



Students' Learning Obstacles on Fractions in Elementary School

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Abstract. Fraction is one of the crucial concepts in mathematics learning. However, fractions learning does not fully involve various representations, which possibly results in learning obstacles. The purpose of this study is to identify and analyze students' learning obstacles in addition and subtraction operations of fractions in grade 5 elementary school. This study used a qualitative method. The participants of this study were 22 elementary school students with different characteristics. Data was collected using tests, interviews, observation, and document study techniques. The data were analyzed qualitatively to determine the types of learning obstacles experienced by students. The results of the research obtained are a description of learning obstacles with the ontological, epistemological, and didactical types of students on fractions. These results can be used as a basis for compiling a hypothetical learning trajectory.

Keywords: fractions · learning obstacles · learning trajectory

1 Introduction

Fractions can be interpreted as part of an entire object. A fraction can be illustrated in a drawing, where the part in question is usually marked with shading. This part is called the numerator. Meanwhile, the whole part is a unit and is commonly referred to as the denominator (Heruman, 2017:43).

Fraction is one of the essential materials in mathematics learning. Fractions and decimal arithmetic are essential for future mathematical achievement and the ability to succeed in many professions (Lortie et al., 2015). As mandated in the Regulation of the Minister of Education and Culture of the Republic of Indonesia no. 37 of 2018, fractions learning starts from grade 2 to grade 6 of elementary school (Permendikbud: 2018). Therefore, mastery of fraction material is highly essential for elementary school students. However, these students frequently experience difficulties in learning fractions. Warsito et al. (2019) reported that fractional problems are inherent in elementary school mathematics learning activities, while students' mastery of fractions bears effects on their learning outcomes and future skills.

Students' difficulty in learning fractions results in obstacles during the learning process. Brousseau (1997, p. 86) reported numerous factors for these obstacles, such as obstacles of ontogenic origin (mental readiness to learn), obstacles of didactical origin (due to the education system), and obstacles of epistemological origin (students' knowledge with limited application context). Ontogenic obstacles (obstacles of ontogenic origin) occur because the learning process does not follow the student's readiness level. Therefore, ontogeny obstacles are closely related to students' mental development, age, and level of development. If their obstacle is caused by slow mental development and not a congenital disease, then the obstacle will naturally disappear as the student grows. Didactic obstacles (obstacles of didactical origin) occur due to errors during the learning process commonly caused by the school's learning system. Epistemological obstacles (obstacles of epistemological origin) are essentially caused by a person's knowledge being limited to a certain context. If someone with an epistemological obstacle faces a different context, the person encounters difficulty in using the knowledge, or his knowledge becomes unusable. Epistemological obstacle disjoints students' views, resulting in noncomprehensive conceptual understanding.

Research conducted by Suryana et al. (2012) discovered some learning obstacles that arise during fraction learning, such as 1) obstacles related to students existing conceptual understanding, 2) obstacles related to determining the value of fractions based on images, 3) obstacles in comprehending the symbol of fractional number, and 4) obstacle related to procedural understanding in showing a fraction. Research conducted by Romdani and Suryadi (2016) also summarized a number of learning obstacles observed during fraction learning, including 1) students' ability to understand fractions from the presented geometric figure images (fractional concept image), 2) drawing fractions in geometric shapes, 3) understanding the $<$ and $>$ signs in fraction comparisons, 4) comprehending the meaning of fractions and procedures for comparing two fractions, along with the ability to translate and solve fraction story problems. Meanwhile, Rohmah (2016) discovered that the learning obstacle faced by fourth-grade elementary school students are mostly epistemological and didactical obstacles. The epistemological obstacle is observed due to students' limited context. Consequently, students understanding of the procedures for simplifying fractions, ordering fractions, subtracting, and adding fractions is limited to certain questions. Additionally, the didactical obstacle is found because the teachers mostly only use a single textbook provided by the school as the learning resource, which has a minimum explanation of the fraction material and only focuses on procedural skills.

Based on previous studies, many learning obstacles in fraction mathematics learning still frequently occur in grades 3 or 4. Therefore, we conducted a study on five-grade elementary school students in Indonesia to attain a comprehensive illustration of the problem. The results of observations and interviews with fifth-grade teachers suggested that many students still have difficulty in learning fractions, especially in solving the story question. The same situation was also articulated by students during an interview carried out after they finished their midterm test. The example of students' answer to a fraction question is presented in Fig. 1.

