Probabilistic Assessment of Seismic Bearing Capacity of Strip Footings Seated on Heterogeneous Slopes Using Finite Element Limit Analysis (FELA) and Response Surface Method (RSM)

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Abstract. The paper demonstrates the use of the response surface method (RSM) to carry out probabilistic assessment of the seismic bearing capacity of shallow footings seated near naturally occurring heterogeneous slopes. To this end, a pseudo-static loading is applied to a randomly uniform slope, which is homogeneous in each case but random between realizations. The method substantially reduces the number of Monte Carlo simulations required to carry out cumbersome probabilistic slope stability analyses. A finite element limit analysis model based on the lower bound theorem is developed, which is then used to generate a large synthetic database of numerical results for the seismic bearing capacity of shallow foundations resting on inherently variable natural slopes. To this end, a permutation of the key parameters is formed and lower bound FELA-based limit loads are sought through optimization in MATLAB. A closed-form solution is formulated using RSM-based polynomials. The RSM equations, which are acquired from least squares regression analyses, are used to carry out probabilistic Monte Carlo simulations and the results are presented in forms of cumulative distribution functions. Results from the probabilistic analyses are introduced into some reliability-based design approach to render design loads for different reliability levels.

Keywords: Slope stability · Seismic Bearing capacity · Shallow footing · Surface response method · Finite element limit analysis (FELA) · Reliability index · Random variability

1 Introduction

Bearing capacity of shallow footings is a topic of great significance in the geotechnical engineering practice. This classic problem turns out to be more complicated when the

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foundation is placed on an earth slope. There has been a large volume of research studies focusing on the assessment of the bearing capacity of shallow foundations seated on slopes [1–7]. However, most of these studies have been devoted to the homogenous slopes and the significant contribution of heterogeneity of soil shear strength parameters has been commonly overlooked. Natural soil deposits possess specific degrees of inherent variability, thus giving rise to the uncertainty issues when encountering geotechnical stability problems [8]. Probabilistic evaluation of the bearing capacity of shallow foundations resting on earth slopes has been carried out in some of previous studies in the literature by considering soil non-homogeneity and spatial variability [9, 10].

On the other hand, shallow foundations in earthquake prone areas are very likely to be subjected to seismic excitations. The earthquake excitation is applied on soil-footing geo-systems as an external loading which in turn could influence the stability status of foundations, especially those seated on top of the earthen slopes, depending on the strength parameters of the underlying soil. Unlike the significance of the topic, the probabilistic seismic response of such surface footings resting on slopes has remained unexamined throughout the literature. In this study, the lower bound theorems of finite element limit analysis (FELA) along with the response surface method (RSM) have been adopted to evaluate the influence of random variability of soil shear strength parameters on the seismic bearing capacity of shallow foundations seated on heterogeneous earth slopes. The so-called RSM has been deployed herein to avert the need of using cumbersome numerous Monte Carlo simulations.

2 Finite Element Limit Analysis

The geometry of the problem under study are shown in Fig. 1. As it can be observed, a shallow foundation is resting on an earth slope whose soil behavior has been assumed to follow the associated plastic flow rule by conforming to the perfectly plastic Mohr-Coulomb failure criterion. The lower bound theorem of FELA has been employed in this study to evaluate the seismic bearing capacity of such a surface footing. The corresponding approach seeks the lower bound of the actual collapse load of the foundation by accounting for an admissible stress field within the soil medium underneath using either linear programming (LP) or second-order cone programming (SOCP) techniques [11–25].

Based on the well-established framework of the finite element method for a plane strain problem, three unknown normal and shear stress variables (σ_x, σ_y and τ_{xy}) are assigned to each nodal point in the finite element mesh while the stresses inside the triangular elements are assessed through the well-known linear shape functions. Furthermore, the lower bound FELA theory states that four constraints must be met within the finite element mesh of the soil medium underlying the foundation so as to achieve the desirable admissible stress field. These constraints include element equilibrium, discontinuity equilibrium, boundary conditions, and yield criterion enforcement.

