds, currencies, or commodities [1]. Since then, many new options have emerged, which have injected more vitality into the financial market and provided investors with more flexible investment choices.

Asian option, also known as the average price option, is one of the most active exotic options in the current financial derivatives market. It was first introduced by Bankers Trust in Tokyo on the basis of summarizing the experience and lessons of real options, virtual options, and stock options in 1987. The difference from the vanilla options is that when the option profit is determined on maturity, Asian options adopt the average price of the underlying asset at a certain time during the option contracts period to substitute the market price of the underlying asset at maturity or the strike price. Compared with the vanilla options, the Asian option possesses significant advantages. It not only has a lower option premium but also provides a way to hedge the volatility of the average price of an asset. Furthermore, it effectively mitigates the risk caused by artificial manipulation of the stock prices, the insider trading of employees, and the public opinions that harm the company's interests.

With the wide application of the Asian option, different improvements have been proposed in a lot of literature. while maintaining its basic property. Wu and Zhang put forward the Asian rainbow option, which is a combination of the Asian option and the rainbow option,

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in 1998 [2]. Yang et al. gained the pricing formulas of the Asian barrier option in 2019 [3]. Recently, the financial company Banque Nationale de Paris and Paribas introduced the conditional Asian option. This kind of Asian option uses the average price of the underlying asset, which only depends on the prices above a specific value. Yao et al. expressed the payoff on call and put options for this continuous conditional average price option mathematically as follows,

$$\left(\frac{\int_0^T S_t \cdot l_{\{S_t>b\}} dt}{\int_0^T l_{\{S_t>b\}} dt} - K\right)^+ \text{ and } \left(K - \frac{\int_0^T S_t \cdot l_{\{S_t>b\}} dt}{\int_0^T l_{\{S_t>b\}} dt}\right)^+ \quad (1)$$

where $(x)^+ = max(x, 0)$, S_t is the asset price in time t, K is the strike price, b (b>0) is the specific value and l_A represent an indicator function which equals one if A is true and 0 otherwise [4].

1.2. Related Research

The price of the Asian option is known to be very challenging to compute. Rogers et al. approach the problem of pricing Asian options in two different ways. First of all, they exploit a scaling property, which allows them to simplify this problem into the problem of solving a parabolic PDE in two variables. Secondly, they provide a lower bound. The bound is so precise that it closely approaches the actual price. The heart of this approach is that if the process is conditioned on a Gaussian variable Z with a zero mean, it will remain a Gaussian process [5]. The modern financial market is highly risky. In order to hedge the risk caused by fluctuated prices, foreign exchange rates, and uncertain demand, options are used. The option gives the buyer the entitlement but not the obligation to sell or buy an underlying asset with a strike price. And Gao et al. focus on the pricing problem of the Geometric Asian barrier option under the assumption that the stock price follows an uncertain differential equation, and a pricing formula is derived. Furthermore, some numerical examples are provided to express how this formula can be used to price corresponding options [6].

Bernard et al. explain how they are able to compute the price of Asian options using competitive Monte Carlo algorithms with a careful selection of the scheme. They demonstrate the efficiency of these algorithms in two different ways - analyzing the rate of convergence of these algorithms numerically comparing them with some other existing methods. This article first introduces the mathematical context and presents the Monte Carlo schemes. Then, they prove the convergence in the Lp spaces and briefly describe the variance reduction method used [7]. Instead of using the classical Monte Carlo method, Ben et al. focus on studying how the multilevel Monte Carlo method introduced by Giles can be applied to tackle the problem of valuation of Asian options in a more efficient way. Then, they prove a central limit theorem of the Lindeberg–Feller type for this recently introduced method. A method of this kind requires that the integral of the payoff process be discretized. To do so, they use two widely used second-order discretization systems, which are known as the Riemann scheme and the trapezoidal scheme, respectively [8].

