A Case Study on the Use of Expert Selected Set Method in the Reliability Analysis of an Urban Deep Excavation Project

Ali Fakher[1], Arefeh Arabaninezhad[1](✉), and Minoo Nikghalbpour[2]

[1] University of Tehran, Tehran, Iran
arefeh.arabani@ut.ac.ir
[2] Mohaghegh Ardabil University, Ardabil, Iran

Abstract. The necessity of constructing structures beneath ground level, increases the need of performing deep excavations in urban areas. It is essential to have a reliable estimation about the performance of deep excavation systems. This approach is not accessible via common deterministic analysis methods, since the uncertainty of soil parameters are not considered in these methods. Thus, various probabilistic and non-probabilistic reliability analysis methods are proposed to overcome this shortcoming. Due to the complicated mathematical procedure and lack of thorough database available for soil properties in real projects, which are both required to perform most of the reliability analysis methods, these methods are not welcomed by the engineers in practical projects. Since, it is common to apply deterministic analysis methods, the disadvantage of ignoring the effects of uncertainty on system response still exists in operation. According to the results of researches done in the field of reliability analysis, there are simple proposed methods which can be utilized in practical projects; however, they are not known to the engineers. In the present study a recently developed and simple reliability analysis method called Expert selected set method is applied for the analysis of an urban deep excavation project in Iran. For verification purposes, the results are compared with field measurement values and the reliability analysis results which were obtained using a well-known method called Point estimate method. The comparisons approve the feasibility of investigated method to predict the system performance and providing a reliable estimate of the system response, during the design stage.

Keywords: Deep excavation · Expert Selected Set method · Point Estimate Method · Reliability analysis · Uncertainty

1 Introduction

Rapid growth in population and the desire of people to live in the urban area accentuate the necessity of deep excavation in the world. Several deep excavations are reported in various countries [1–3]. The support system is the essential part of deep excavations and needs to be designed precisely. The design of support system is complicated due to the lack of thorough information about input data, soil behavior and model uncertainties.

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In the most of practical geotechnical problems, engineers use deterministic methods to perform the stability analysis. Deterministic methods in geotechnical engineering includes consideration of a single value for each input variable and represents a single response of the system [4]. However, there is uncertainty as an inherent part of the geotechnical problems as well as lack of knowledge. Uncertainty of concept, field and laboratory measurement methods, sampling process, sample sensitivity and boundary condition are some resources of uncertainties in the geotechnical engineering [5]. Performing a wide range of tests would reduce the uncertainties, however it is not possible as reputing the tests are time consuming and costs a lot. Due to the presence of uncertainties, it is essential to manage them to obtain the risk of critical performance [6]. Considering uncertainties provide the situation in which designers can decide and judge based on the desired performance level of a project [7]. The engineers must understand the nature of uncertainty and probability to develop the appropriate input variables base on the knowledge, profession's practical approach and engineering judgment.

Observational method widely was used to deal with uncertainties in geological material [8–10]. Thereafter by advances in computational modeling, reliability theory as a formal concept were applied by researchers to quantify and handle uncertainties [11]. Several reliability methods including probabilistic and non-probabilistic approaches are developed and modified to treat the uncertainties [6, 12–15]. Among the developed probabilistic methods, the point estimate (PE) method is welcomed by engineers due to its accuracy in comparison with practical project [16]. The basic concept of PE method is to substitute the probability distributions of soil variables by single values considering their predefined probability to perform the solution at various estimation points and apply weighting to consider an approximation of the distribution of the solution [17]. The low computational effort and simplicity of PE method makes it a proper reliability analysis method. The available data in practical works is often insufficient to fit the precise probability distribution for each soil parameter, hence the first assumptions to perform the PE method might be deceptive. Because of the complex and time-consuming procedure and lack of thorough data, the well-known reliability analysis methods are not appreciated by engineers in real projects. In order to consider the uncertainty in geotechnical practical projects by professional engineers, it is necessary to propose simple, accurate and time efficient methods. Recently, one simple and rapid method was developed by Arabaninezhad and Fakher, called expert selected set (ESS) method which was evaluated and verified for five case studies in Tehran [18].