According to the element equilibrium constraint in the seismic loading condition, all triangular elements as described above must be in the dynamic equilibrium state, expressed as follows:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = k_h \gamma \tag{1a}$$

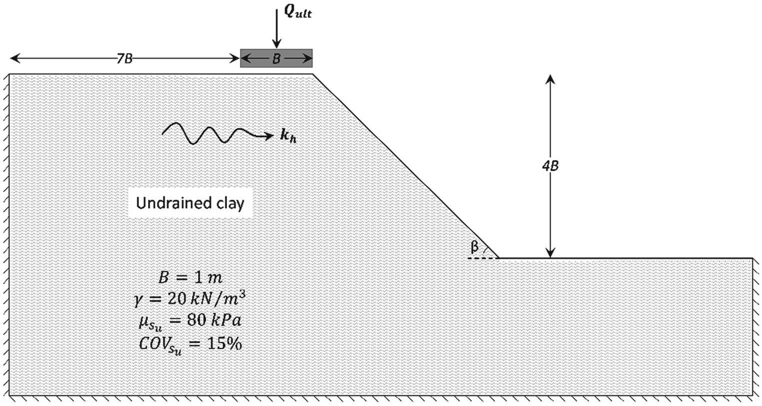

Fig. 1. Schematic illustration of the problem under study, $\gamma = 20\ \text{kN/m}^3$, B = 1 m.

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = \gamma \tag{1b}$$

where k_h is the coefficient of pseudo-static horizontal acceleration. The discontinuity equilibrium constraint states that the normal and shear stresses must be continuous throughout the problem medium including the allowable discontinuities along the interfaces of adjoining triangular elements. Based on the boundary conditions constraint, the induced stresses are bound to take particular values at the boundary edges of the problem domain as well as the soil-footing interface. As the fourth constraint, the yield criterion enforcement expresses that the stress state within the soil medium beneath the surface footing must not exceed the Mohr-Coulomb yield envelope. In order to convert the corresponding yield function to the standard form of SOCP, three auxiliary variables, including $z_1 = (\sigma_x - \sigma_y)$, $z_2 = 2\tau_{xy}$ and $z_3 = 2c.\cos\varphi - (\sigma_x + \sigma_y)\sin\varphi$, are introduced to the FELA formulations by considering the conic quadratic constraint (Q_c^3), defined as:

$$Q_c^3 = \left\{ z \in R^3 : \sqrt{z_{1,i}^2 + z_{2,i}^2} \le z_{3,i} \right\} \quad i = 1, 2, 3, \ldots, \textit{number of nodes} \tag{2}$$

The abovementioned constraints were all assembled into a total matrix of $\{b_l\} \le [A]\{X\} \le \{b_u\}$ and the well-defined SOCP optimizer was then exploited to determine the unknown vector $\{X\}$ containing all the nodal normal stresses (σ) throughout the problem domain. The vertical load component exerted on top of the footing is the objective function in the current problem, which is reached through integration of the vertical normal stress values at the interface between the footing and the sloped soil. A maximum was sought to constitute the ultimate seismic bearing capacity of the overlying shallow foundation:

$$V = Maximize\left\{ -\int_S \sigma\, dx \right\} \tag{3}$$

Table 1. Values of the relevant parameters considered for the reliability-based calculations of the ultimate seismic bearing capacity of a shallow foundation resting on an earth slope

Parameter	Value
Undrained shear strength, $s_u/\gamma B$	2, 3, 4, 5, 6, 7, 8
Slope angle, α (°)	45, 60, 75
Earthquake horizontal acceleration coefficient, k_h	0, 0.1, 0.2, 0.3
Distance ratio from the foundation to the slope, a/B	0

3 Internet Variability of Soil Shear Strength Parameters

Inherent variability of soil shear strength parameters, stemming from the deposition process and sedimentation history, is a crucial feature of soil deposits which must be taken into account while performing stability analyses [26]. The common approach to account for this important feature in lower bound FELA simulations is the realization of the substantiated spatial variability through performing a great number of deterministic stability analyses. Monte-Carlo simulations in conjunction with random field theory are canonically implemented to generate realizations. To shed more light on the problem, a sufficient number of stability analyses should be performed, considering the variation of a typical stochastic/probabilistic parameter, so as to render a clear description of the uncertainties embedded in the calculations. Owing to the time-consuming nature of the Monte Carlo technique, which demands to carry out numerous random simulations, the surface response method (RSM) has recently garnered great attention by practitioners as it provides with enormously efficient approach for the quick random stability analyses of a variety of geo-structures. The corresponding method will be further elaborated later in this paper.