Wang focuses on studying how counterparty risk affects the prices of Asian options. He first introduces the dynamics of the market index. Then, using betas to represent the sensitivity of the underlying asset to systematic risk, he builds stochastic volatility models to connect the dynamics of the market index and the issuer's assets. The stochastic volatility models capture leverage effects and stochastic correlation between two assets. Next, he derives an explicit pricing formula for Asian options with counterparty risk and illustrates how Asian option prices are affected by systematic risk. Moreover, when the issuer's assets and the underlying asset have constant total risk but different proportions of systematic risk, the prices of the Asian option will also be different. And he investigates the differences in this article [9]. Yao et al. studied the problem of pricing the conditional Asian option, which is a recently introduced market product and is cheaper than the regular Asian option. Under a local volatility model, they develop some new short maturity asymptotic features for the pricing of conditional Asian options using the large deviation theory. Furthermore, by explicitly expressing the explicit definition of the appropriate rate function, Yao et al. establish a novel approach for conditional Asian options under the Black Scholes framework. Instead of depending on the Monte Carlo simulations or other numerical approaches, they use the theoretical findings to generate reasonable approximations for the conditional Asian options [4]. Gao et al. aim to study the valuation of the Asian rainbow option under an uncertain financial environment. In this framework, the underlying assets' prices are considered stochastic processes that follow uncertain differential equations driven by independent Liu processes which are uncertain processes with stationary and independent increments. Some numerical examples are provided to illustrate how to compute the prices of the corresponding options using the derived pricing formula [10].

Zhang and Zhou briefly describe two theoretical types of the Asian option: the arithmetic average Asian option and the geometric average Asian option. The pricing method of the arithmetic-average option is more complicated than that for the geometric-average option. They take the continuous-time case as an example and illustrate how the analytical pricing formula of the Asian option is obtained [11]. Liu introduces a new uncertain index Ornstein Uhlenbeck process model. The option pricing problem is studied assuming that the interest rate follows an uncertain mean recovery process. The pricing formulas of the Asian call option and put option are derived, respectively. Ultimately, a numerical algorithm for calculating option prices is designed, and a numerical example is provided [12]. Yang et al. propose a new

hybrid modeling method, which combines the nested long-term and short-term memory neural network model (NLSTM) with the Heston model to realize the dynamic correction of ETF option pricing deviation. With the use of the high-frequency option data of Huaxia Shanghai Stock Exchange 50ETF, Jiashi Hushen 300etf, and Huatai Bairui Hushen 300etf, some tests are conducted to verify the effectiveness of the proposed method [13].

1.3. Objective

In this paper, we will focus on the functions of this new type of Asian option. The proposed option is similar to the conditional Asian option above but different from it in that the proposed option has both upper and lower price boundaries. In Section 2, the methods used in the research will be elaborated. And in section 3, we will conduct testing and generate relative results with the use of Stata. The results will be analyzed in section 4, and we will also explore how the proposed option outperforms the normal Asian option in the contemporary financial market situation in this section. Finally, some brief conclusions are given in section 5.

2. METHOD

2.1. Definition of the proposed option

An Asian option becomes the proposed option if and only if the option belongs to the average strike option, which uses the average price of the underlying asset at a certain time during the option contracts period to replace the strike price when it is due for delivery.

The average price is only based on prices that lie between the upper and lower boundaries. Mathematically, the payoff model for the call option and put option from the proposed option can be established in the discrete form below, respectively.

$$\left(S_T - \frac{\sum_0^T S_t \cdot l_{\{|S_t - \underline{S}| < \underline{S} \cdot \alpha\}}}{\sum_0^T l_{\{|S_t - \underline{S}| < \underline{S} \cdot \alpha\}}}\right)^+ \text{and } \left(\frac{\sum_0^T S_t \cdot l_{\{|S_t - \underline{S}| < \underline{S} \cdot \alpha\}}}{\sum_0^T l_{\{|S_t - \underline{S}| < \underline{S} \cdot \alpha\}}} - S_T\right)^+ \quad (2)$$

$$\underline{S} = \frac{\sum_0^T S_t}{\sum_0^T l_{\{S_t > 0\}}} \quad (3)$$

Where S_t is the asset price in time t, α is a percentage that determines the size of the two upper and lower price boundaries and l_A represents an indicator function which equals 1 if A is true and 0 otherwise.