In this study, ESS method is applied to analyze the reliability of a practical project in Tehran. The horizontal displacement at the excavation top point and the factor of safety (FOS) are considered as the main system responses. For verification, the results of reliability analysis obtained by ESS are compared with field observations and measurement values of horizontal displacement as well as the results obtained by PE method as a well-known reliability analysis method. In addition PLAXIS.2D as a finite element (FE) software is used to model the project and obtaining the system output [19].

2 The Expert Selected Set Method

In the ESS method, each soil variable is assigned by one range; without defining a specific probability share. The steps to perform ESS method are as follows [18]:

- Step 1: Considering the geometry of the system and according to the geotechnical investigation the main finite element model is generated.
- Step 2: Only one range is assigned to each soil parameter. The input sets are selected by expert judgment; although the statistical knowledge may help suggesting more appropriate ranges.
- Step 3: Sensitivity analysis is applied to assess the most influential input parameters; and to decrease number of required finite element runs.
- Step 4: Considering the lower and upper bounds of the input sets, various combinations of soil parameters are developed; and the relevant system responses are recorded.
- Step 5: Applying statistical software such as EasyFit [20], the best distribution function is fitted to the system response values calculated in step 4.
- Step 6: An acceptable value is determined for the system response, and the probability of occurring the unsatisfactory system performance is estimated.

In this study the horizontal displacement at the top of excavation is considered as one of the system performance functions which could help the engineers predict the probability of unsatisfactory performance of a deep excavation. The acceptable displacement value depends on national codes and engineering judgment [21].

Various values could be considered as the acceptable limit for horizontal displacement of the wall, based on the project constraints. These values are generally estimated by semi-empirical relationships proposed in the technical standards and codes [22–25].

It is worth mentioning that excessive deformations would not always result in system failure [26]; Hence, the probability of excessive movement for a deep excavation is not equal to the probability of system's collapse. It is obvious that FOS = 1 is the governing criteria for stability control. The threshold range between 10^{-6} to 10^{-4} is suggested by many researchers for failure probability [27–29]; But due to the disastrous consequences of ultimate failure in comparison with excessive movement, the critical probability of excessive deformation (APED) is definitely higher. In this study the value of 0.10 is considered for APED as proposed by Momeni et al. [21].

3 Implementing the ESS Method for a Deep Excavation Project in Iran

The investigated deep excavation, with an area of 1800 m^2, was located in Mirdamad Street in Tehran. Figure 1 and Fig. 2 show the excavation location and neighboring facilities, including buildings and roads respectively.

The wall section between points A and B as illustrated in Fig. 2 is investigated. The excavation and stabilization process has been completed in this zone. The deterministic methods had been utilized in the design stage of the support system and a combination of a Berliner column with ground anchors was used as shown in Fig. 3. The excavation depth is 23.7 m and the ground water level is below the excavating level.

3.1 Numerical Modeling

PLAXIS 2D as a finite element software was applied for numerical modeling. Stage construction was utilized for the analysis, and Hardening Soil (HS) model was selected