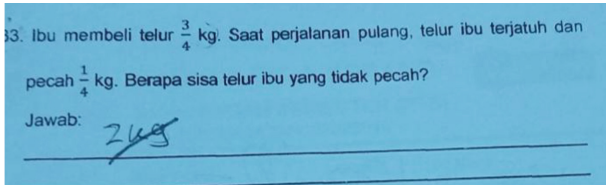


Fig. 1. Student's Answer Showing Their Problems in answering questions

In answering the question presented in Fig. 1, students understood that 3 minus 1 equals 2, while 4 minus 4 is zero, so they answered 2 kg. This answer illustrates students' minimum understanding of the concept of fractions. Therefore, we conducted in-depth research on grade 5 elementary school students by giving story questions about fractions to identify students' difficulties in understanding fraction material.

2 Method

This study used the descriptive qualitative method. This method was chosen as this study described the written or spoken words from people and observed their behavior to explore learning obstacles in fractional material in grade V elementary school. In addition, this study aimed to identify their types of learning obstacles, fractions, and the influencing factors of learning obstacles. We collected data in the form of words and pictures. This research was started by giving a fraction mathematical test, followed by the construction of a clinical interview instrument to reveal learning obstacles and their factors. Then, we analyzed students' learning obstacles and composed hypothetical learning trajectory (HLT) fractions that can be applied to fraction learning in fifth grade of elementary school.

3 Results and Discussion

In this study, students were given four questions, consisting of two fill-in questions and two fractional story questions. Each question was an addition and subtraction of fractions. The questions were used to observe students' ability to solve direct fraction problems and problems in the form of stories. In solving the problems, the students should have the ability to understand the prerequisite material, the meaning of the question, the operations used, and the procedure for calculating fractional numbers. From the obtained 22 answer sheet, we only analyzed the incorrect answers. Some students provided no answer, while others gave students wrong answers. This finding signifies students' low understanding of the given fraction questions. In the discussion session, we deliberated our findings showing learning obstacles experienced by students when working on the fraction questions.

3.1 Ontogenic Learning Obstacles

Ontogenic learning obstacles were observed in students' work on adding and subtracting fractions. These questions were given to see the students' ability to understand fractions, as presented in Fig. 2.

What is the result of the sum

Fig. 2. Question no. 1

Berapakah hasil dari penjumlahan dari $\frac{6}{11} + \frac{5}{7} = \frac{11}{115}$

Fig. 3. Answers of Students who Experience Learning Obstacles

What is the result of subtracting =

Fig. 4. Question no. 2

Berapakah hasil dari pengurangan dari $\frac{7}{11} - \frac{3}{4} = \frac{4}{7}$

Fig. 5. Answers of Students who Experience Learning obstacles

The questions presented in Fig. 2 aimed to determine students' understanding of adding fractions. In that question, 15 students were suspected of having ontogenic learning obstacles. Meanwhile, three students did not answer the question because they forgot the procedures to solve it, while 12 students answered by adding the numerator with the numerator and the denominator with the denominator. This is presumably caused by their low understanding of the proper procedure for working on the questions. The answer from students with ontogenic learning obstacles is illustrated in Fig. 3.

Question no. 2 is about fraction subtraction problems. This question was given to examine students' ability to solve fraction subtraction problems (Fig. 4).

In the second question, 17 students were suspected of having ontogenic learning obstacles. Four students did not answer the question since they forgot the procedures, while 12 students answered it by subtracting the numerator with the numerator and the denominator with the denominator. This is presumably because the students did not understand the procedure for working on the questions. The answer from students who experienced an ontogenic learning obstacle is presented in Fig. 5.

Furthermore, question no 3 aimed to examine students' understanding of fractions in the form of story questions. Question no. 3 is shown in the Fig. 6.

In question no. 3, 13 students experienced ontogenic learning obstacles. Besides, four students did not give an answer because they did not know the procedure for answering the question, while nine students worked carelessly without understanding the questions. After being confirmed, the students answered the question by multiplying the numbers

There is reserve of grain in the warehouse tons, brought again tons. How many tons of grain must be added to make it 10 ton?