According to the definition illustrated above, the difference between the proposed option and the regular average strike option is that the average price that the proposed option uses is on the basis of the prices that are filtered by a certain price range. From the payoff model, the price range takes the average price before filtering ($\underline{S}$) as the center point, and its size can be adjusted by modifying the value of α. Therefore, the size of the price range is not fixed. The exchange or other relevant institutions can decide the value of α by taking the factors such as the market conditions, the economic trend of the country, the investors' psychological expectations, and so on into consideration, which make the option circulate better in the financial market.

2.2. Optimization of the regular Asian option

In this optimized Asian option, the return on the expiration date of the option does not depend on the stock price on the exercise date but on the average price of the stock in the past period of time. Compared with the traditional Asian option, the Asian option has better resistance to risk. We believe that today's financial market is becoming more and more complex, China's financial market still lacks effective supervision, and the phenomenon of artificial manipulation is common. As a result, we hope to construct an option with lower volatility compared to the original Asian option. By removing extreme values of stock prices in the option contract period, our goal is achieved. And the price of the proposed option is lower than other identical ordinary options. By effectively minimizing the interference of extremes and emergencies, this optimized option enhances the financial market efficiency and improves the enthusiasm of investors.

2.3. Data

In the database of Choice, we obtained the weekly trading data of these 20 stocks from May 2018 to May 2019, including opening price, closing price, and average price. Based on these data, Asian options and improved Asian options are constructed. Finally, we selected Jia yuan International and A-Zenith Home Furnishings Co. Ltd as our research objects. After filtering out the extreme values of stock prices through Stata, we get data that can be used for testing directly.

3. RESULTS

The stock price of A-Zenith Home Furnishings Co. Ltd rose violently due to human manipulation in the selected time range, which started on January 2nd, 2018, and ended on October 31st, 2018. After screening, the average value of stock prices decreased slightly. On the other hand, the stock price of Jia yuan International fell abnormally within the time period we selected. The period started on July 4th, 2018, and ended on June 28th, 2019. After screening, the average value of this stock rebounded to ¥13.74457 per share, as shown in Table 1.

Table 1. Test Results

Listed Company	Stock Number	The average value of stock prices (¥)	
		Before Screening	After Screening
Jia yuan International.	02768.HK	9.858844	13.74457
A-Zenith Home Furnishings Co. Ltd	603389.SH	11.8507	11.8032

4. DISCUSSION

Judging from the above test results, the proposed option issued in early May 2018 and expired at the end of October 2018, with A-Zenith Home Furnishings Co.Ltd. as the underlying asset, will yield less profit to the option holder than the normal Asian option, which meets the expected standard. According to news reports, from April to September 2018, one person controlled 55 securities accounts, concentrated on capital advantages and shareholding advantages to carrying out continuous trading, and artificially manipulated the stock price of A-Zenith Home Furnishings Co.Ltd. by means of reverse selling after intraday pull-up and reverse trading, causing the stock price of the company to keep rising during this period. For the normal Asian options assumed above, the average price of the shares in the option period is significantly higher than the conventional expectation due to the continuous increase in the company's stock price. During the option period, the stock price increased from ¥11.04 per share to ¥16.60 per share, and the increase was 50.36%. Therefore, the payoff of the option holder at the time of maturity delivery has also been significantly increased. This eventually led to unexpected losses for the sellers. At the same time, it also made the people who artificially controlled the stock price obtain a larger profit, which prompted such illegal behaviors to occur more frequently. In contrast, since the prices above the upper bound and below the lower bound are filtered out before calculating the average price of the stock, the holder's payoff at the final delivery of the proposed option is lower. Thus, the optimized option effectively alleviates the problems caused by such events in the normal Asian options.