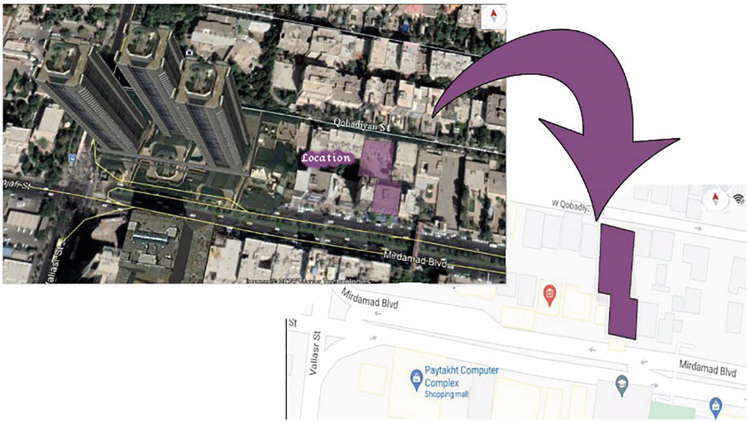

Fig. 1. Aerial view of the project location

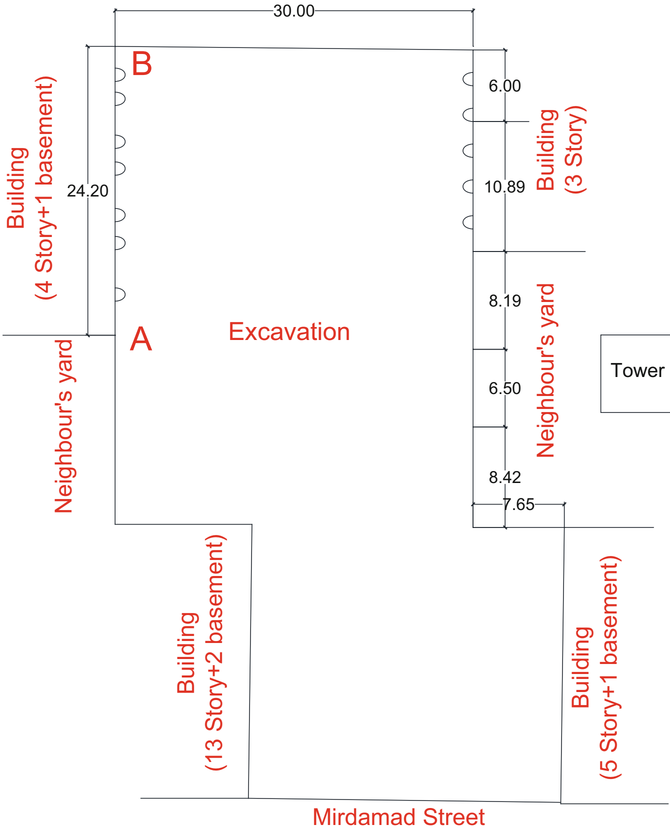

Fig. 2. Neighboring facilities

Fig. 3. The support system used for stabilization

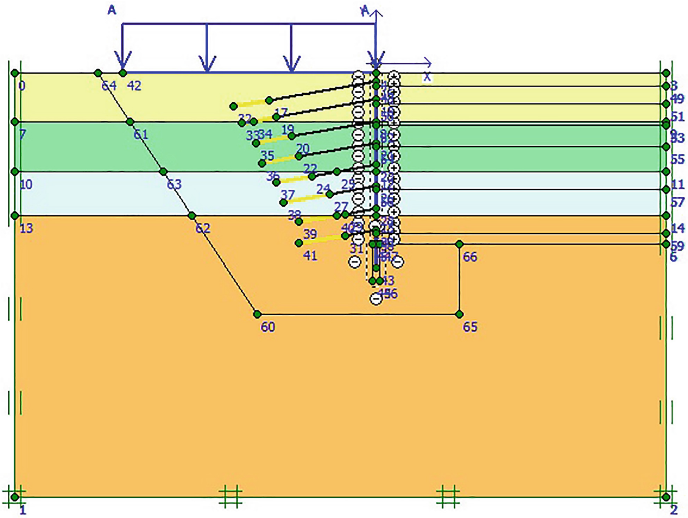

Fig. 4. Cross-section of the model

as the constitutive model for the soil. Figure 4 shows a cross-section of the system and the values assigned to soil parameters in the main model are presented in Table 1. These values are defined according to the geotechnical reports available for the project. The parameters representing the support system of the wall are also shown in Table 2.

