Probabilistic Analysis of a Slope Using RLEM and Cross-Correlated Conditional Random Field

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Abstract. Probabilistic analyses of slopes using Random Limit Equilibrium Method (RLEM) have been extensively reported in literature. However, in these types of analyses, the generated random fields are based on assumed values of horizontal and vertical correlation lengths. In practice, horizontal and vertical correlation lengths can be measured using CPT data and the data can be used to condition the generated random fields. Conditioning random fields reduces the level of uncertainty in the analysis and helps the simulations to render more reasonable results. In this study, the stability analysis of a simple slope is used to investigate the influence of conditional and unconditional random fields. To generate spatially variable fields, first, some artificial borehole data are employed to correlate the spatially variable friction angle field. Then, considering some typical values for the variability of the cohesion random field and the possible cross-correlation between the two fields, a couple of scenarios are defined to synthesize the spatially variable realizations of the cohesion field. Then, the results of cross-correlated conditioned and unconditioned random fields are compared. The results show that conditioning random field and considering the cross-correlation between soil input parameters significantly reduce the probability of slope failure.

1 Introduction

About 300 slope failures occur annually in Hong Kong causing significant sums of money being spent on slope stabilisation. Hong Kong statistics show that 5% of slopes that have been constructed and stabilised using the classical deterministic approach would ultimately collapse. Neglecting the fluctuations in the many parameters influencing the stability of slopes is one of the causes of this phenomenon (Huang et al. 2019). An important source of uncertainty in the analysis of slope stability is the inherent spatial variability of soil parameters. The impacts of soil spatial variability on slope reliability analysis have been thoroughly examined using random field theory throughout the past few years (Griffiths and Fenton 2004; Griffiths et al. 2009; Hicks and Spencer 2010;

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Li et al. 2015; Jiang et al. 2014; Hicks et al. 2014; Liu et al. 2017; Jamshidi Chenari and Alaie 2015; Javankhoshdel et al. 2017; Liu et al. 2018; Cami et al. 2018; Jamshidi Chenari and Izadi 2019; Shah Malekpoor et al. 2020; Javankhoshdel et al. 2020; Mafi et al 2020).

In order to evaluate the dependency of an embankment slope in spatially variable soils to available CPT data, Liu et al. (2016a) presented a numerical strategy combining Subset Simulations (SS) with the Kriging method. Yang et al. (2017) also used Kriging approach to generate correlated random field (CRF) utilizing CPT data for probabilistic stability study of slopes. Li et al. (2016a) noted that if the number of known data points was vast, the calculation time for the Kriging approach would turn out to be unreasonably high.

Li et al. (2016b) presented a Markov Chain Monte Carlo (MCMC) approach for producing CRFs from borehole data so as to describe the variability of geologic profiles. Additionally, to create CRFs of soil parameters for a probabilistic investigation of tunnel longitudinal performance, Gong et al. (2018) used the Hoffman technique (2008). The Hoffman technique, while theoretically straightforward and computationally effective, is unable to account for the measurement uncertainties in site investigation data. While modeling multivariate geotechnical random fields, it is also important to appropriately account for the cross-correlation between various parameters in addition to the autocorrelation for a given geotechnical parameter. In a recent study, Tang et al. (2020) proposed a generic method for producing multivariate cross-correlated geotechnical random fields.

Random Limit Equilibrium Method (RLEM) was originally introduced by Javankhoshdel et al. (2017). RLEM is a combination of random field theory to create a spatially variable field and limit equilibrium slope stability analysis to calculate the factor of safety. In the current study, two cases of RLEM analysis with and without conditional and cross-correlated random fields are studied and compared.

2 Conditional Random Fields

To generate spatial variability fields, horizontal and vertical spatial correlation lengths should be measured. These two parameters are measured using CPT data (Cami et al 2020). Using these two measured parameters is a starting point of generating the spatially variable random field. However, the process of random field generation is still random and does not consider the known values of the data in the CPT locations.

