Innovation in Teaching Methodology for Mathematical Beauty in the Aesthetic Education

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Abstract. This paper proposes an innovative thinking model for teaching practice based on the previous qualitative study of mathematical beauty, with the original intention of guiding practice with theory, and filling the gap of few existing teaching models of beauty in the discipline. This paper provides a theoretical account of mathematical beauty and mathematical aesthetic education, outlining the results of relevant academic research. In addition, based on the findings of mathematician Mr Xu Lizhi and Mr Li Zhengyin, the core contributor to mathematical beauty theory, this paper innovatively introduces a three-dimensional conceptual model based on condition-nature-level, aiming to provide mathematics pedagogues with theoretical references in the actual teaching process. In addition to this, the final chapter of this paper introduces a thinking model that can be applied to the practice of teaching mathematics and extended to various disciplines in its core concept.

Keywords: mathematical beauty · aesthetic education · teaching methodology

1 Introduction

The researches on mathematical beauty in education sector mainly center on the following two modules: First, they discussed the essential attribute of mathematical beauty, based on Xu Lizhi's concept of aesthetic consciousness, and the method of discovering mathematical beauty in practice. [1] Zhang Dianyu et al.discussed the four levels of aesthetics education, namely, beauty, aesthetics, wonder, and perfection, and makes a pair of arguments that beauty in form is not necessarily true beauty; nor is beautiful form necessarily unaesthetic. [2] Zhang Yufeng and others held that the mathematical beauty is a free form of affirming practice reflecting the beauty of reality, and has dual nature of sociality and materiality. [3] Secondly, the research methods of mathematical beauty are discussed from the cross perspectives of philosophy and history. Liu Ping and others believed that the mathematical beauty is the free value of and originates from mathematical practice, embodies the essential power of mathematicians and is the link connecting Popper's three worlds [4].

Chen Huanbin et al.pointed out that the mathematical beauty is a concept relative to the cognitive subjects, essentially reflecting the subjects' understanding of the deep structure of mathematical objects and their mutual internal relationships. [5] Wang Qinmin further analyzed Liao Lie's main ideas about the essence of beauty and aesthetic opinions in the east and west in history, and by comparing people's cognition of mathematical beauty in different periods, promoted the cognition to be more in line with the essence of mathematical beauty. [6] Xu Benshun put forward that mathematical beauty is an essential force, showing a motivating mathematical thinking structure.[7] The existing researches of mathematical beauty present that the theoretical researches account for the majority, and only a few researches of higher mathematics teaching considering specific teaching modes, methods and means. Huo Wenting regards that mathematical beauty is people's aesthetic perception of mathematical objects, and the six properties of mathematical beauty can be divided into general attributes of aesthetics and attributes of mathematics discipline as the source.[8] Chen Huanbin explained the four essential attributes of mathematical beauty.[5] Yang Zezhong formulated the basic requirements for teaching mathematical beauty, which included students' familiarity, vision's broadening, appropriate association, aesthetic guidance and aesthetic review, and put forward three points for teachers' attention. [9] Overall, the existing theoretical research on mathematical beauty lacks in-depth analysis of the function of higher education, and has not yet discussed the concrete countermeasures for the application of mathematical beauty in combination with the characteristics of mathematics.

This paper aims to establish a model for the application of mathematical classroom teaching based on previous qualitative studies of mathematical beauty, in the hope of giving practical significance to the theories, and providing teaching with reference. In this research, previous theories of mathematical beauty were systematically and categorically sorted out, especially under the theoretical guidance of mathematicians Mr. Liji Xu and Mr. Zhengyin Li, and therefore a trinity thinking model of condition- nature- level is established to help the teacher community to have a basis to follow in teaching mathematical beauty, and also to promote students' deep understanding of the aesthetic value of subjects. The significance of this paper lies in the breakthrough of integrating and refining multiple theories to build a thinking model for teaching practice, instead of focusing on the study of a specific mathematical beauty theory, and the model can be further modified and extended to other subjects and aesthetic education.

2   Integrate the Mathematical Beauty into Teaching Practice

2.1   A Multi-dimensional Perspective on the Mathematical Beauty

Students should be guided to explore the core qualities of mathematical beauty, such as "simplicity", "unity", "harmony", "symmetry" and "singularity" as shown in Fig. 1. Current research has focused on the learning practice of establishing different connotations of a property with the basic definitions as one important way to build a theory of mathematical beauty. However, while research usually takes mathematical uses or categories as a static research perspective, this paper shifts the perspective from the history of development to appreciate the mathematical beauty of knowledge points based on the
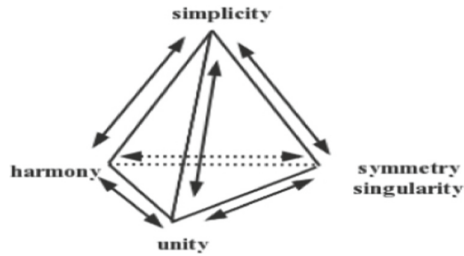
Fig. 1. Five orientations of mathematical beauty by Xu Lizhi (Self Drawing)

level of specific mathematical stories, aiming to provide new ideas on the educational function of mathematical beauty.