Table 1. Soil properties used in the main PLAXIS model

Layer	Depth (m)	γ (kN/m^3)	c (kN/m^2)[a]	$\varphi°$[b]	E (MN/m^2)[c]
1	0–8	20	35	37	650
2	8–15	20	40	38	750
3	15–21	20	50	39	800
4	Below 21	20	60	40	900

[a] Effective cohesion
[b] Effective friction angle
[c] triaxial loading stiffness.

Table 2. Parameters for structural elements in numerical model

Structural element	Behavior	EA (kN/m)	EI $\left(kN.m^2/m\right)$	Tensile Capacity (kN/m)	Moment Capacity $\left(kN.m^2/m\right)$	Horizontal spacing (m)
5-strands anchor	Elastoplastic	4.77E+04	-	260	-	3.0
Berliner Column (2IPE240)	Elastoplastic	5.21E+05	5187	375.4	31.1	3.0

3.2 Input Sets Assigned to Soil Variables

As previously stated, the input sets are assigned to each soil variable based on the expert judgment. Utilizing the statistical knowledge about various geotechnical properties could help suggesting more appropriate input sets. One of the most important statistical specifications, investigated by researchers, is the coefficient of variation (COV). This item can be applied to calculate the standard deviation (σ) while a mean value (μ) is considered for the soil variable.

$$COV = \left(\frac{\sigma}{\mu}\right) \quad (1)$$

The suggested values of COVs for different soil variables are presented in Table 3.

Previous studies suggest that considering the $[(\mu - \sigma), (\mu + \sigma)]$ range as the input set leads to a reliable estimation of the system response [18]. According to the main values for soil parameters in Table 2 and the standard values of COV in Table 3, the selected sets for input variables are presented in Table 4.

3.3 Sensitivity Analysis

In order to select the most effective soil variables on the performance functions i.e., FOS or horizontal displacement, the sensitivity analysis is performed. The main purpose

Table 3. Recommended ranges for COV

Soil property	Reported COV (%)	Standard COV (%)	Source
Cohesion (un-drained clays)	25–50	30	Singh 1971 [30]; Lumb 1974 [31]
Cohesion (un-drained sands)	25–30	30	Lumb 1974 [31]
Friction angle (various soil types)	9	9	Lumb 1966 [32]
Stiffness modulus	2–42	30	Kennedy 1978 [33]; Otte 1978 [34]

Table 4. The range of input soil parameters for ESS method

Soil property	c (kN/m^2)		$\varphi°$		E (MN/m^2)	
	Lower	Upper	Lower	Upper	Lower	Upper
1	24.5	45.5	33.67	40.33	455	845
2	28.0	52.0	34.58	41.42	525	975
3	35.0	65.0	35.49	42.51	560	1040
4	42.0	78.0	36.40	43.60	630	1170

of sensitivity analysis is to decrease the required FE runs. in this study the method provided by the US Environmental Protection Agency was utilized [35]. The procedure of performing sensitivity analysis is not explained here for brevity; but it can be found in references [18]. Finally, based on the sensitivity analysis ratios and the acceptable value recommended in literature [36] the following four variables were determined as the most effective input variables: soil stiffness and friction angle of Layers 3 and 4.

3.4 Reliability Analysis Results Applying ESS

2^4 combinations were generated considering the lower and upper bounds of the most influential variables. Table 5 shows the possible combinations and the relevant model outputs.

Table 5. Input values for x = (φ_3, E_3, φ_4, E_4) and relevant FE model outputs (ESS method)

