Analysis of Students Problem Solving Ability in Solving the Volume of a Solid of Revolution

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Abstract. This study aimed to determine the students' problem solving ability in solving the volume of a solid of revolution questions in online Integral Calculus Learning through zoom implementing Student Centered Learning. The research method used in this study was descriptive qualitative method. The subjects were 32 students from class B in the 3rd semester of Mathematics Study Program FKIP UNIB Academic Year 2020/2021. The test was conducted online via Google Classroom on 8th September, 2020. The analysis in Polya stages consisted of: P1) Understanding the Problem; P2) Making a Plan; P3) Carrying out the Plan; and P4) Looking Back. The instruments used to collect datas were midterm test sheets including volume of a solid of revolution questions. Analysis tool using in this research was Polya Analysis Instrument. The result showed the average score of the students' problem solving ability was 77. The mean value in each stage: P1) = 84.15; P2) = 81.56; P3) = 71.79; and P4) = 71.9. It was concluded that students had good abilities in understanding the problem and planning the solution but they still had difficulties in drawing graphs and calculating the volume.

Keywords: Problem Solving Ability · Volume of a Solid of Revolution Questions

1 Introduction

Integral Calculus is one of the compulsory subjects in the Undergraduate Mathematics Education Study Program, FKIP UNIB with 4 (3–1) credits. This subject becomes important to understand because it is connected to many subject in Mathematics Study Program in case of solving the question of calculus or differential. Besides, the use of integrals can be used in various fields such as engineering, economics, and biology such as to determine the surface area and volume of an area [1]. One of the chapters studied in integral calculus class is the volume of a solid of revolution object. If taken $y = f(x)$ is a non-negative continuous function on an interval [a, b] the area between the x-axis and the curve $y = f(x)$, $a \le x \le b$, is rotated about the x-axis, a three-dimensional area is obtained, which is referred to as a solid of revolution [2]. The volume of the solid can be gained by rotating the part of the curve between $x = a$ and $x = b$ about the x-axis which is stated by $V = \int_a^b \pi y^2 dx$ [3].

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Integral calculus learning was carried out online through zoom meetings, google classroom, and email for assignment collection. Lecturer applied student centered learning in learning to make students able to construct their own understanding with the guidance of lecturer assistants and lecturers. Student-Centred Learning (SCL) is a learning approach in higher education institution which acts for both a mindset and a culture and is mainly supported by constructivist theories of learning. Innovative teaching methods are used to support learning in interaction between teachers and students that make students actively involved in learning activity. Thus, this action was expected to make students conduct their own problem-solving, critical thinking and reflective thinking skills [4].

In practice, students had been active in making presentations, discussions, and questions and answers. During learning, students were given a quiz about the material being studied at that time. However, some problems were found through the student quiz answer sheets. From the results of student quizzes, it was found that students still experienced errors in calculating integrals, determining integral area formulas, and drawing area graphs. Regarding learning difficulties, the results showed that students' mathematics learning difficulties were 12.2% factual knowledge, 19.7% conceptual knowledge, 20.7% procedural knowledge, and 47.4% metacognitive knowledge [5]. The research according to Tall stated the reasons for learning difficulties in learning mathematics in general were as follows: (1) insufficient basic concepts possessed, (2) inability to formulate problems orally, (3) inadequacy, ability in algebra, geometry, and trigonometry) [6].

Based on the argument above, it can be assumed that students faced some struggles in solving mathematics problem. Besides, this ability is important to be mastered by students during learning mathematics or in daily life phases. From the described problems above, the researchers decided to conduct research on the analysis of students' problem solving abilities regarding the volume of a solid of revolution based on polya steps. The stages of polya consist of: P1) Understanding the problem; P2) Making a Plan; P3) Carrying out the Plan; and P4) Looking Back [7]. This was done to analyze what and how the steps taken by students in solving mathematical problems.