Fig. 6. Question no. 3

Terdapat cadangan gabah di gudang $5\frac{1}{2}$ ton, didatangkan lagi $3\frac{1}{2}$ ton. Berapa ton gabah yang harus ditambahkan agar menjadi 10 ton? $1\frac{1}{4}$

Fig. 7. Answer from Student with Learning Obstacles

Mother has 2 flour sack. If each sack contains quintals of flour, how many quintals of flour mom in total?

Fig. 8. Question no. 4

Ibu memiliki $2\frac{1}{2}$ karung tepung. Jika setiap karung berisi $\frac{2}{5}$ kuintal tepung, berapa kuintal tepung Ibu seluruhnya? $2\frac{2}{10}$

Fig. 9. Answer from Student with Learning Obstacles

in the questions. The example of a student's answer to question no 3 is presented in Fig. 7.

In this question, 15 students' answers were incorrect. During the interview, those students confirmed that they attained the answer from the numbers in the question. This signifies that students do not understand the meaning of the question. They also explained that they just simply multiplied those numbers. Further, the students also used the same procedures to answer the question no 4. Question no 4 is presented in Fig. 8.

Our analysis results showed that 15 students were suspected of experiencing ontogenic learning obstacles, while nine students did not answer the questions because they did not know the procedures or the formulas to answer the question. Besides, eight students answered the question without providing the calculation process. The answers of students who experience learning obstacles to question no 4 is presented in Fig. 9.

Figure 9 shows students' wrong answers to question no 4. Consequently, our findings can be interpreted in various ways. Thus, we carried out an interview with students who gave wrong answers. During the interviews, the students admitted that they had never worked on story questions. Thus, they were confused about answering the questions since they did not understand the meaning of the questions, so they just answered the

questions randomly. Further, the students also stated that their inability to solve the regular question worsened their confusion in solving the story question.

Based on the above, it can be ascertained that students experience ontogenic learning obstacles. Brousseau (1997, p. 86) explained that learning obstacles could be caused by several factors, such as the obstacle of ontogenic origin (mental readiness to learn). Ontogenic obstacles (obstacles of ontogenic origin) occur because the learning process does not follow the students' preparedness. Therefore, ontogeny barriers are closely related to students' mental development, age, and level of development. The ontogeny obstacle induced by slow mental development will disappear following the students' growth, unlike the obstacle which is caused by a congenital disease.

3.2 Didactical Learning Obstacles

Didactical learning obstacles were observed in students' answers to questions no. 1 and 2, showing their wrong conceptual understanding of fraction operations. In solving the summation question, students added the numerator with the numerator and the denominator with the denominator. The same procedure was also applied to the problem of subtracting fractions. An example of students' wrong answers is shown in Fig. 10.

In question no 1, 12 students experienced didactical obstacles. Students added the numerator with the numerator and the denominator with the denominator. During the interview, the students admitted that their difficulty emerged as they learned the material through the limited online learning conducted via Wattshap, without video conferences. Consequently, they could not understand the teacher's explanation. The same issue was also observed in students' answers to question no 2, as presented in Fig. 11.

In question no. 2 about the subtraction of fractions, 13 students gave wrong responses. These students answered the question by subtracting the numerator from the numerator, the denominator with the denominator, without equating the denominator first. This finding indicated students' low understanding of the online learning carried out through WhatsApp during the COVID-19 pandemic. Based on the observed problems, the students experience didactic learning obstacles. Brousseau (1997, p. 86) divided students learning obstacles into three. One of them is the obstacle of didactical origin (due to the education system). Didactic barriers (obstacles of didactical origin) occur due to the

Berapakah hasil dari penjumlahan dari $\frac{6}{11} + \frac{5}{7} = \frac{11}{18}$.

Fig. 10. Answer from Student with Didactical Learning Obstacle

Berapakah hasil dari pengurangan dari $\frac{7}{11} - \frac{3}{4} = \frac{4}{7}$.

Fig. 11. Answer from Student with Didactical Learning Obstacle

errors in the learning process caused by the school learning system. The didactic barrier occurrence depends on the selected project in an education system.