Pythagorean theorem and Euler's formula are applyed to appreciate in different ways, either by analyzing the angle of appreciation of the beauty of the formula or by trying to explain the history or anecdotes of the development of the theorem or formula. The beauty of the Pythagorean theorem can be appreciated by explaining the simplicity of the expression and the singularity of the Pythagorean numbers, or by recounting the origin of the Pythagorean theorem as a sidelight through a standing pole in ancient astronomical observations. For example, it is mentioned to the students that as early as the Zhou Dynasty, the mathematician Shang Gao first proposed the theorem Gousan, Gusi, Xianwu that the sum of the squares of two right angles in a right triangle is equal to the square of the hypotenuse. In comparison, for Euler's formula, the singularity and simplicity properties of 0, 1, i, e, pi, linked together in a simple form based on its expression, can also be explained by elaborating on Euler's process of simplifying and refining the great theorem in a step-by-step discovery. Yang Zezhong has conducted comparative experiments with high school students to perceive the beauty of the two formulas, and data indicated that the beauty of the Pythagorean theorem was more easy to be sensed than that of Euler's formula due to the students' lack of knowledge of higher mathematics [10].

The static or dynamic perspectives on the understanding of a formula or theorem are all internally contrasting, as both illustrating the nature of its mathematical beauty. This paper finds that analyzing the properties of formulas and theorems from five basic properties of mathematical beauty, expanding the specific developmental history and short stories behind mathematical theorems from a cross-temporal perspective, and recognizing the connotations of formulas and theorems from multiple dimensions can all contribute to students' ability to appreciate beauty, while the teacher community should also integrate specific levels of basic aesthetic competence in their teaching to enhance students' deeper understanding of their learned knowledge, thereby realizing the beauty of mathematics educational function.

As a way of styling specific formulas and theorems, properties themselves contain a wealth of information capable of characterizing the mathematical beauty of specific points of knowledge. Through the analysis of properties, different modules can be linked together in a point-by-point manner. For example, one can combine the inner product space Schwartz inequality's simplicity

$$(\alpha, \beta)^2 \leq (\alpha, \alpha) \cdot (\beta, \beta) \tag{1}$$

analyzing its nature in different dimensions, thus discovering

$$[\int_a^b f(x)g(x)dx]^2 \leq \int_a^b f^2(x)dx \cdot \int_a^b g^2(x)dx \tag{2}$$

$$[E(\varepsilon_n)]^2 \leq E\left(\varepsilon^2\right) \cdot E\left(\theta^2\right) \tag{3}$$

$$\left(\sum_{i=1}^n a_i b_i\right)^2 \leq \sum_{i=1}^n a_i^2 \cdot \sum_{i=1}^n b_i^2 \tag{4}$$

these three equations represent specific properties. Therefore, teachers can consider developing the lecture in terms of the unity of the formula, incorporating the underlying equations to give students a dynamic understanding of the nature of the Schwarz formula. Another example is combining series with Euler' s series and Euler' s product formula

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!}(\pi t)^{2k} = \prod_{n=1}^{\infty} 1 - \frac{t^2}{n^2}, \varepsilon(s) = \frac{\pi^6}{6} \tag{5}$$

$$\sum_n \frac{1}{n^s} = \prod_p \frac{1}{1 - p^{-s}} \tag{6}$$

has a continued product on one side, a summation on the other, and a historical development of Euler's formula can be simultaneously illustrated to show the singular beauty.

2.2   Modelling the Teaching of Mathematical Beauty

Basic approaches to theoretical research and different horizons of understanding aesthetics are most commonly graded from three levels as shown in Fig. 2: (1) The ability to feel beauty, i.e. the numerical, symbolic and graphic views; (2) The ability to appreciate beauty, i.e. the discriminatory, appreciative and evaluative views; (3) The ability to create beauty, i.e. the discovery and creative views.