Inherent variability of soil shear strength parameters as described above is manifested in the current simulations in form of spatially variable parameters. The statistical properties of any random soil parameter include the coefficient of variation, the correlation length and the probability distribution function. In this study, an infinite correlation length with a COV = 15% for the undrained shear strength is considered so as to be on the safe side. Typical ranges of variation for different parameters considered in the FELA analyses are selected according to Table 1. As observed, two deterministic parameters, namely the earth slope angle (α) and the horizontal earthquake acceleration coefficient (k_h), as well as a random parameter, namely the undrained shear strength of the clay deposit (s_u), have been adopted for the sample reliability-based design calculations.

4 Response Surface Method (RSM)

In the current study, a simple but at the same time considerably efficient regression-based methodology is exploited to acquire the RSM closed-form solutions for the assessment of the ultimate seismic bearing capacity of the shallow foundation on a sloped ground. In this regard, **Myers and Montgomery** [27] presented a quadratic polynomial containing

Table 2. Coefficients of the response surface equation for the prediction of the ultimate seismic bearing capacity of shallow foundations resting on an earth slope

No.	Coefficient	Value
1	β_0	0.218578319293210
2	β_1	0.311752281008330
3	β_2	6.73476162743733
4	β_3	0.501120667310003
5	β_4	-14.4094785245479
6	β_5	0.0711617879717456
7	β_6	-0.0364060203612900
8	β_7	0.470916737961426
9	β_8	0.0968542035235331
10	β_9	-0.724010050392814

"*m*" variables, as presented in Eq. (4), which in turn significantly cuts down the need to perform an extensively great number of Monte Carlo simulations. To be more specific, using a very small number of FELA analyses based on the ranges of parameters shown in Table 1 leads to the derivation of a desirable and accurate formula which will be indispensable for subsequent reliability-based stability analyses. For this purpose, a complete database of 84 lower bound FELA simulations has been compiled in this study. Table 2 summarizes the values of the unknown coefficients, estimated through the numerical analyses performed in MATLAB.

$$y = \beta_0 + \sum_{i=1}^{m} \beta_i x_i + \sum_{i=1}^{m} \beta_{ii} x_i^2 + \sum_{i=1}^{m-1} \sum_{j=+1}^{m} \beta_{ij} x_i x_j \tag{4}$$

Figure 2 shows the comparison between the predictions rendered by the numerical (FELA) and RSM (Eq. 4) approaches. As observed, there exists a quite satisfactorily good agreement between the RSM predictions and numerical analysis results, which is quantitatively shown by the bias value being equal to one and the coefficient of variation of bias values equal to 4%. Note that bias is the ratio of numerical (lower bound FELA) to RSM values. The excellent consistency between the FELA and RSM predictions corroborates the very fact that the developed closed-form model is a robust formula which can be readily and effectively deployed in subsequent Monte Carlo simulations and the corresponding reliability-based design calculations.

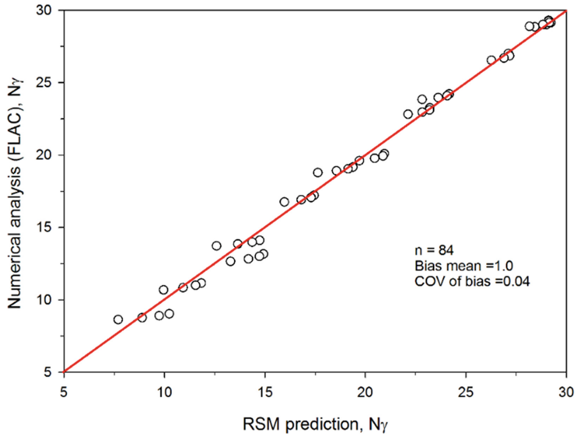

Fig. 2. Comparison between the predictions of the numerical (FELA) and RSM (Eq. 4) approaches

5 Reliability-Based Design of Shallow Footing on an Earth Slope

In this study, 1,000,000 realizations of the undrained shear strength of the clay deposit were generated through the well-established random field theory, which were then introduced into the developed RSM model. In this regard, it was assumed the mean random variable was 80 kPa while its COV was considered to be 15%. The adopted statistics for the undrained shear strength, representing the random variable under study, are only an example and were chosen to render random numbers falling within the range of parameters in Table 1. The undrained shear strength random field was described by a lognormal distributed random field. It was also assumed that the undrained shear strength field below the foundation is homogenous spatially but random through realizations. This assumption clearly expresses that for each individual realization, the undrained shear strength would take equal values at various zones of clay deposit having different coordination; however, the corresponding undrained shear strength value would undergo changes through consecutive realizations.

