Student Problem Solving Skill in Linear Algebra Subjects Based on Polya Solving Steps

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Abstract. One of the general skills that must be achieved by undergraduates is being able to make appropriate decisions in the context of solving problems in their field of expertise, based on the results of information and data analysis. The importance of problem solving skills was the background of this research which aimed to determine the mathematical problem solving ability of students in Linear Algebra courses based on Polya's solving steps. This research was a descriptive research with a qualitative approach. The subjects in this study were second semester mathematics education students at Bengkulu University in 2020/2021 who took Linear Algebra courses. The research data studied in this study were student answer sheets for the Final Semester Examinations in the 2020–2021 academic year using video conferences. The results showed that students with high scores were able to understand the problem well, make and implement plans properly and re-examine the steps that have been taken. Students with average scores understood most of the problems but there were still things they didn't understand. Students with low scores didn't understand the problem so they were unable to make and carry out plans or re-examine.

Keyword: problem solving skill · linear algebra · polya

1 Introduction

Mathematics plays an important role in improving students' thinking skills. Mathematics learning is a teaching and learning process built by teachers to develop students' creative thinking that can improve students' thinking skills, and can improve the ability to construct new knowledge as an effort to improve good mastery of mathematical material [1]. One of these thinking skills is problem solving ability.

Problem solving ability is an important skill possessed by students at all levels, even undergraduate candidates [2]. One of the general skills that must be achieved by undergraduates is being able to make appropriate decisions in the context of solving problems in their field of expertise, based on the results of information and data analysis. Mathematics is needed in various aspects, one of which is in the world of work, the level of mathematics required for intelligent society has increased as well as the level of mathematical thinking and problem solving [3].

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Problem solving is known as an important life skill, where the process includes analyzing, interpreting, reasoning, predicting, evaluating and reflecting. Problem solving [4, 5] is a thought process to determine what to do when not knowing what to do. Problem solving is a subject of learning from and in its own process, as an approach to a problem and as a way of teaching [6]. Problem is a situation that requires resolution in which the steps towards the solution are not yet known. The descriptions of the experts above show that problem solving is a thinking process or skill in finding solutions to a problem that can be learned as well as a way of teaching [6].

There are four steps of problem solving, namely understanding the problem, making plans, carrying out plans and re-examining the results of the work [7]. At the stage of understanding the problem students need to understand information, data, conditions and what is not known or is the problem. At the planning stage, students need to connect data and questions to the problems presented and then make a settlement plan. At the stage of carrying out the planning, students need to implement the settlement strategy that has been made and ensure the completion steps taken are appropriate. At the stage of re-checking, students need to check whether the results obtained are correct. Polya's problem-solving strategy is a strategy that has been widely used in learning because it is seen as simple and practical.

Several previous studies have been conducted to see the problem solving ability of students. The results of previous research generally indicate that the problem solving ability of students is not fully good or is still in the sufficient category [8–10]. This ability can be measured based on several aspects such as prior knowledge, student test scores and cognitive abilities. The better the initial knowledge, test scores and cognitive abilities of the student, the better his problem solving ability is.

Linear Algebra is a course that contains a lot of material related to problem solving, both problems in the form of daily life story questions and problems not in the form of story questions. Potential to analyze students' problem solving skills in this course. Research can be done by analyzing problem solving skills using problem solving steps by Polya so that the distribution of students' problem solving ability levels in this course is illustrated.

Since November 2019 the lifestyle of the world community has changed due to the emergence of the Covid-19 virus outbreak. This epidemic is forcing people to change all habits including study habits that were previously done at school or on campus, are now done from their respective homes. Learning is carried out in the internet network using various supporting applications including video conferencing applications. Linear Algebra lectures are also carried out online. This new learning pattern directs students to learn from their own homes and demands independence in order to be able to achieve thinking skills, especially good problem solving skills. Analysis of students' problem-solving abilities needs to be done in the midst of new learning styles during this pandemic.

Based on the description above, the researcher conducted a research with the formulation of the problem "How is the student's mathematical problem solving ability in Linear Algebra courses based on Polya's solving steps?". The purpose of this study was to determine the distribution of students' mathematical problem solving ability levels in Linear Algebra courses based on Polya's solving steps.

2 Method

This research is a descriptive study with a qualitative approach which aims to explain qualitatively the problem solving abilities of mathematics education students in solving Linear Algebra problems. The subjects in this study were second semester mathematics education students at Bengkulu University in 2020/2021 who took Linear Algebra courses. The research data studied in this study are student answer sheets for the Mid-Semester Examination (UTS). The UTS implementation time is on Monday, March 29, 2021 online using a video conference application.