The nature of mathematical beauty as proposed by Xu is divided into five categories: (1) The awe of simplicity for easy answers to difficult questions; (2) The love of symmetrical patterns, buildings, etc.; (3–4) The love of harmony and regularity as a natural nature, and the delight in discovering the universality and regularity of things that are chaotic and disordered; (5) The enjoyment of beauty that arises from the singularity of exotic flowers or extremely beautiful shells or stones. [1] This qualitative notion, well-organized and realistic, is highly persuasive for teachers to better characterize formulas
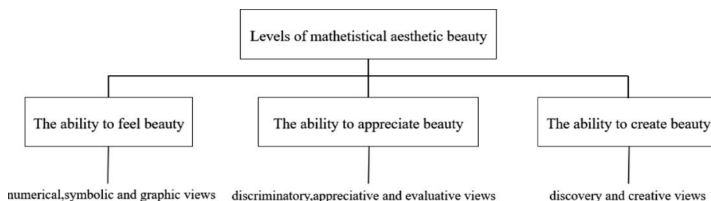
Fig. 2. Mathematical aesthetic ability in three levels by Li Zhengyin (Self Drawing)

and theorems in terms of mathematical beauty, and to guide students in building up their notions of mathematical beauty.

The abstract nature of higher mathematical theorems creates barriers to the students' understanding, resulting in conceptual ambiguity. Therefore, this paper applies the three basic levels of Li Zhengyin aesthetics to break through the teaching difficulties of core concepts. Hierarchical analysis is an effective learning tool for the analysis of important and difficult points through the history of subject development and images of life situations. [10] For example, students have difficulty understanding the meaning of "approximation but never coincidence" embodied in the limit, the teacher can explain ancient Chinese Mathematician Liu Hui's use of internal polygons to cut the circle to calculate the area of the circle, therefore students are allowed to understand that when polygons are internally connected to the circle in turn, the area is constantly approaching but never reaching the area of the circle. Visual and vivid mathematical vignettes and connections to familiar objects in life can avoid lengthy textual narratives and convey otherwise complex and abstract scientific concepts in an intuitive and concise manner. Deep and abstract mathematical theorems are often disturbing students' learning of advanced mathematics. If unfamiliar formulas and theorems are replaced by basic mathematical situations and placed in problem situations, they can be brought to life, reinforcing students' understanding of basic concepts and appreciation of mathematical connotations.

The above analysis can be summarized in three points: (1) Conditions. Proper historical and disciplinary stores of the point of knowledge should be found in lesson preparations; (2) Nature. The search for *conditions* needs to be in line with one or more of the properties of mathematical beauty proposed by Mr. Xu Lizhi; (3) Levels. The level at which students are expected to reach in their experience of mathematical beauty.

This paper argues that the three basic ideas are highly desirable for teachers to address the needs of teaching mathematical beauty from the level of pedagogical principles, where *conditions* are primary and foundational, *nature* is based on *conditions* and is a quick way of screening, and *levels* are outcomes based on the gains brought to students by the model, thus building an interconnected, interacting, triadic model of mathematical ideas.

2.3 Create a Teaching-Oriented Mathematical Beauty System Based on Core Ideas

The realization of the aesthetic education function of mathematics and the improvement of students' aesthetic ability both need to undergo processes, from mastering the
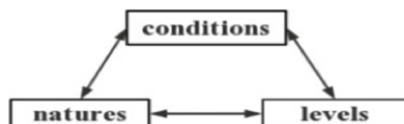
Fig. 3. Conditions-natures-levels three-dimensional Model in mathematical beauty teaching (Self Drawing)

knowledge students have learned, combining ability improvement and idea innovation, to forming knowledge reserve, etc. In these processes, teachers are required to pave ways to teach at different learning levels and gradually achieve the leveled innovative ability. Xu's creativity formula C*reativity = Effective knowledge base*Divergent thinking ability*Abstract thinking ability* Aesthetic ability* [11] steers students to improve their self-learning and thinking ability in the right direction. Mathematics beauty, as the core component of mathematics theory, takes up a large portion in the cultivation of subject literacy.

This paper sorts out three elements of nature, condition and level as shown in Fig. 3, balances aesthetic thoughts to complete Xu's view that mathematics is the science of research mode which requires ideal, simplicity, and harmony, therefore mathematics has the internal and external properties to be beautiful, meaning the internal beauty can only be realized by meeting the goodness of external requirements.

3   Conclusion

Achieving innovation in the teaching practice requires long-term research, profound background knowledge, transformative conceptualisation of ideas, and theoretical underpinnings for teaching and learning. All of which are elements that further enhance the understanding of mathematical beauty. Therefore, the introduction of a theoretical model allows the innovation in aesthetic education to happen. In addition, the model can be extended and applied to various disciplines in the field of educational beauty. The condition-nature-level trinity is both instrumental and humanistic in its approach to teaching and learning, constructing a new way of thinking about the subject and cultivating students' thinking skills and subject literacy in multiple levels.

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