The goal in the probabilistic analysis is to reduce the level of uncertainty of the problem. Considering spatial variability of soil properties instead of having homogenous material in the soil profile is one of the steps. The second way of reducing the uncertainty of the problem is to consider the cross-correlation between soil input parameters (Javankhoshdel and Bathurst 2015).

The last step that can be taken into account to reduce this uncertainty is to consider the available data to generate a random field, which is called conditional random field. Conditioning of a random field comprises a process that uses an unconditional random field along with the statistics calculated from the available data. The random field is then post-processed in order to condition it at the known conditioning points (Loret-Cabot et al. 2012).

There are different approaches to condition random fields. The most recent approach presented by Ching et al. (2021) uses sparse Bayesian approach to provide site characterization and conditional random fields. The purpose of site characterization is to measure the statistical parameters (mean, standard deviation, and spatial correlation length) and use them together with the cross-correlation between the soil parameters so as to generate conditional cross-correlated random fields. The same algorithm combined with the RLEM analysis is utilized in this study to investigate the influence of conditioned and unconditioned random fields. Slide2 (Rocscience 2022) software is used for the RLEM analyses.

3 Methodology

Figure 1 shows the steps followed in this study to generate cross-correlated conditional random fields. First, a rectangular random field of 100 m by 45 m is generated for friction angle. This random field was random, and the statistical parameters of friction angle will be measured later. Then, for one of the fields, 20 artificial boreholes were generated to extract data every 5 m horizontally and every 0.2 m vertically. An example of one of the vertical data points is shown in Fig. 2.

These artificial boreholes were then used to measure the statistical parameters of the friction angle. These parameters can be found in Table 1.

Next, using the artificial boreholes and statistical parameters of friction angle, unconditional and conditional random fields are generated.