3.3 Epistemological Learning Obstacle

The epistemological obstacle is related to students' knowledge in certain contexts. This learning obstacle occurs because students are not used to dealing with different contexts. Of the 22 students, 16 students experienced epistemological learning obstacles, as shown in Fig. 12.

As shown in Fig. 12, students worked on adding fractions by cross-multiplication or multiplying the numerator by the denominator, multiplying the denominator by the numerator, and then dividing by two. This error occurs because students do not understand the meaning of addition operations on fractions. Students assumed that all fraction operations could be solved with cross-multiplication. During the interview, students admitted that they learned that procedure from their classroom learning. However, the teacher explained that cross multiplication was taught in dividing fractions.

Similarly, some students also used cross-multiplication in solving the subtracting fraction, but they wrote the numbers wrong, as presented in Fig. 13.

As presented in Fig. 13, the students also used cross-multiplication in solving the question. In fact, in solving the summation and subtraction fraction, the students should equate the denominator first.

Students' limited knowledge in certain contexts can be seen in their answers to the story question. Students were confused about solving the story problem. They did not understand the meaning of the questions while also having minimum knowledge of the correct procedures to answer the question. Thus, they gave wrong answers, as illustrated in Fig. 14.

Students' errors due to their understanding in a single context are called epistemological learning obstacles. The answer from students with the epistemological obstacle is presented in Fig. 15.

In this obstacle, students made mistakes due to their limited understanding of a certain context. Thus, it can be concluded that students experience epistemological obstacles.

Berapakah hasil dari penjumlahan dari $\frac{6}{8} + \frac{5}{7} = \frac{26}{8} \times \frac{5}{7} = \frac{6}{8} \times \frac{5}{7} = \frac{40}{42} : 2 = \frac{20}{21}$

Fig. 12. Answer from Student with Epistemological Learning Obstacles

Berapakah hasil dari pengurangan dari $\frac{7}{8} - \frac{3}{4} = \frac{7}{8} - \frac{3}{4} = \frac{5 \times 3}{8 \times 10} = \frac{10}{100}$

Fig. 13. Answer from Students with Epistemological learning obstacles

Terdapat cadangan gabah di gudang $5\frac{1}{2}$ ton, didatangkan lagi $3\frac{1}{2}$ ton. Berapa ton gabah yang harus ditambahkan agar menjadi 10 ton? $5\frac{1}{2} + 3\frac{1}{2} = \frac{11}{2} + \frac{7}{2} = \frac{11+7}{2} = \frac{18}{2} = \frac{10}{2} = \frac{8}{2}$

Fig. 14. Answer from Students with Epistemological Learning Obstacles

Ibu memiliki $2\frac{1}{2}$ karung tepung. Jika setiap karung berisi $\frac{2}{5}$ kuintal tepung, berapa kuintal tepung Ibu seluruhnya? $2\frac{1}{2} \times \frac{2}{5} = \frac{5}{2} \times \frac{2}{5} = \frac{25-9}{10} = \frac{21}{10}$

Fig. 15. Answer from Students with Epistemological Learning Obstacles

According to Duroux (Suryadi, 2011), Epistemological barriers (obstacles of epistemological origin) are essentially a person's limited knowledge in a certain context. In a different context, the knowledge becomes unusable. In this case, students have disjointed views of one concept with another concept.

The observed students learning obstacles in this study are summarized in Table 1.

As listed in Table 1, the students experienced three types of learning obstacles. To improve the quality of learning, it is necessary to anticipate the learning obstacles. The alternative solutions for resolving the three learning obstacles are discussed in the following session.

First, to anticipate ontogenically obstacles in understanding fraction operations, it is necessary to create a didactic situation to enhance raise students' understanding of the basic concept of fractions. Besides, the teachers should also pay attention to the prerequisite material that is closely related to fractional operations, such as the multiplication of integers, division of integers, and the concept of fractions. Prerequisite material is essential because, without mastery of those prerequisite materials, students will have difficulty comprehending the next material. According to Widiarto (2004), to avoid learning obstacles, we have to ensure students have mastered the prerequisites of knowledge and ability. This method is used to identify students who fall short of a given basic competency's knowledge requirements. Before students understand new basic competencies, they must understand the prerequisite basic competencies, both vertically and horizontally. Thus, teachers have to ask students to re-learn these prerequisite materials independently. In addition, to anticipate ontogenically obstacles in learning, the learning material should be prepared following the students' learning experiences.

