Scaffolding Appropriation for Students' Metacognitive Failure in Solving Mathematical Problems

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Abstract. This article determines the provision of scaffolding on students' metacognitive failure in solving mathematical problems. Students' metacognitive failure in solving mathematical problems may be simplified into three levels of scaffolding: (1) environmental provisions, (2) explaining, reviewing and restructuring and (3) developing conceptual thinking. This qualitative research had 15 students as prospective research subjects and were divided into two groups: those who half-finished the task, and the others who finished the task completely, though both groups were resulting wrong answer. From each group, a research subject was taken. Data were collected as the subject did the task with think-aloud method, then the subject was interviewed based on his work. This study found that the metacognitive failure occurred thrice in those who half-finished the task, and scaffolding level 1 was given. On the contrary, metacognitive failure in another group was occurred 5 times, and scaffolding level 1 and 2 were given.

Keywords: Scaffolding Appropriation · Students' Metacognitive Failure · Solving Mathematical Problems

1 Introduction

Problem solving is a process that entails systematic observation and critical thought in order to identify the best answer or path to the desired outcome [1]. Problem solving in mathematics education has become a leading research area that aims to understand and relate the processes that occur in problem solving [2]. Furthermore, [3] stated that problem solving is a part of student-centered curriculum. According to Hamda [4], solving mathematical problems requires the ability to emerge many concepts into one as a key concept.

Furthermore, Polya [5] claimed that issue solving is a method for gaining insight into things that aren't yet obvious and transforming them into something that is. To arrive at a correct answer, it is necessary to comprehend, solve, and reflect on the situation. Polya said that problem solving is a unique skill that necessitates intellect.

Metacognition is intimately linked to problem-solving activities. Metacognition, or the awareness, monitoring, and management of one's cognitive processes, is a key feature

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in problem solving [6]. Hassan and Rahman [7] suggested that problem-solving abilities and metacognitive awareness have a crucial role in boosting high school students' mathematics proficiency.

Metacognition means thinking to think [8] and the component of metacognition consists of metacognitive knowledge and metacognitive regulation [9]. Oszoy and Ataman [10] stated that metacognition can be used as a useful tool to develop students' problem-solving skills and metacognitive processes can improve problem solving outcomes [11]. According to Magiera and Zawojewski [12], there was a positive relationship between metacognitive activities and the implementation of problem solving.

Three metacognitive behaviors are listed in: (1) metacognitive awareness refers to a decision on the effectiveness of an individual's thinking about the strategies they choose, (2) metacognitive evaluation refers to a decision on the effectiveness of an individual's thinking about the strategies they choose, and (3) metacognitive regulation refers to when individuals modify their thinking in solving problems [12]. Also, Goos [13] believes that when it comes to solving mathematical issues, there are three types of metacognitive failures: (1) metacognitive blindness, (2) metacognitive vandalism, and (3) metacognitive mirage.

According to Huda et al. [14], metacognitive failure was noticed in students in mathematical proofs that are observed using the assimilation and accommodation paradigm. According to Huda, Subanji and Rahardjo [15], pupils' metacognitive failure was caused by metacognitive assessment faults. The categories of metacognitive failure observed in metacognitive blindness, metacognitive vandalism, and metacognitive mirage were then recognized by Huda, Subanji and Rahardjo [16]. The Artzt and Armor-Thomas processes were also used to illustrate the process of student metacognitive failure in problem solving.

Based on the results of initial observations that the researchers conducted on 15 students of the Mathematics Education Study Program, FKIP Jambi University in October 2020, there was metacognitive failures found in problem solving. Metacognitive vandalism occurred during metacognitive regulation's activities while metacognitive blindness occurred during metacognitive evaluation's activities. Additionally, metacognitive mirage occurred during metacognitive awareness's activities.

One of the solutions to apply regarding to metacognitive failure in solving mathematical problems is the provision of scaffolding [17]. Scaffolding can influence student learning through the support and encouragement throughout metacognitive activities that monitor and regulate metacognitive activity. Thus, it can be concluded that scaffolding can overcome students' metacognitive failure in solving mathematical problems. According to Anghileri [18], scaffolding consists of 3 levels: (1) Environmental Provisions, (2) Explaining, Reviewing and Restructuring, and (3) Developing Conceptual Thinking.