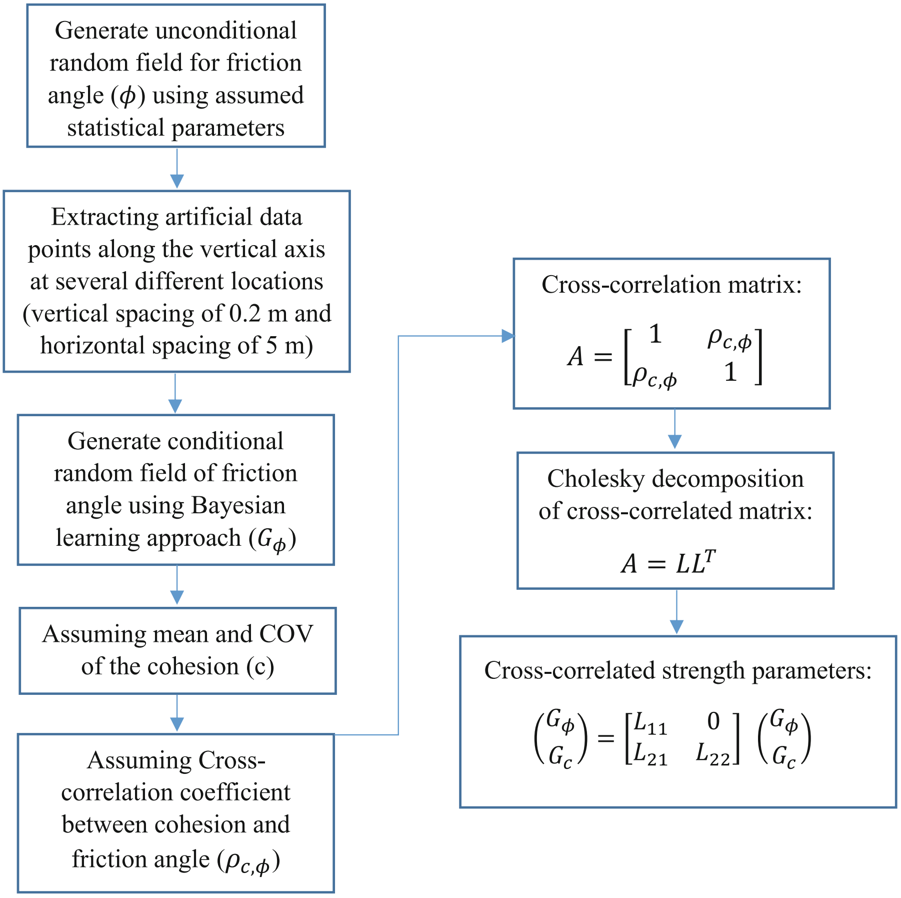


Fig. 1. Flowchart of the steps of generating conditional cross-correlated random fields (X = 80 m).

Then, with the assumed statistical parameters of cohesion (see Table 2), a cross-correlation of −0.5 between cohesion and friction angle, and also an algorithm suggested by Sasanian et al. (2019), unconditional and conditional random fields for cohesion were generated. The procedure suggested by Sasanian et al. (2019) is presented in Fig. 1.

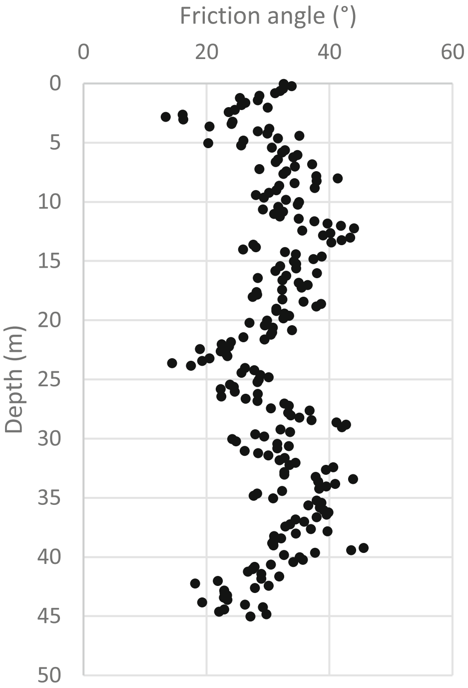


Fig. 2. The information of one of the artificial boreholes for friction angle (X = 80 m).

Table 1. Measured statistical parameters of friction angle

Parameter	Mean	Standard deviation	Horizontal correlation length	Vertical correlation length
Friction angle (°)	32.25	5.65	1.65	0.97

Table 2. Assumed statistical parameters of cohesion.

Parameter	Mean	Standard deviation
Cohesion (kPa)	26.42	13.21

4 Illustrative Example

Figure 3 shows the model used in this study. The geometry is modified version of a case study in Oman presented by Dastpak et al. (2022).

Figure 4 shows an example of unconditional random field for friction angle generated using the statistical data from Table 1.

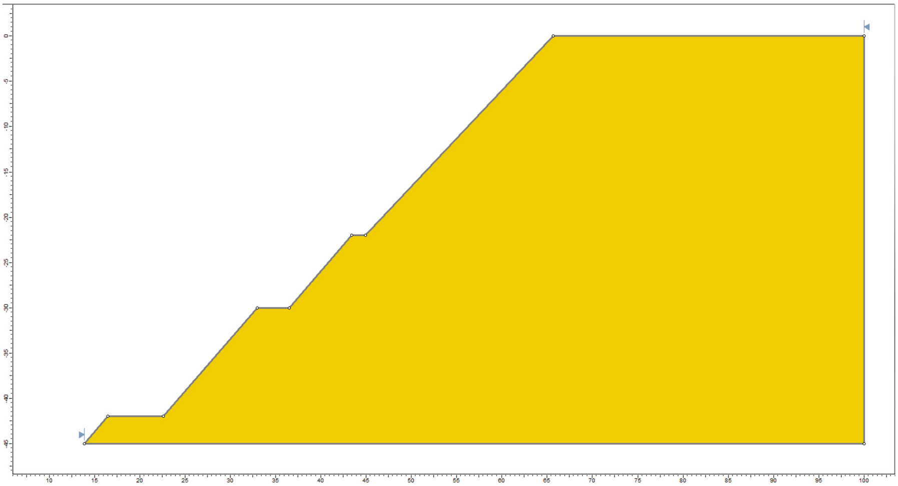


Fig. 3. An example model used in this study.