Furthermore, from the dataset, it can be observed that the stock price of Jiayuan International Group Ltd. plummeted on January 17th 2019. On January 17th, Jiayuan International's share price saw a flash crash, falling as low as HK$1.4 per share. The ending stock price was HK$2.52 per share, down 80.62% compared to the previous day, which evaporated the company's market value by more than HK$26 billion. It is reported that the day was the maturity date of Jiayuan International Group Ltd.'s US$350 million debt. Some short-selling firms released false news that the company was failing to repay its debt. They used self-media to spread fabricated content to damage the company's reputation. For most investors, the media is their primary source of information, and they make investment decisions based on the information on social media. Thus, when these firms posted the fabricated content that damaged the company's reputation on social media, many investors blindly sold off their stock, causing the share price to fall sharply. In the testing process, when Jiayuan International's stock is chosen to be the underlying asset of both the regular Asian option and the proposed options, the issued date of both options is assumed to be July 4th, 2018, while their maturity date is assumed to be June 28th, 2019. The strike price of the crude Asian options is calculated by taking the arithmetic average of the daily stock price of the underlying assets. The arithmetic average of Jiayuan International's share price in the options period is calculated to be ¥9.86. This value is significantly lower than conventional expectations due to the impact of the flash crash. As a result, the payoff for the put option holders will significantly decrease. When the strike price becomes less than the stock price on the delivery day, these options expire worthlessly. By plummeting stock prices through market manipulation, the option seller can earn premiums upfront without any loss on the delivery day. In contrast, the new mechanism introduced to calculate the strike price in the proposed option allows us to filter out the data after the stock plunge and before the recovery. Therefore, it effectively mitigates the risk arising from the plunge in the stock price.

Nowadays, despite the continuous development of financial markets all over the world, there are still some problems, such as imperfect supervision system, diversified securities insider trading, lack of effective checks and balances of information, etc. Many people have taken illegal actions such as artificially manipulating stock prices and insider trading in order to pursue excess returns, breaking the fair and just trading environment in financial markets, and making many small and medium-sized investors become victims of the trading. The above test example shows that the proposed option can better minimize the adverse impact caused by the abnormal sharp change in the stock price than the normal Asian option and can effectively limit the occurrence of such illegal acts. In general, we believe that the improved Asian option reduces the effect of abnormal stock price fluctuation to a certain extent. Therefore, investors can minimize unnecessary losses in the face of emergencies. It can also avoid the influence of artificially

manipulated stocks on ordinary investors. The overall financial market will become more effective.÷

5. CONCLUSION

This paper has explained how the proposed option can be used to mitigate the risk of a sharp change in stock price caused by market manipulation, the insider trading of employees, public opinions, and so on. It's known that the regular Asian option can mitigate the risk of market manipulation on the delivery day. However, this protective mechanism will be much less effective if the stock prices soar or plunge prior to the maturity date during the option contract period. In contrast, despite calculating the arithmetic average of stock prices naively to get the strike price, the new mechanism introduced in the proposed option allows us to filter out values outside the boundaries. Therefore, when the stock posts a sharp gain prior to the delivery date, the proposed option effectively reduces the payoff for option holders and protects the option sellers from unexpected loss. On the other hand, when there is a sudden and significant drop in stock prices, the payoff for the put proposed option holders will significantly increase compared to the conventional one. Conclusively, the proposed option effectively alleviates the problems caused by market manipulation and therefore provides both option sellers and holders with more protection from unexpected loss compared to the normal Asian options. Because this paper mainly focuses on exploring the advantages and feasibility of the proposed option, it does not analyze the specific pricing problem of the option. The overall nature of the proposed option needs further research and improvement in the future.

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