Based on the description above, the researchers need to provide scaffolding on students' metacognitive failure by conducting a study entitled "Scaffolding on Students' Metacognitive Failure in Solving Mathematical Problems".

2 Method

This research was conducted in September 2021 on students of the Mathematics Education Study Program, FKIP Jambi University. This research is qualitative research. The 15 students who experienced metacognitive failure were grouped into two groups. The first group is students who half-finished the task and the second group consist of students who complete the task, though both groups were resulting wrong answer. A student was taken from each group and became research subject. Data were collected using a worksheet for the subject to do with think-aloud method and followed by an interview based on the subject's work. The sheets and interview guidelines have been validated by the validator. The research data was analysed by the following steps: (1) transcribing the results of think aloud and student interviews, (2) conducting data reduction by making abstractions, (3) coding each thought process carried out by students, (4) describing the structure of students' thinking processes. (5) analysing what happened during the study, and (6) concluding.

3 Result and Discussion

3.1 Student Metacognitive Failure Data Exposure in Problem Solving and Scaffolding was Given

First Subject Metacognitive Failure (S1)

S1 solved the problem for 5 min 35 s. S1 experienced activities metacognitive regulation by thinking back and writing if $(\boldsymbol{x} - 1)^2$ was the divisor of $\boldsymbol{ax}^4 + \boldsymbol{bx}^3 + 1$ then the equation coefficient $(\boldsymbol{a}, \boldsymbol{b}, 0, 0, 1)$ divided by 1 will produce $\boldsymbol{a}, \boldsymbol{a}+\boldsymbol{b}, \boldsymbol{a}+\boldsymbol{b}$. . The coefficient of the equation $(\boldsymbol{a}, \boldsymbol{b}, 0, 0, 1)$ minus $(\boldsymbol{a}, \boldsymbol{a}+\boldsymbol{b}, \boldsymbol{a}+\boldsymbol{b}, \boldsymbol{a}+\boldsymbol{b})$ gave $(\boldsymbol{a}, \boldsymbol{a}+\boldsymbol{b}, \boldsymbol{a}+\boldsymbol{b}, \boldsymbol{a}+\boldsymbol{b}+1)$. Next, s1 continued the calculation using horner's rule, which was to divide the coefficients of the equation $(\boldsymbol{a}, \boldsymbol{a}+\boldsymbol{b}, \boldsymbol{a}+\boldsymbol{b}, \boldsymbol{a}+\boldsymbol{b}+1)$ by 1 and get $(\boldsymbol{a}, 2\boldsymbol{a}+\boldsymbol{b}, 3\boldsymbol{a}+2\boldsymbol{b}, 4\boldsymbol{a}+3\boldsymbol{b}+1)$. It can be seen from the work of s1 as shown in Fig. 1.

Furthermore, S1 looked back at the results that have been obtained. S1 conducted activities on metacognitive evaluation and rethought to get a and b scores. S1 read the answer again and wrote the equations 4a + 3b = -1 and a + b = -1. S1 writes 4a + 3b = -1. Therefore, S1 did not give rise to "error detection" by ignoring the errors he had made. Thus, S1 experiences metacognitive blindness (Figs. 2 and 3).

