Probabilistic Assessment of Reinforced Soil Walls Using Fourth Moment Normal Transformation

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Abstract. Fourth Moment Normal Transformation (FMNT) is an efficient method to probabilistically scrutinize the stability of reinforced soil walls (RSW). The methodology doesn't require the assumption of the distribution of involved random variables to estimate the probability of failure (P_f) and subsequently the safety of RSW. Rather, the P_f is calculated using the first four moments (mean, standard deviation, skewness, and kurtosis) associated with the random variables. The present study is focused on assessing the stability of RWS using FMNT. The internal friction angle (ϕ), soil unit weight (γ), and the allowable reinforcement strength (T_{ult}) are chosen as random variables. A detailed and straightforward procedure for using FMNT is presented besides comparing its accuracy with the results obtained using the first-order reliability method (FORM). Further, a clear comparison of the efficiency of FMNT is showcased by comparing the obtained results with those calculated using the Monte Carlo simulation (MCS). The results depict that the proposed concept can be smoothly used by researchers and practicing engineers in the field to obtain the probability distributions and consequently the P_f associated with RSW.

Keywords: FMNT · Reinforced soil wall · Probability · Random Variables · Moments · Distribution

1 Introduction

Probabilistic analysis is becoming popular for the stability assessment of unreinforced and reinforced soil walls (RSW). The prime reason behind this is its capability to address the associated uncertainties which generally cannot be included while performing the deterministic analysis. New methods for probabilistic analysis are being formulated to improve their respective previous counterparts in accuracy, efficiency, and simplicity. A comprehensive review of the methods used for probabilistic slope stability analyses is provided by Zhang et al. [1]. The researchers have discussed various methods such

S. Javankhoshdel and Y. Abolfazlzadeh (Eds.): TMIC 2022, AHE 13, pp. 55–60, 2023.
https://doi.org/10.2991/978-94-6463-104-3_6

as FORM, MCS, response surface methods (RSM), machine learning methods, subset simulation, and their applications in slope stability analysis in both 2D and 3D domains. However, from a rigorous literature review, it is seen that distributions of the input random variables are usually defined by fitting them to the available histogram data. These distributions are largely dependent upon the mean and standard deviation which may lead to erroneous conclusions. Also, there are many field cases where the distributions are not even available to perform the probabilistic analysis. Therefore, developing a distribution-independent method to analyze the stability of geotechnical structures in the probabilistic framework becomes important. FMNT employs the first four moments associated with a random variable to calculate the probability of failure (P_f) of an event. It can be used in various fields of engineering to obtain different distributions and even design when the distribution data is unavailable. Recently, Xu et al. [2] used it to design unreinforced slopes. However, still there is a scarcity of literature on the use of FMNT for both, unreinforced and reinforced walls, and slopes. Therefore, in an honest attempt to fill this gap, the present study employs FMNT to calculate the P_f of RSW choosing the internal friction angle of soil (ϕ), unit weight (γ), and allowable reinforcement strength (T_{ult}) as random variables. The results are compared with the FORM and Monte-Carlo simulation (MCS) to judge the accuracy and efficiency of the proposed formulation, respectively. For the convenience of the readers, a straightforward step-to-step methodology is also explained in the present study.

2 Deterministic Formulation

The $(5N - 1)$ formulation of the horizontal slice method (HSM) is used for the deterministic analysis of the RSW. HSM optimizes the total normalized required tensile reinforcement strength (K) by considering horizontal, vertical, and moment equilibrium equations.

$$K = \frac{\sum_{j=1}^{N} T_{rj}}{0.5\gamma H^2} = \frac{T_r}{0.5\gamma H^2} \tag{1}$$

where, T_{rj} is the mobilized tensile force in j^{th} reinforcement layer, H is the wall height, γ is the unit weight, and T_r is the total required reinforcement force.

For simplicity, the following approximation is followed.

$$T_{rj} = \gamma z_{r,j} S_{vr,j} \tag{2}$$

where, $z_{r,j}$ is the distance of the j^{th} reinforcement layer from the top of the slope and $S_{vr,j}$ is the vertical distance between two reinforcement layers.