2 Method

This type of research was descriptive qualitative research. The subjects of this study were 32 students of Mathematics Education in the third semester Academic Year 2020/2021. To collect data research, data collection techniques were carried out as follows: Written test which consists of 2 questions about the volume of a solid of revolution.

Analyzing the data, the datas obtained were collected and then analyzed using descriptive analysis by representing the results of student answers in the form of students' problem solving abilities according to Polya's stages when solving test questions. The tools used in the students' problem solving ability analysis were Table 1 as follows:

The points that had been obtained were then searched for the final score using the formula [8] below:

$$Score = \frac{Obtained\ Score}{Maximum\ Score} \times 100 \quad (1)$$

Table 1. Student Problem Solving Ability Analysis Tool

Questions	Indicator	Score
Draw and determine the	P1: Be able to write down what is known and	2
volume of the above function	what is	
if it is rotated around the x-	asked	2
axis.	P2: Be able to plan steps for completion	2
	Be able to plan to draw graphics.	
	Be able to determine volume formula	2
	P3: Be able to calculate and execute a two-step	2
	plan	2
	Be able to draw graphics	
	Be able to calculate volume	
	P4: Recheck answers	
Draw and determine the	P1: Be able to write down what is known and	2
volume of the above function	what is	
if it is rotated around the y-	asked	2
axis..	P2: Be able to plan steps for completion	2
	Be able to plan to draw graphics.	2
	Be able to determine volume formula	2
	P3: Be able to calculate and execute a two-step	2
	plan	
	Be able to draw graphics	
	Be able to calculate volume	
	P4: Recheck answers	

Table 2. Qualification of Problem Solving Ability

Score	Qualification
$80 < x \leq 100$	Very Good
$65 < x \leq 80$	Good
$55 < x \leq 65$	Enough
$40 < x \leq 55$	Not Enough
$0 < x \leq 40$	Very Less

The scores that had been obtained were then qualified using the following adapted Table 2 [9].

3 Result and Discussion

3.1 Result

The sub-analysis data in this study were collected from the results of the Mid-Term Examination for third semester students specifically number 5 and question number 6 about the volume of a solid of revolution. The data were collected from students' written answers on the answer sheet and then collected through email and Google Classroom. The students' problem solving abilities were analyzed based on Polya's steps. The following Table 3 was the problem of the volume of a solid of revolution that was given to students.

The questions in Table 3 were made based on students' absent numbers with the aim that each student gets a different question. It also minimized cheating that could occur during online learning. Through the questions in Table 3, students' problem solving ability was analyzed according to the scores and indicators that had been set previously. So that the results were obtained as shown in Fig. 1.

Table 3. Questions

Number	Question
	a and b are two consecutive absentee numbers, for example sequence absenteeism 12, $a = 1$ and $b = 2$ $f(x) = (a+1)x + b, for\ b = 0, 1, 2, 3, and\ 4$ $f(x) = (a+1)x + (b-3), for\ b = 5, 6, 7, 8, and\ 9$ $x = 0, x = 4, x\ axis$
5	Draw and determine the volume of the above function if it is rotated around the x-axis.
6	Draw and determine the volume of the above function if it is rotated around the y-axis.

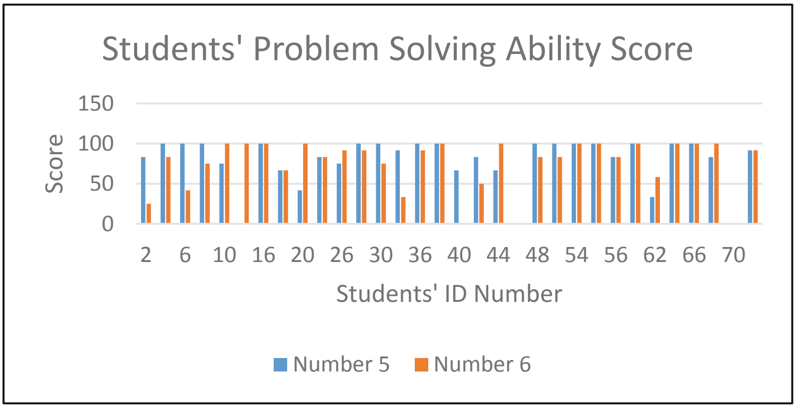

Fig. 1. Students' Problem Solving Ability Score

Table 4. Result of Students' Problem Solving Ability

Mean of P1	84,15
Mean of P2	81,56
Mean of P3	71,79
Mean of P4	71,9
Mean in Total	77