Analysis number	Possible combinations	φ_3	$E_3 (MPa)$	φ_4	$E_4 (MPa)$	Model output for horizontal crown displacement (mm)	Model output for FOS
1	LLLL[a]	35.49	56	36.4	63	34	1.42
2	ULLL	42.51	56	36.4	63	30.4	1.55
3	LULL	35.49	104	36.4	63	29.38	1.45
4	LLUL	35.49	56	43.6	63	28.64	1.52
5	LLLU[b]	35.49	56	36.4	117	28.09	1.45
6	UULL	42.51	104	36.4	63	24.91	1.54
7	ULUL	42.51	56	43.6	63	26.02	1.62
8	ULLU	42.51	56	36.4	117	25.09	1.53
9	LUUL	35.49	104	43.6	63	23.94	1.51
10	LULU	35.49	104	36.4	117	22.92	1.44
11	LLUU	35.49	56	43.6	117	23.31	1.51
12	UUUL	42.51	104	43.6	63	21.11	1.62
13	UULU	42.51	104	36.4	117	20.286	1.54
14	ULUU	42.51	56	43.6	117	20.509	1.63
15	LUUU	35.49	104	43.6	117	18	1.50
16	UUUU	42.51	104	43.6	117	15.29	1.66

[a] Lower bound of set
[b] Upper bound of set

Utilizing the EasyFit software, the best fitting probability distribution function representing the system response values (shown in columns 7 and 8 of Table 5), was determined. Figure 5 and Fig. 6 display the reliability analysis results in the form of PDF (probability distribution) and CDF (cumulative distribution) curves. According to mentioned criteria in literature, the value of 47 mm (0.002H) is considered as the acceptable value for horizontal displacement at the top of deep excavation.

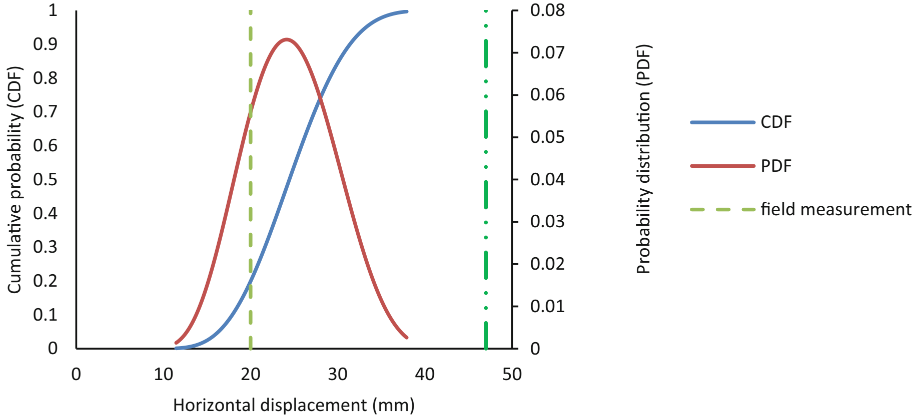

Fig. 5. ESS method results considering the horizontal displacement as the performance function

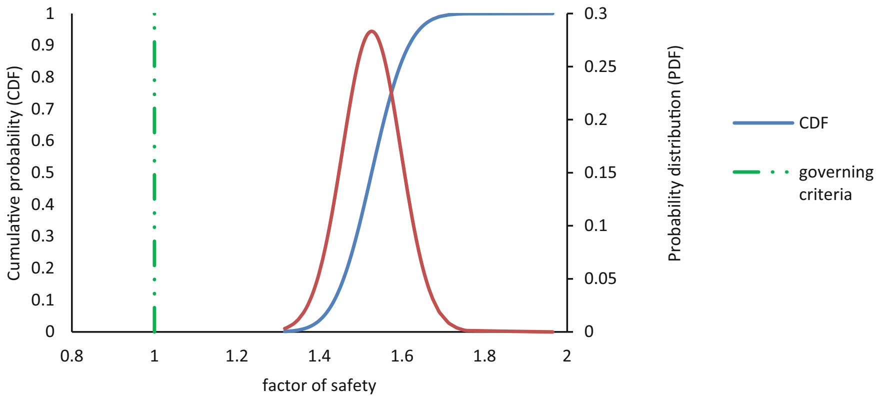

Fig. 6. ESS method results considering FOS as the performance function

Table 6. Statistical specification of ESS method results

System Response	Mean (μ)	Standard deviation (σ)	Best fitted distribution
Horizontal displacement at the top of excavation (mm)	24.49	4.887	Johnson SB
Factor of safety	1.5297	0.07139	Normal

Table 6 shows the statistical specifications of the distribution function fitted on the values of system responses.