Second, based on our findings, the factor of the epistemological obstacle is limited students' knowledge of a certain context. Therefore, they face difficulties in solving a problem in a different context. This can happen due to teachers or students. This may occur due to the teacher only providing one way or one piece of knowledge to students during the learning process. Besides, this issue may also occur as a result of students' sluggish comprehension of teachers' explanations. Thus, teachers may have

Table 1. Recapitulation of learning obstacle

Question number	Types of learning obstacles	Identified error
1.	Ontogenic learning obstacle	Don't understand the meaning of the question
	Didactical learning obstacle	Misunderstanding the teacher's explanation
	Epistemological learning obstacle	Don't understand the meaning of the question
2.	Ontogenic learning obstacle	Don't understand the meaning of the question
	Didactical learning obstacle	Misunderstanding the teacher's explanation
	Epistemological learning obstacle	Don't understand the meaning of the question
3.	Ontogenic learning obstacle	Don't understand the meaning of the question
	Epistemological learning obstacle	Don't understand the problem with a different context
4.	Ontogenic learning obstacle	Don't understand the meaning of the question
	Epistemological learning obstacle	Don't understand the problem with a different context

numerous different explanations, but students only understand one type of explanation. According to Widiarto (2004), learning obstacles can be identified through a material profile approach. Each material has a different level of difficulty, and every student has different difficulties with each mathematical concept they obtain from the learning process.

4 Conclusion

Learning obstacles observed in fraction materials among fifth-grade elementary school students include ontogenic obstacles, didactic obstacles, and epistemological obstacles. Students' ontogenic was indicated by the student's confusion in working on the questions. Didactical obstacles were observed from their errors due to the non-maximized learning process during online learning. The epistemological obstacle was revealed when students could not use their obtained knowledge in a different context. The obtained results provide an overview for the teacher that can serve as a source for making hypothetical learning trajectories or learning designs to improve the learning process. In making a didactic design, it is necessary to analyze learning obstacles and their factors so that the learning is in accordance with the stage of student development. Then in giving fractions

questions, the teacher should ensure that students already understand the concept so that students can work on the problem to avoid epistemological obstacles.

References

- Brousseau. (1970–1990). *Theory of Didactical Situations in Mathematics*. The Netherlands. Kluwer Academic Publishers.
- Creswell. (2018). *Penelitian Kualitatif dan Desain Riset*. Yogyakarta: Pusaka Pelajar.
- Heruman. (2017). *Model Pembelajaran Matematika di Sekolah Dasar*. Bandung: Remaja Rosdakarya.
- Romdhani dan Suryadi. (2016). Desain Didaktis Konsep Pecahan Untuk Kelas III Sekolah Dasar. *EduHumaniora: Jurnal Pendidikan Dasar*. ISSN 2085–1243 Vol. 8. No.2 Juli 2016 | Hal 198–210.
- Purnomo. (2015). *Pembelajaran Matematika Untuk PGSD*. Jakarta: Erlangga.
- Suryadi, Didi. (2010). Didactical Design Research (DDR) Dalam Pengembangan Pembelajaran Matematika 1, *Jurnal ilmu pendidikan*, (Online), (<http://ejurnal.fkip.upi.ac.id>, diakses 15 Februari 2016).
- Suryana, Y. dkk. (2012). Desain Didaktis Pengenalan Konsep Pecahan Sederhana Pada Pembelajaran Matematika Untuk Siswa Kelas III Sekolah Dasar, *Jurnal ilmu pendidikan*, (Online), (<http://ejurnal.fkip.upi.ac.id>, diakses 15 Februari 2016).
- Warsito, Nuraini dan Sukirwan. (2019). Desain Pembelajaran Pecahan melalui Pendekatan Realistik di Kelas V. *Mosharafa: Jurnal Pendidikan Matematika* 25 Volume 8, No. 1, Januari 2019.
- Widdiharto, R. (2004). *Model-model Pembelajaran Matematika SMP*. PPPG Matematika. Yogyakarta.

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