From Fig. 1, it could be seen that not all students who could work on Question Number 5 could work on Question Number 6, and vice versa. Problems Number 5 and Number 6 were questions about the volume of a solid of revolution with different methods and different volume formulas. Based on the results of the analysis, it was found that the errors made by the students included choosing the method wrongly, error in determining the volume formula, inaccuracy in calculations, and imperfect images. Those analysis results showed that different types of question need different way of solution. Students need to precisely understand which steps they need to work on and which method they need to apply. This was in line with the results of research which stated that the biggest type of error in solving the problem of the volume of a solid of revolution was understanding the problem, understanding the boundaries of the area to be rotated, planning errors for completion, errors carrying out planning errors re-examining the process and results [10].

Table 4 was the result of the analysis of each indicator based on polya steps, namely: P1) be able to write down what was known and what was asked; P2) be able to plan the steps of completion; P3) be able to calculate and execute the second phase plan; and P4) Rechecking the answers.

Table 4 showed that the average problem-solving ability of students in the topic of volume of a solid of revolution was 77 with good qualifications with each indicator obtained: P1) 84.15 with very good qualifications; P2) 81.56 with very good qualifications; P3) 71.79 with good qualifications; and P4) 71.9 with good qualification.

3.2 Discussion

Through the results of the analysis of students' problem-solving abilities, it was found that most of the students already had good skills in solving the problem of rotating object volume. However, some students still get a score of 0 and have not gotten maximum score. Some of them already determined the perfect method for each question whereas some picked the wrong method causing them falsely solving the problem. The mistakes made by students include incorrectly determining the formula, incorrectly determining the rotation area, imperfect images, and inaccuracy in calculating. Figure 2 is the result of student work who can draw graphs well.

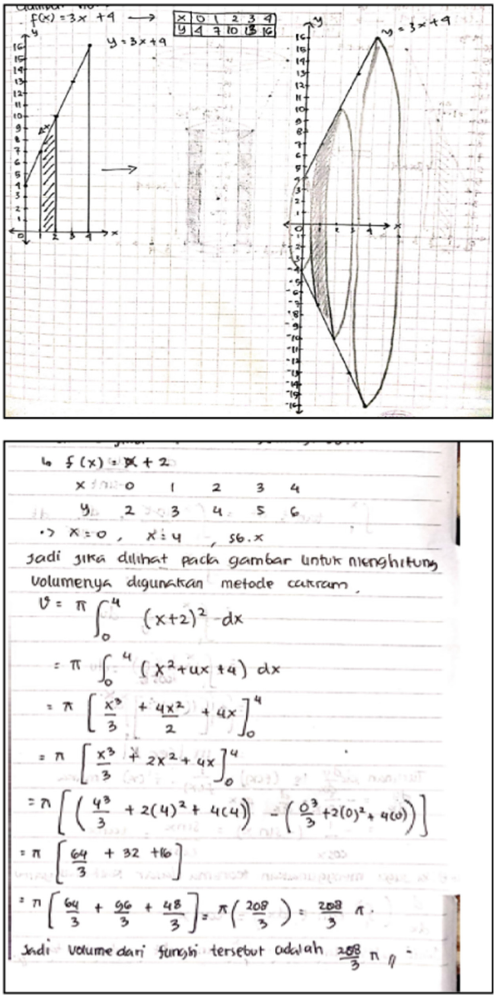

Fig. 2. Graph and Answer of Volume of a Solid of Revolution Problem

In Fig. 2 the student had written down the polya steps correctly, namely P1) be able to write down what was known which consists of function formulas and limits; P2) be able to write a plan that was used, what method was chosen and planned a graphic depiction; P3) be able to draw graphs and calculate volumes; and P4) Rechecked the answer because they could answer correctly.