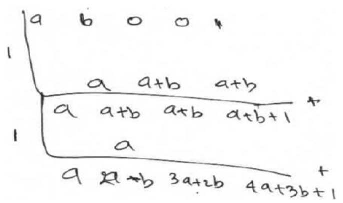

Fig. 1. Results of S1's Work on Metacognitive Regulation Activities

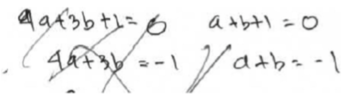

Fig. 2. The result of S1's work is to get 4a + 3b = -1 and a + b = -1

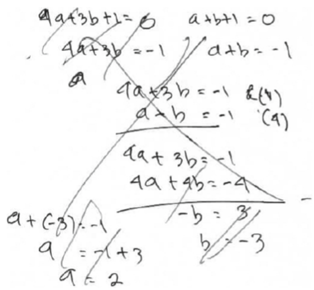

Fig. 3. The results of S1's work are to determine the results of a times b

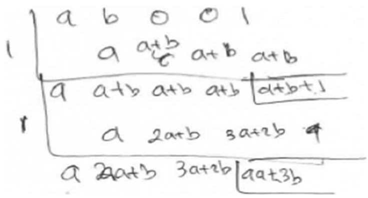

Fig. 4. S1's Work in Using Horner's Rule at the Time of Scaffolding

Scaffolding on Metacognitive Failure First subject (S1)
S1 made Provision of scaffolding after S1 resolve a given problem by conducting interviews between investigators with S1. Interviews were conducted based on the S1 answer sheet. From S1's answer, it is clear that S1 experienced metacognitive blindness when determining the remainder of the Horner's rule. Researchers provided scaffolding Level 1 by asking S1 to rework the problems given. This is evident from the results of interviews with researchers with S1 as follows.

Researcher: Now you solve this problem again
S1: OK

S1 repeated the problem again by using a horner. From the results of the S1 work, the scaffolding in Environmental Provisions level provided was able to overcome the metacognitive blindness S1. Next, S1 produces the remainder of the division of $ax^4 + bx^3 + 1$ divided by $(x - 1)^2$ as the work of S1 in Fig. 4.

By providing scaffolding of Environmental Provisions in S1, S1 could overcome S1 experiencing metacognitive vandalism inactivities metacognitive regulation. S1 could determine the values of a and b by solving the equations a + b + 1 = 0 and 4a + 3b = 0, so that a = 3 and b = 4. This was obvious in the results of student work in Fig. 5.

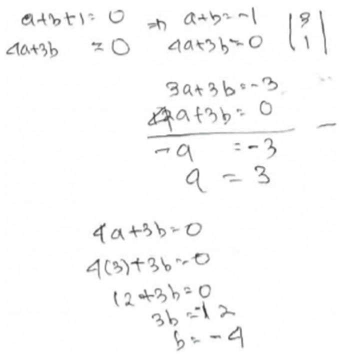

Fig. 5. Work Results S1 Determines the Value of a and b at the time of Scaffolding

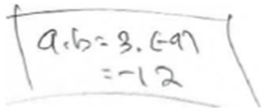

Fig. 6. Results of S1's Work in Determining the Value of a times b When Giving Scaffolding

S1 determined the value of *a* times *b* by substituting the value of a and *b*, so they equal to -12. This was visible from the work of S1 in Fig. 6.

Next, the researcher asked S1 to express his idea in a different way, but S1 immediately said that he could not solve it any other way. This can be seen from the results of interviews with researchers with S1 as follows.

Researcher: Do you have any other ideas or ways to solve it?
S1: No

Second Subject Metacognitive Failure (S2)
S2 solved the problem for 4 min 53 s. S2 read the problem from the end using think aloud.. S2 performed activities metacognitive awareness by remembering that this material had been studied before. Then S1 did a metacognitive evaluation and thought that $(x - 1)^2$ wass the same as $x^2 + 2x + 1$ dividing $ax^4 + bx^3 + 1$, asked for the result of a times b. This can be seen from the results of think aloud S2 as follows.

S2: It is known that (x-1)2 is simplified to $x^2 + 2x + 1$ divides ax4 + bx3 + 1. Determine the result of ab…. Which is asked for the result of a times b.

Next, S2 experienced metacognitive evaluation of how to divide $x^2 + 2x + 1$ with $ax^4 + bx^3 + 1$. In this case, S2 experienced a metacognitive failure, namely metacognitive blindness, because S2 did the division by specifying the number to be divided as the divisor. This can be seen from the results of the S2's work in Fig. 7.