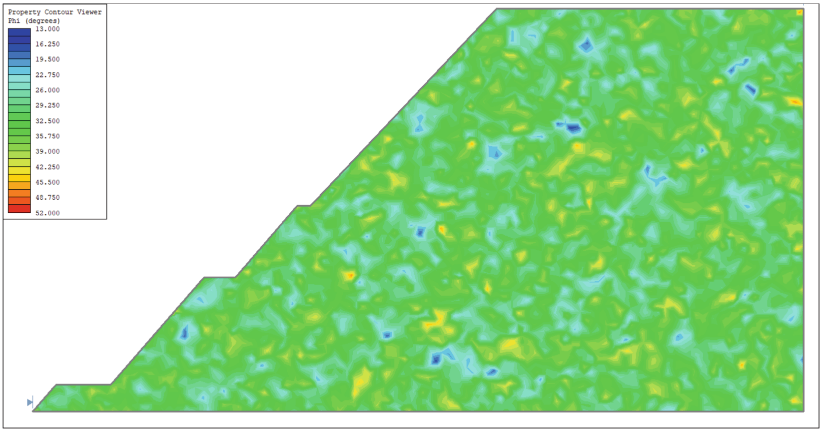
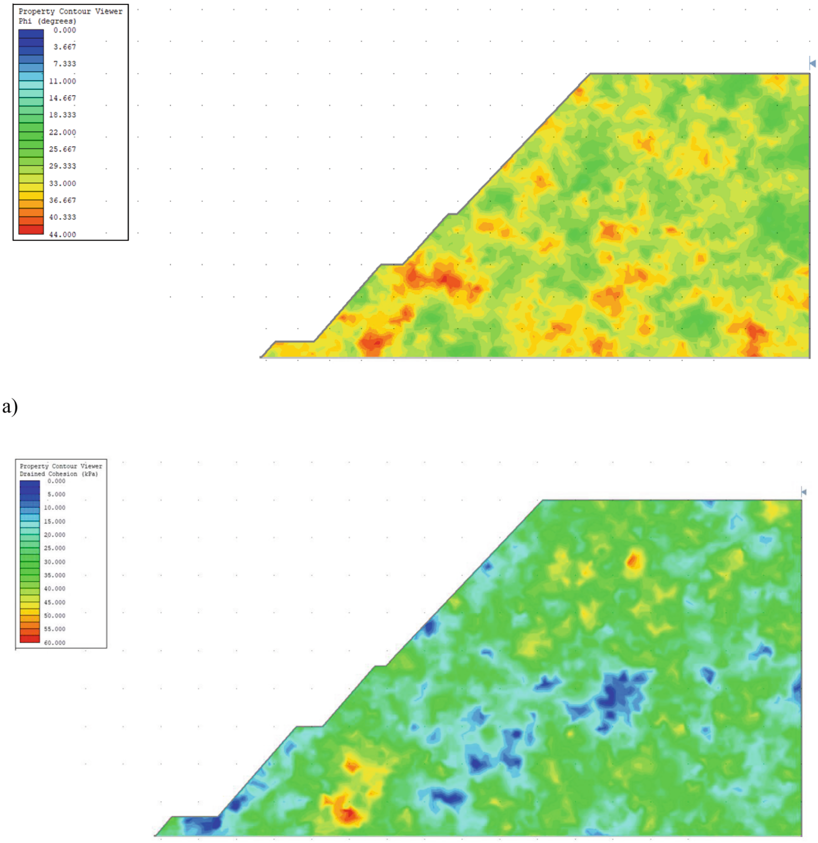


Fig. 4. A sample random field generation for friction angle.