The method is accurate owing to a minimum number of assumptions considered. The detailed theory, assumptions, equations involved, steps undertaken, and validation are comprehensively explained in Agarwal and Pain [3]. The explanations are not provided here to avoid word similarity with the mentioned literature, adhere to the word limit, and focus primarily on the probabilistic technique. The readers may follow Agarwal and Pain [3] and Nouri et al. [4] for more insights into HSM and its implementation.

3 Probabilistic Analysis Using FMNT

FMNT is used in the present study to analyze the safety of RSW and further calculate the P_f. As already stated, rather than the distributions, the first four moments of the input random variables are required. The formulation is based on the cubic normal distribution which is defined using the cubic polynomial of u (standard Gaussian variable) [5].

$$Z = c_1 + c_2 u, + c_3 u^2 + c_4 u^3 \quad (3)$$

where, $Z = \frac{x - \mu_x}{\sigma_x}$ is the standardized random variable with a mean (μ) and variance (σ) equal to 0 and 1, respectively, μ_x and σ_x, respectively are the mean and standard deviation of x. The skewness (α_{3x}) and kurtosis (α_{4x}) of Z are the same as x. c_1, c_2, c_3, and c_4 are the coefficients calculated according to Fleishman [6].

Usually, in case of non-availability of distribution of random variables, the first four moments are available. These moments are used to transform the standard normal variable, x to u using Eq. (3). This transformation is accurate owing to the cubic normal distribution which is highly flexible and has a wide range in skewness-kurtosis plane. The transformed variable is then used in FORM to estimate the P_f and further related statistics. For more details and basic information on FMNT readers may refer to Xu et al. [2]. A detailed procedure illustrating the steps to use FMNT for the probabilistic analysis of a RSW is also given in the form of a flowchart in Fig. 1.

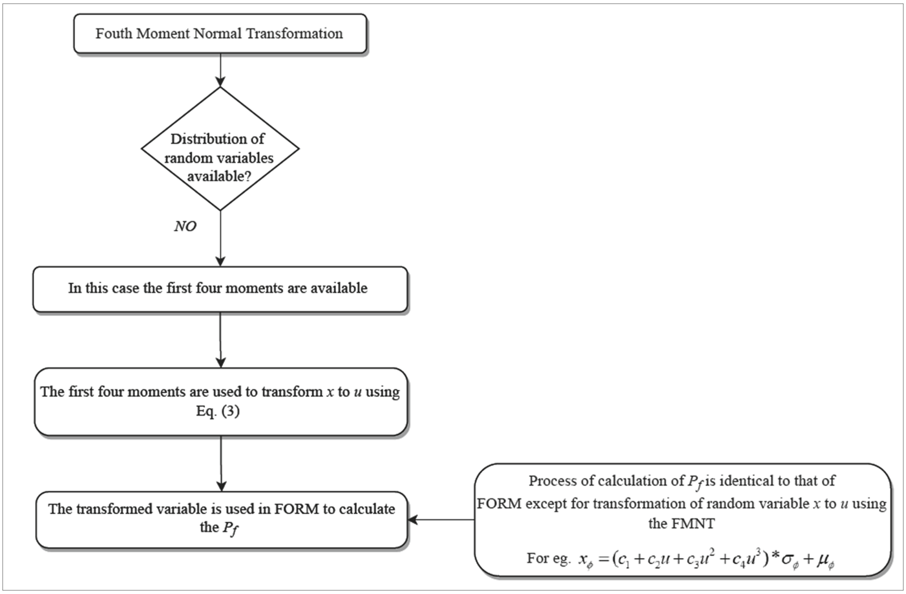

Fig. 1. Procedure to use FMNT for probabilistic analysis of RSW

4 Discussion

4.1 Validation of FMNT

The validation of the proposed technique is performed by comparing the results with Agarwal and Pain [7] and Basha and Babu [8]. Both the mentioned literature had used the first order reliability method (FORM) to calculate the reliability index against tensile mode of failure (β_t) for a RSW (Fig. 2) with height, $H = 5.5$ m, slope inclination angle with the horizontal, $\beta = 78.7°$, horizontal coefficient of acceleration, $k_h = 0.216$, soil-reinforcement interface friction angle, $\delta/\phi = 0.8$, and vertical coefficient of acceleration, $k_v = 0$. The random input parameters used in the validation and the parameters of FMNT are given in Table 1. The comparison of results is given in Table 2. The accuracy of FMNT is evident from Table 2; the obtained value of β_t is in line with the established literature where a small difference is attributed to the difference in deterministic methodologies used. This highlights that FMNT may be used to predict stability for a variety of RSW.