Table 7. Soil properties used in the main PLAXIS model

Variable	Distribution function	Mean	Standard dev.
Soil stiffness layer 3	Normal	80767	14021
Friction angle layer 2	Normal	38.84	1.977
Soil stiffness layer 4	Normal	91480	15599
Friction angle layer 4	Normal	40.13	2.120

4 Reliability Analysis Applying Point Estimate Method

In this study the suggested approach by Zhou and Nowak (1988) [37] was utilized in order to implement PE method. In this method the $2n^2 + 1$ (n is the number of basic variables) integration rule is utilized [38]. For the purpose of brevity, the concept of PE method is not presented in this paper, but it could be found in the literature [37, 39–42]. The procedure of implementing PE reliability analysis are as follows.

At the first step the best distribution function representing each basic input parameter is determined, applying the EasyFit software. Table 7 shows the statistical specifications of the effective input variables.

33 combinations of the soil parameters were generated based on the integration rule in $2n^2 + 1$ method. Table 8 shows the input combinations and the relevant FE model outputs.

In the last step, the best fitting probability distribution function representing the system response values was determined. The reliability analysis results were depicted in Fig. 7 and Fig. 8; and the statistical specifications are shown in Table 9.

5 Comparison and Discussion

The reliability analysis results obtained by ESS method were verified by comparing with the results of PE method along with the field measurement and the acceptable values considered for system performance functions.

The CDF curves for the crown horizontal displacement are illustrated in Fig. 9.

According to Fig. 9 for horizontal displacement as the performance function:

- A satisfying similarity was observed between the results of the two methods.
- The reliability analysis results cover the value of horizontal deformation measured in the field.
- The probability of excessive deformation (representing the system unsatisfactory performance) equals to zero according to both methods, which is less than the acceptable probability of 0.1. This finding is confirmed by the monitoring sheets which report no crack around the deep excavation.

The CDF curves for the FOS are illustrated in Fig. 10.

According to Fig. 10 for FOS as the performance function:

Table 8. Input values for x = (φ_3, E_3, φ_4, E_4) and relevant FE model outputs (PE method)

Analysis number	φ_3	$E_3(MPa)$	φ_4	$E_4(MPa)$	Probability of happening	Model output for horizontal crown displacement (mm)	Model output for FOS
1	38.84	80.77	40.13	91.48	0.333	23.395	1.526
2	38.84	80.77	40.13	129.70	0.000		
3	38.84	80.77	40.13	53.27	0.000		
4	38.84	115.11	40.13	91.48	0.000		
5	38.84	46.42	40.13	91.48	0.000		
6	38.84	80.77	45.32	91.48	0.000		
7	38.84	80.77	34.94	91.48	0.000		
8	43.68	80.77	40.13	91.48	0.000		
9	34.00	80.77	40.13	91.48	0.000		
10	38.84	105.05	40.13	118.49	0.028	19.19	1.538
11	38.84	105.05	40.13	64.46	0.028	23.93	1.528
12	38.84	56.48	40.13	118.50	0.028	23.46	1.531
13	38.84	56.48	40.13	64.46	0.028	28.81	1.521
14	38.84	80.77	43.80	118.49	0.028	18.66	1.543
15	38.84	80.77	43.80	64.46	0.028	24.79	1.572
16	38.84	80.77	36.46	118.50	0.028	23.32	1.509
17	38.84	80.77	36.46	64.46	0.028	28.93	1.512
18	38.84	105.05	43.80	91.48	0.028	18.99	1.542
19	38.84	56.48	43.80	91.48	0.028	23.77	1.560
20	38.84	105.05	36.46	91.48	0.028	23.28	1.502
21	38.84	56.48	36.46	91.48	0.028	28.53	1.499
22	42.26	80.77	40.13	118.50	0.028	19.46	1.579
23	42.26	80.77	40.13	64.464	0.028	24.60	1.571
24	35.42	80.77	40.13	118.50	0.028	22.56	1.479
25	35.42	80.77	40.13	64.46	0.028	27.99	1.469
26	42.26	105.05	40.13	91.48	0.028	19.96	1.611
27	42.26	56.48	40.13	91.48	0.028	24.61	1.571
28	35.42	105.05	40.13	91.48	0.028	22.68	1.475
29	35.42	56.48	40.13	91.48	0.028	28.11	1.468