Figure 3a depicts the relative frequency plot of the realized undrained shear strength random field following a lognormal distribution. Incorporation of the generated random undrained shear strength values into the RSM closed-form model (Eq. 3) will facilitate accurate assessments of the ultimate seismic bearing capacity of overlying shallow footing. Figure 3b illustrates a typical probability density function (PDF) of the evaluated bearing capacity values for the case of $k_h = 0°$ and $\beta = 75°$. As it can be noticed, the estimated ultimate bearing capacity field follows a plausible lognormal frequency distribution; the observation which can be attributed to the very fact that the ultimate bearing capacity of strip footing founded near a clay slope is simply originated from the bearing capacity of the clay deposit per se as well as other influential factors, such as the slope angle and footing-slope distance. Consequently, it is not unexpected that the bearing capacity inherits the lognormal distribution of the random field pertaining to the undrained shear strength of the underlying clay deposit.

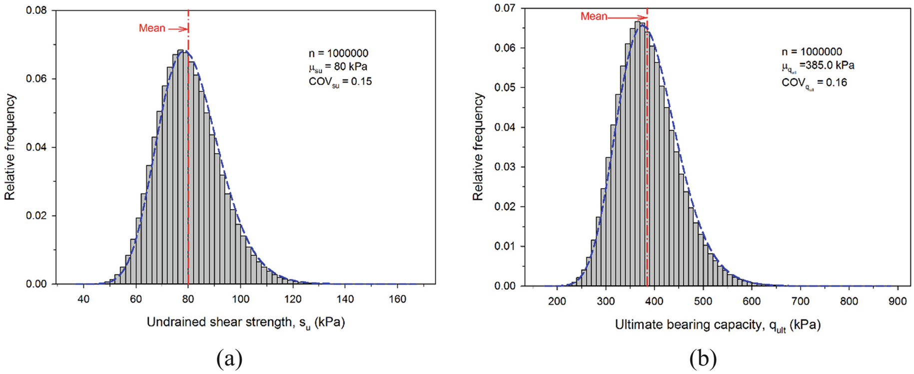

Fig. 3. Relative frequency distribution of RSM probabilistic simulations; (a) undrained shear strength of the underlying clay deposit, and (b) ultimate seismic bearing capacity of shallow foundation ($k_h = 0$, $\beta = 75°$)

The assessed bearing capacity of shallow foundation will be introduced into the reliability-based design calculations. To this end and to make the comparison between the load and resistance sides possible, a performance function needs to be defined. To be more specific, for a given design safety factor, the individual evaluated random bearing capacity values are compared with the service load (defined as the deterministic bearing capacity divided by the design factor of safety). It is worth mentioning that the deterministic bearing capacity is estimated on the basis of mean random parameters. Accordingly, for the problem under study, $\mu_{su} = 80$ kPa has been introduced into the RSM model to give such bearing capacity estimations. The probability of failure (p_f) and the reliability index (β) could be calculated using Eqs. (5) and (6), respectively, Note that the former allows the estimation of failure probability for a wide range of undrained shear strength configurations, whereas the latter introduces an equivalent index by a simple conversion.

$$p_f = P(q_{ult} < \frac{q_{u(det)}}{FS_d}) \quad (5)$$

$$\beta = \phi^{-1}(1 - p_f) \quad (6)$$

In these equations, q_{ult} is the individual random bearing capacity obtained via the RSM model estimations, $q_{u(det)}$ is the deterministic bearing capacity acquired by considering the undrained shear strength in the average sense, and FS_d is the design safety factor.