4.2 Illustrative Example

To highlight the flexibility of FMNT, an illustrative example is presented. The input parameters considered in Sect. 4.1 are used except for a small change that T_{ult} is now considered as a deterministic variable. The steps given in Fig. 1 are followed to calculate the P_f and β_t for the RSW, which are stated below.

$$\beta_t : 3.2372; P_f = 0.06\%$$

Owing to the consideration of T_{ult} as a deterministic variable, the reliability index of the RSW has increased due to the reduction in variability of T_{ult}. The results depict

Table 1. Input parameters used for validation & Parameters of the FMNT

Variable	First four moments				Parameters for the FMNT			
	μ_x	COV (%)	α_{3x}	α_{4x}	c_1	c_2	c_3	c_4
γ (kN/m^3)	18	5	0	3	0	1	0	0
ϕ (°)	37	5	0.150	3.040	-0.025	0.998	0.025	0
T_{ult} (kN/m)	20	5	0	3	0	1	0	0

Table 2. β_t from FMNT compared with Agarwal and Pain [7] and Basha and Babu [8] for the Seiken wall

Seiken wall (Toho-Oki earthquake)	β_t
Present Study	2.7564
Agarwal and Pain [7]	2.806
Basha and Babu [8]	2.2230

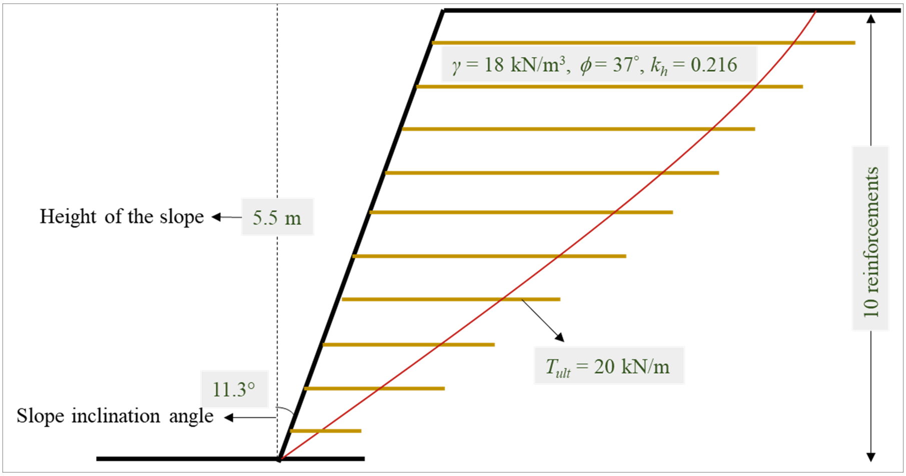

Fig. 2. Labelled RSW diagram for $H = 5.5$ m, $\beta = 78.7°$, $k_h = 0.216$, $\delta/\phi = 0.8$, and $k_v = 0$

Table 3. Computation time taken by MCS (100 samples) and FMNT to complete one analysis for parameters considered in Sect. 4.2

MCS	FMNT
170 s	2 s

that FMNT may be used with different sets of input variables to analyze the stability of RSW.

4.3 Efficiency of FMNT

The efficiency of the FMNT is estimated by comparing the computation time taken to populate the results with MCS (100 samples). The same input parameters chosen in Sect. 4.2 are used for the present comparison of efficiency. Table 3 illustrates the considerable efficiency of FMNT over MCS which makes it feasible to be practically used in the field. FMNT is almost 98.8% more efficient than MCS for the present case.

5 Conclusions

The present study highlights the importance of FMNT in performing the probabilistic analysis of RSW. The analysis and discussion lead to the following conclusions.

1. FMNT is a useful and efficient technique that may be employed for an accurate stability analysis of RSW in the absence of distributions of input random variables. It can also be easily applied in the field.
2. FMNT can be used with different possible cases of RSW.

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