(*continued*)

Table 8. (*continued*)

Analysis number	φ_3	$E_3(MPa)$	φ_4	$E_4(MPa)$	Probability of happening	Model output for horizontal crown displacement (mm)	Model output for FOS
30	42.26	80.77	43.80	91.48	0.028	20.39	1.609
31	42.26	80.77	36.46	91.48	0.028	24.40	1.528
32	35.42	80.77	43.80	91.48	0.028	23.10	1.507
33	35.42	80.77	36.46	91.48	0.028	27.24	1.408

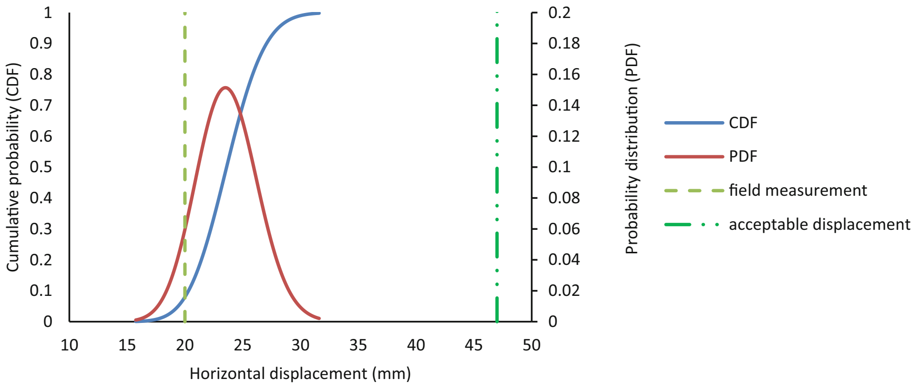

Fig. 7. PE method results considering the horizontal displacement as the performance function

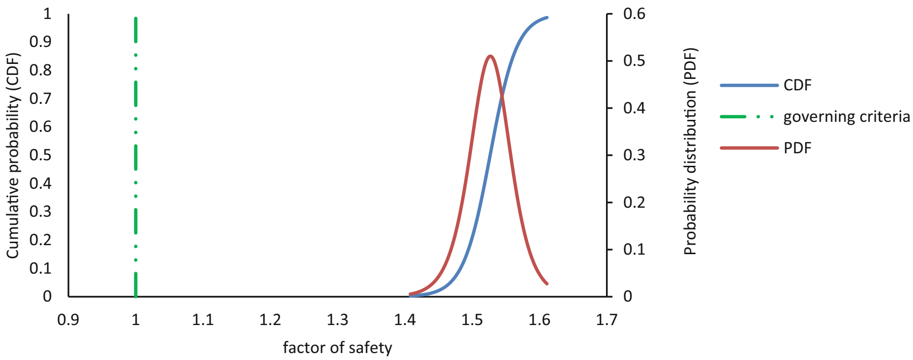

Fig. 8. PE method results considering FOS as the performance function

Table 9. Statistical specification of reliability analysis results using PE method

System Response	Mean (μ)	Standard deviation (σ)	Best fitted distribution
Horizontal displacement at the top of excavation (mm)	23.653	2.612	Nakagami
Factor of safety	1.5262	0.03818	Burr (4P)