Overall, not all students could answer correctly as shown in Fig. 2. Figure 3 was the imperfection of the graph sketched by the students.

From Fig. 3, it could be seen that students were still wrong in sketching the volume of a solid of revolution. Starting from the misrepresentation of Cartesian coordinates, inaccuracies of intersection points, and imperfections in rotating the graph about the

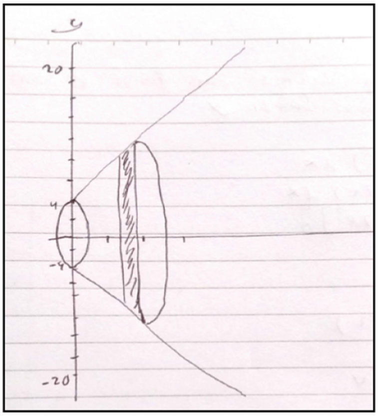

Fig. 3. Imperfect Graph

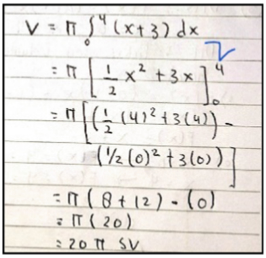

Fig. 4. Inaccuracy Calculation

x-axis. Some students were also still made some mistakes in determining the formula of the volume of a solid of revolution as shown in Fig. 4.

In Fig. 4, it could be seen that the students were wrong in determining the volume formula so that it resulted in incorrect solutions to existing problems. Other problems such as arithmetical errors, errors in integration, and errors in method selection were also found in student midterm answer. Figure 5 was an example of an error in choosing a method by students.

In Fig. 5, it could be seen that the students misunderstood the method that should be chosen. If the area being rotated was below the graph and was bounded by the x-axis, then the method used was the shell of the tube. The error resulted in the wrong formula determined by the student. Errors made by students in solving the problem of rotating object volume could be grouped into 3 types of difficulties, namely difficulty understanding concepts such as incorrectly using the method used, incorrectly electing the upper and lower limits of specified integrals that were relevant to the problem and incorrectly choosing the right function for the problem, integrated, difficulty of calculation accuracy, and difficulty of image representation [11]. Another study states that this error was classified into 4 types of errors, namely factual, procedural, conceptual errors and errors due to carelessness [12].

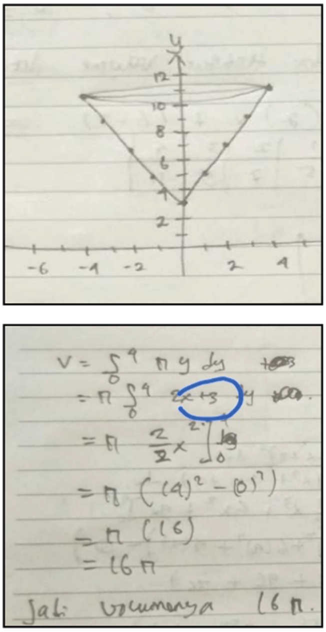

Fig. 5. Misunderstood the method

4 Conclusion

From the results of the analysis of students' problem-solving abilities on the material of the volume of a solid of revolution, it was found that students had an average problem-solving ability score that was 77 with good qualifications. The average of each indicator was: P1) 84.15 with very good qualifications; P2) 81.56 with very good qualifications; P3) 71.79 with good qualifications; and P4) 71.9 with good qualifications. These results indicate that students in general had had a fairly good problem solving ability in the volume of a solid of revolution. However, some students still experienced errors in determining the completion strategy both in determining the formula and in determining the method, and made some errors in carrying out the plan, errors in calculations and drawing graphs, besided that some students also did not recheck the answers, resulting wrong final results obtained by students.

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