This study uses several data collection techniques, namely (1) observation. Observations in this study were carried out to observe the UTS answer sheets for Linear Algebra courses and those that matched the problem solving indicators according to Polya. (2) interviews (interviews), interviews conducted are needed to obtain in-depth information and support what is obtained from the results of the analysis of students' UTS answers. (3) Documentation, which is the main data in this research is student UTS answer sheet data. The validity of this research data using triangulation. In this study, triangulation was carried out by comparing the observations of students' answer sheets with the results of interviews. The data analysis technique used is data reduction, presenting data and making conclusions.

3 Results and Discussion

3.1 Understanding the Problem

3.1.1 Students with High Problem Solving Ability Scores

Students with high problem-solving ability scores are able to understand the problem well. Students fully understand the information in the question, namely what is known and what is being asked. In question number 1, students understand that a matrix is known and asked about the determinant of the matrix using the sarus rule, cofactor expansion and row reduction. In question number 2, students understand that a matrix is known and asked for the inverse of a matrix using adjoins and row operations. In question number 3, students understand that a system of linear equations is known and asked to solve it using Gauss Jordan elimination, Cramer's rule and inversion method.

3.1.2 Students with Moderate Problem-Solving Ability Scores

Students with moderate problem-solving ability scores are able to understand the problem quite well. Students understand the information on most of the questions, namely what is known and what is asked. In question number 1, students understand that a matrix is known and asked about the determinant of the matrix using the sarus rule, cofactor expansion and row reduction. In question number 2, students understand that a matrix is known and asked for the inverse of a matrix using adjoins and row operations. In question number 3, students understand that a system of linear equations is known and is asked to solve it using Gauss Jordan elimination, Cramer's rule, but students do not understand that question number 3 point c asks the solution of a system of linear equations using the inversion method. This is proven by the wrong steps taken by students (Fig. 1).

Fig. 1. Misunderstanding the Problem

3.1.3 Students with Low Problem-Solving Ability Scores

Students with low problem-solving ability scores are only able to understand a few problems. Students are only able to understand exactly 1 of 8 problems. Even students do not understand the 5 problems at all. This happens because students do not understand the material being tested.

3.2 Making Plans

3.2.1 Students with High Problem Solving Ability Scores

After understanding the problem, students with high problem-solving ability scores were immediately able to make plans. This plan was in their minds. Students understand very well what they have to do to solve the problem. In question number 1, students understand that in order to determine the determinant of a matrix with the sarus rule, the steps that need to be taken are to rewrite the first two columns of the matrix and then subtract the number of product items diagonally from the left side to the right side. To determine the determinant of a matrix with cofactor expansion, the steps that must be taken are to select the row or column that will be used as a benchmark, then find the minor based on the selected row or column, then multiply each element in the selected row or column by its minor determinant, then operate addition and the reduction. To determine the determinant of a matrix with row reduction, the steps that must be taken are swapping rows or adding multiples of rows or multiplying rows by a scalar until a scalar is obtained that multiplies the matrix with diagonal 1, then multiplies the scalar by the product of the elements on the diagonal.

In problem number 2, students understand that to determine the inverse of a matrix using an adjoin, the steps that need to be taken are to determine the determinant of the matrix, determine the minor and cofactor, determine the adjoin and apply it to the inverse matrix formula. To determine the inverse of a matrix using row operations, the step that needs to be done is to perform row operations on two adjoining matrices, namely the matrix to be searched for the inverse and the identity matrix, in such a way that an identity matrix is obtained instead of the initial matrix. In problem number 3, students understand that in order to determine the solution to a system of linear equations, the first thing to do is to convert the system into a matrix form Ax = b and then apply the Gauss Jordan elimination step, namely by using row operations to obtain a reduced row echelon

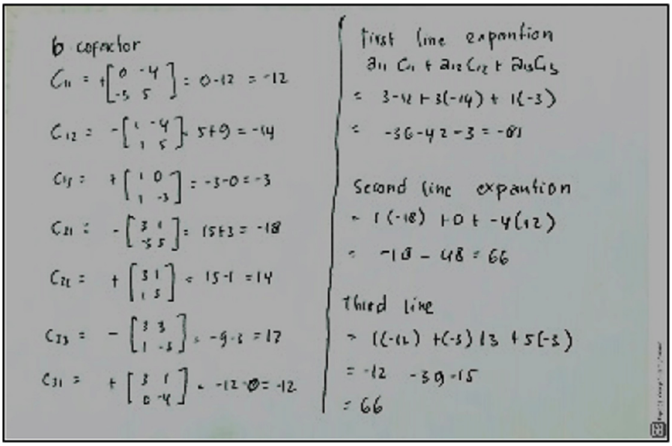

Fig. 2. Mistake In Making Plan

matrix. Meanwhile, for the Cramer's rule step, it is necessary to find the determinant of matrix A and the determinants of the modified matrix A matrix which has replaced its column elements with matrix b, then apply it to the Cramer's rule formula. Finally, to determine the solution of a system of linear equations using the inversion method, the steps that need to be taken are to determine the inverse of matrix A and multiply it by matrix b.