On the other hand, using the generated internal friction angle field, as shown in Fig. 4, the corresponding conditioned cross-correlated random field of c and ϕ using the adopted artificial boreholes are shown in Fig. 5. Note that weak regions in Fig. 5a correspond to strong regions in Fig. 5b since c and ϕ fields are negatively cross-correlated.



a)

b)

Fig. 5. Conditional and cross-correlated random fields of a) friction angle b) cohesion

The probability of failure for the unconditional case is 7.6% with the reliability index (RI) of 0.83. However, probability of failure for the conditioned case reduces to 1.2% with the RI of 2.4. As it was expected, conditioning the random fields even with some random artificial data, increases the level of certainty in the problem and reduces the probability of failure significantly (increases the RI value).

In this study, an advanced RLEM approach is used which utilized the Surface Altering Optimization technique for local optimization in order to find a better factor of safety (Mafi et al 2020). Figure 6 shows a critical slip surface for one of the failed cases (FS = 0.996). It is clear in this figure that the failure mechanism goes through the blue region which is indicative of the weak material in this problem. This shows the advantage of the adopted advanced RLEM approach that seeks out the weakest failure path in the model.

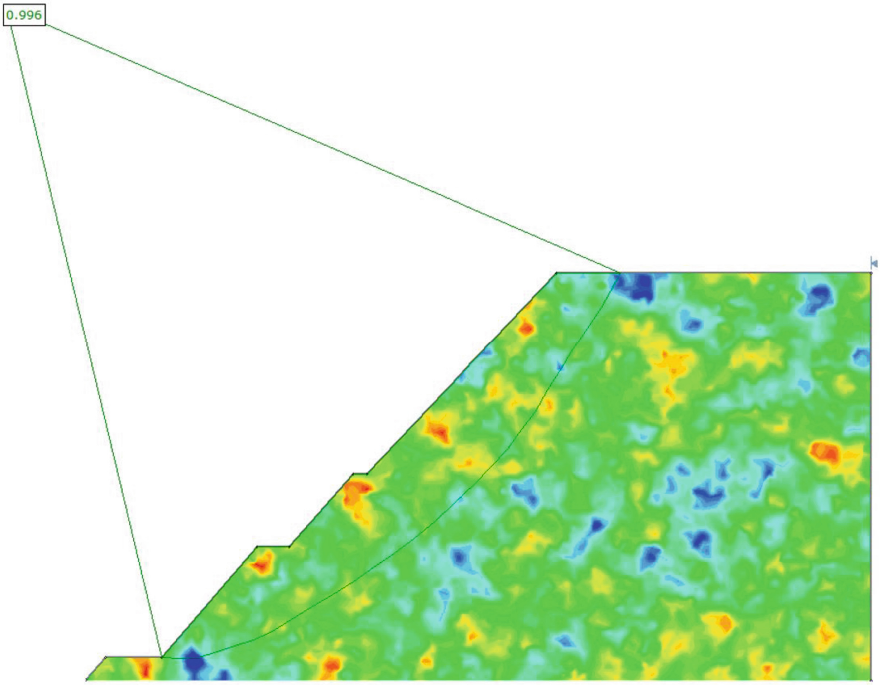


Fig. 6. Critical slip surface for one of the failed cases of the model.

5 Conclusion

In this study, the influence of conditional and unconditional random fields was examined through the stability analysis of a simple slope using random limit equilibrium method (RLEM). To this end and to generate spatially variable fields, first, some artificial borehole data were exploited to correlate the spatially variable friction angle field. In the next step, some typical values for the variability of the cohesion random field along with a possible cross-correlation between the soil friction angle and cohesion fields were considered so as to generate unconditional and conditional random fields for cohesion. Comparing the results of cross-correlated conditioned and unconditioned random fields show that conditioning random fields and considering the cross-correlation between soil input parameters, including the shear strength properties, would significantly reduce the probability of slope failure. The described procedure was demonstrated via an illustrative example. It was shown in this example that conditioning the random field even with some random artificial data, increases notably the level of certainty in the problem and thus reduces significantly the probability of failure (increases the RI value).

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