Next, S2 experienced activities metacognitive regulation in determining the results of the division of $ax^4 + bx^3 + 1$ with $x^2 + 2x + 1$. . In this case S2 is deadlocked (lack

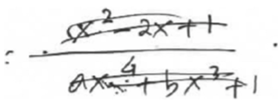

Fig. 7. The results of S2's work in determining the results of the division

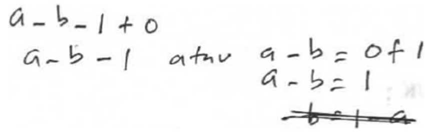

Fig. 8. Results of S2's Work in Changing the Context of Problems

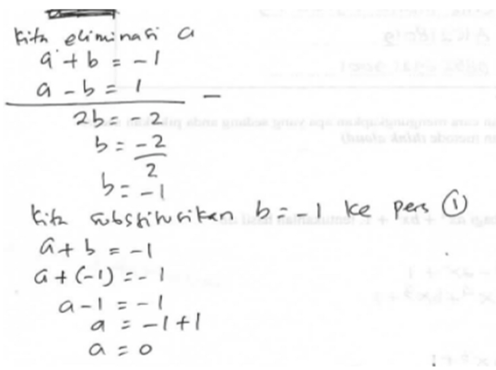

Fig. 9. Work Results S2 experienced metacognitive blindness in determining the value of a and value of b

progress). S2 changed the problem so that the existing problem is in accordance with the concept of knowledge possessed, for example $a + b + 1 = 0$ and $ab - 1 = 0$ and results achieving undesired solution. Thus, S2 experienced metacognitive vandalism during metacognitive evaluation's activities. To solve the next problem, S2 experienced metacognitive regulation's activities by changing the context of the problem to match the concept of knowledge he had. As a result, S2 did not get the right solution (anomaly result). Thus, it can be said that S2 experienced metacognitive vandalism. This was visible from the results of the S2's work as shown in Fig. 8.

At the end of solving problems, S2 performed metacognitive regulation activities by solving $a + b = -1$ and $ab = 1$. In this case, S2 made a mistake in solving the problem and did not realize there was an error. Thus, it can be said that S2 has metacognitive blindness. This can be proven from the results of the S2's work as shown in Fig. 9.

Next, S2 conducted a metacognitive evaluation by re-checking the results of problem solving obtained. S2 stated that the product of a and b is 0 times -1 equals 0. Therefore, S2 ignored the error he made. In other words, S2 experienced metacognitive mirage when checking the final results. This can be proven from the results of his work in determining the product of a by b, as shown in Fig. 10.

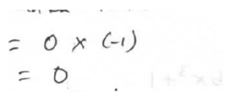

Fig. 10. Work Results S2 experienced metacognitive mirage in determining the product of a by b

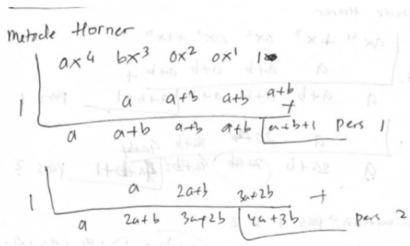

Fig. 11. Results of S2's Work in Using Horner's Rule at the Time of Scaffolding

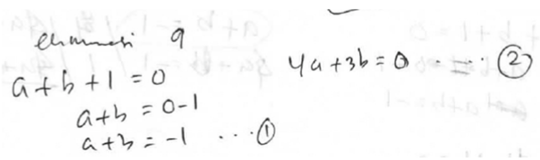

Fig. 12. Results of S2's work to get a + b + 1 = 0 and 4a + 3b = 0

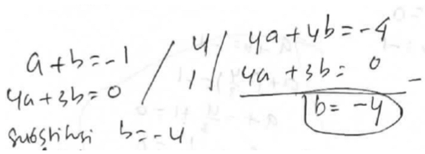

Fig. 13. The results of the work of S2 get a value of a times b

Scaffolding on Metacognitive Failure Second Subject (S2)
S2 experienced metacognitive blindness and metacognitive vandalism during metacognitive evaluation's activity. S2 also experienced metacognitive vandalism in the activity of metacognitive regulation. Based on the metacognitive failure of the Master's Degree, the researcher provided scaffolding Environmental Provisions, namely asking S2 to rework the problems given. S2 again solved the problem with Horner's rule. Next, S2 divides $ax^4 + bx^3 + 1$ by $x^2 + 2x + 1$ using Horner's rule. This was obviously seen from the results of student work as shown in Fig. 11.