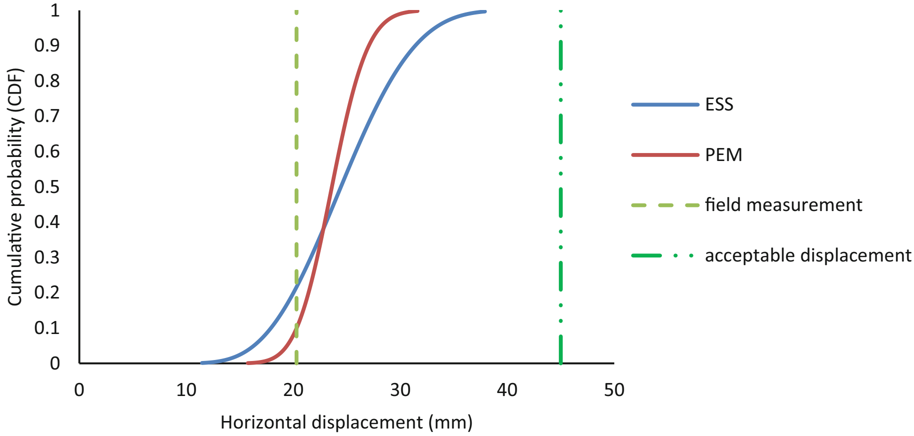

Fig. 9. Results of the ESS method compared to the PE along with the field measurement and acceptable values of horizontal displacement

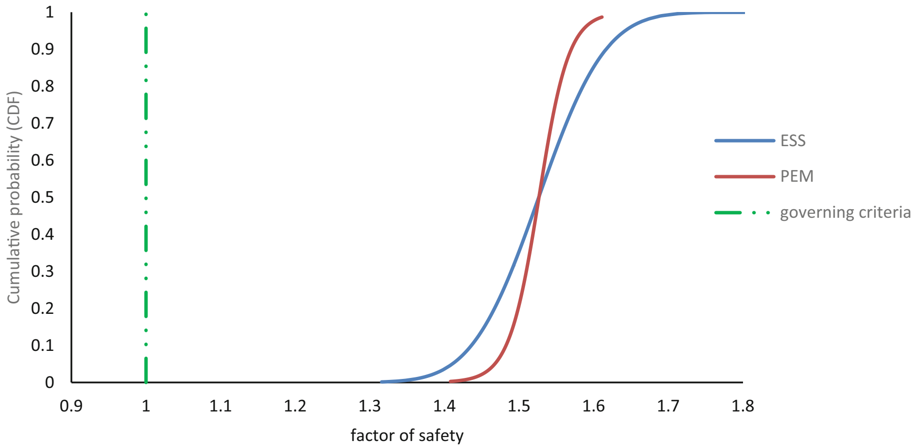

Fig. 10. Results of the ESS method compared to the PE overlaid with acceptable value for FOS

- A satisfying accordance was observed between the two methods.

- The system failure occurs when the FOS is less than the critical value. The probability of failure equals to zero based on both ESS and PE methods, which is less than the acceptable probability of 10^{-4}. This conclusion is confirmed with the field observations where no failure was reported.

6 Conclusion

The major goal in this study was to evaluate the feasibility of a recently developed reliability analysis method called expert selected set (ESS) method. The main conclusions of the study are as follow:

a) The mathematical procedure of the ESS method is simple. Also, the available geotechnical data in most of the real projects is sufficient to implement the ESS method; hence, this method could be easily utilized by experts in real projects. The probabilities of excessive horizontal displacement at the excavation top point and ultimate failure were calculated and were less than the acceptable criteria. This finding shows the satisfactory performance of the system which is also confirmed by the filed measurement and site observations.
b) Since the ESS method has been recently developed, in order to verify, the results were compared to what obtained by point estimate method as a well-known probabilistic technique. A good agreement was observed between the results of the two methods.
c) The reliability analysis results cover the field measurement values. Hence, the ESS method can be utilized in the design stage of real projects to provide a reliable estimation of the system performance in reality.

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