Figure 4 demonstrates the variations of probability of failure and reliability index with the assumed design safety factor for different slope angles. On each figure, different target limits have been depicted so as to facilitate the selection of the design safety factor corresponding to each limit. To put it in a nutshell, the designer could assume a target probability of failure equal to 1/1000 or even 1/5000 corresponding to different design safety factors, depending primarily on the importance of overlying superstructure and life cycle of the whole project. Figure 4 also clearly shows that the earth slope angle of

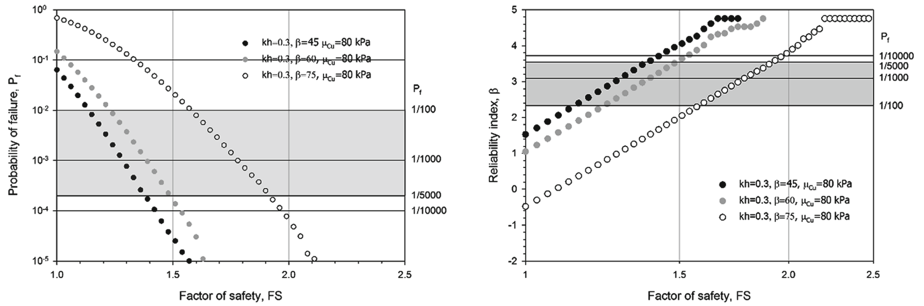

Fig. 4. Effect of the earth slope angle on reliability-based design of shallow foundations near an earth slope; (a) probability of failure, and (b) reliability index.

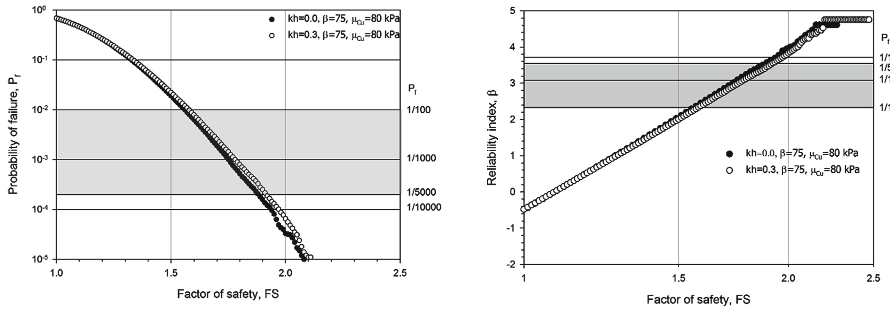

Fig. 5. Effect of the earthquake acceleration intensity on reliability-based design of shallow foundations near an earth slope; (a) probability of failure, and (b) reliability index.

75° would give rise to the diminished factor of safety, or in other words, the substantially augmented probability of failure compared to the other two slope angles considered. On the other hand, the slope angle of 60° demands higher design factor of safety in order to maintain a specific level of safety as compared to the slope angle of 45°.

Figure 5 illustrates the influence of the earthquake horizontal acceleration coefficient on the variations of probability of failure and reliability index with the design factor of safety. As it can be observed, both the probability of failure and reliability index envelops are almost identical for the two extreme acceleration coefficients. At first blush, the trends of variation seem counter intuitive. In other words, it was expected the earthquake acceleration application to have a major impact on the bearing capacity of the overlying shallow footing. However, due to the large cohesion value of the underlying natural slope and high shear strength in comparison to the static mobilized shear stress values, applying an earthquake acceleration coefficient even as large as 0.3 has failed to have a destructive impact on the stability of the overlying shallow footing. This implies that providing cohesion to the underlying soil deposit either naturally or synthetically through grouting, could impart noticeable seismic strength to the overlying geo-structures.

The results presented in this article are only meant to corroborate the robustness of the SRM calculations in probabilistic stability analyses and the way it could be implemented into reliability based designs. In other words, this study has shown a simple procedure on how to use the SRM calculations in seismic bearing capacity analysis of shallow footings resting on a sloped ground. Therefore, the numeric values of the parameters adopted in this study are not the main focus herewith.

6 Conclusions

This study illustrated the efficient use of the response surface method (RSM) to perform a great number of probabilistic evaluations of the bearing capacity of shallow footings founded on a naturally occurring heterogeneous clay slope. A lower bound finite element limit analysis (FELA) model was developed, which was then utilized to generate a large synthetic database of bearing capacity calculations for the shallow foundation resting on inherently variable natural slopes. A closed-form solution was proposed using a RSM-based polynomial which in turn was shown to be sufficiently robust to be implemented in Monte Carlo simulations and the corresponding reliability-based design calculations. Results generally revealed that the response surface method can be an efficient tool when encountering cumbersome and massive numerical-based stability analyses of geo-structures. Moreover, increasing the slope inclination proved to have a tremendous influence on the safety levels and reliability-based design of the overlying footing. Last but not least, it was shown the cohesion of the underlying sloped ground to have a suppressing effect when a horizontal earthquake acceleration loading would be present.

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