By using Horner's rule, S2 obtained two equations, namely $a + b + 1 = 0$ and $4a + 3b = 0$. This was proven from the results of S2 work as shown in Fig. 12.

Next, S2 thought again to determine a times b, firstly S2 determined the values of a and b. By using the elimination method and the substitution method, S2 obtained $a = 3$ and $b = 4$ This was in accordance with the results of the S2's work in Fig. 13.

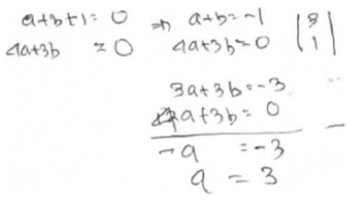

Fig. 14. Results of S2 Work Determine the Value of a and Value of b at the time of Scaffolding

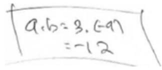

Fig. 15. Results of S2's Work in Determining the Value of a times b When Scaffolding is Given

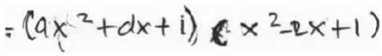

Fig. 16. S2's work results when doing metacognitive evaluation of the quotient multiplied by the divisor

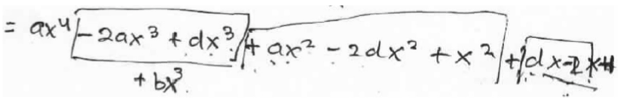

Fig. 17. Results of S2's work in determining the multiplication result between quotient and divisor

Next, the researcher provided scaffolding of Environmental Provisions for S2. This overcame metacognitive vandalism in the activity of metcognitive regulation of S2. This was proven by S2 being able to determine the values of a and b by solving the equations $a + b + 1 = 0$ and $4a + 3b = 0$, so that $a = 3$ and $b = 4$. This was clear in the results of student work on Fig. 14.

S2 determined the value of a times b by substituting the values of a and b, so that a times b = 12. This can be seen from the work of S2 in Fig. 15.

To find out the Zone of Proximal Development (ZPD) of S2 in problem solving, the researcher asked S2 to express ideas in a different way. In this case, the researcher provided a scaffolding of Explaining, Reviewing and Restructuring. S2 rethought and did metacognitive awareness to find out other ways that were used in solving mathematical problems. S2 performed a metacognitive evaluation by assuming the quotient of $ax^4 + bx^3 + 1$ with $x^2 + 2x + 1$ was $ax^2 + dx + 1$. This is in accordance with the results of the S2's work in Fig. 16.

Next, S2 performed metacognitive regulation, rethought again to determine how to solve mathematical problems by using an example as shown in Fig. 17. S2 multiplied $ax2 + dx + 1$ by $x2 + 2x + 1$, so we get $ax^4 - 2ax^3 + dx^3 + ax^2 - 2dx^2 + x^2 + dx^2 - 2x + 1$. This was clear as from the results of the S2's work in Fig. 17.

S2 rethought and did metacognitive regulation to solve mathematical problems by making the equation $ax^4 + bx^3 + 1 = ax^4 - 2ax^3 + dx^3 + ax^2 - 2dx^2 + x^2 + dx^2 - 2x + 1$,

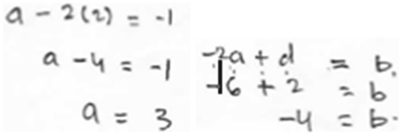

Fig. 18. The results of S2's work in a and b values

so he got $a = 3$ and $b = -4$. This is in accordance with the results of the S2's work as shown in Fig. 18.

By doing a metacognitive evaluation, S2 obtained a times b equal to -12 and S2 was sure of the answer.

4 Conclusion

S1 experienced metacognitive failure in solving mathematical problems thrice (metacognitive blindness, metacognitive vandalism and metacognitive mirage) with the scaffolding provided at the Environmental Provisions level. S2 experienced metacognitive failure in solving math problems 5 times (metacognitive blindness 2 times, metacognitive vandalism 2 times, and metacognitive mirage) The scaffolding given to S2 is the Environmental Provisions level scaffolding and the Explaining, Reviewing and Restructuring level scaffolding.

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