3.2.2 Students with Moderate Problem-Solving Ability Scores

Students with moderate problem-solving ability scores are able to make plans quite well even though there are some plans that are still wrong. This plan is reflected in the steps taken. For example in number 1 point c. Students should determine the determinant of the matrix with cofactor expansion, the step that must be taken is to choose the row or column that will be used as a benchmark, then find the minor based on the selected row or column, but in fact students use all rows and columns (Fig. 2).

In the picture, it can be seen that students use all rows and columns as if to determine the inverse of a matrix. In fact, they simply choose one row and one column.

3.2.3 Students with Low Problem Solving Ability Scores

Students with low problem-solving ability scores are only able to plan a few questions. For example, PR is only able to plan 3 out of 8 questions. Only one plan is correct. This happens because students do not understand the material being tested.

3.3 Carrying Out Planning

3.3.1 Students with High Problem Solving Ability Scores

Students with high problem-solving ability scores are able to carry out well-thought-out plans. However, there are still some errors, for example in question number 3, students

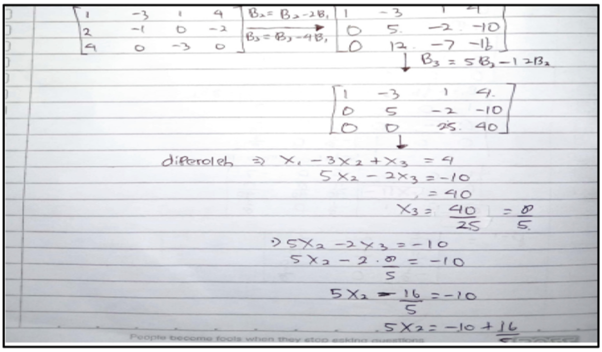

Fig. 3. Mistakes in Carrying Out Plan

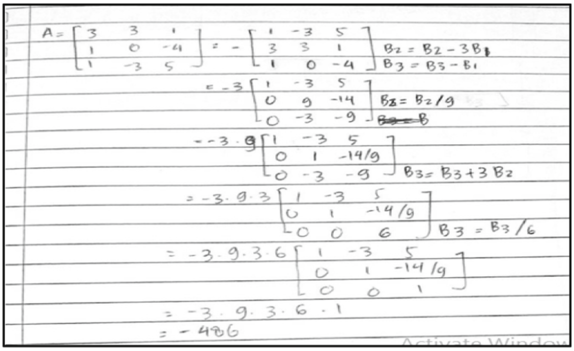

Fig. 4. Mistakes in Calculating

do not perform line operations until they are finished, but replace them with algebraic operations (Fig. 3).

This shows an error in the implementation of the plan for Gauss Jordan Elimination, namely not carrying out the plan until it is finished. Another mistake that students make is a calculation error. The steps taken were correct, but a calculation error occurred due to lack of accuracy.

3.3.2 Students with Moderate Problem-Solving Ability Scores

Students with moderate problem-solving ability scores are able to carry out well-thought-out plans. However, there are still some errors, for example in question number 1c, students made a calculation error.

This is an elementary row operation for finding determinant. This question has the answer -66 but the student answered -486. The figure shows that the answer is wrong.

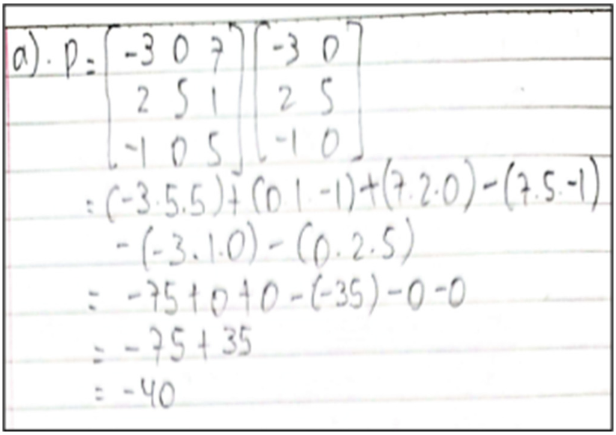

Fig. 5. The Correct Answer

3.3.3 Students with Low UTS Scores

Based on a proper plan, students are able to carry out the plan well. Proven students can answer the question correctly (Fig. 5).

This is the only question that students understand best. It is proven that students are able to understand problems, make plans and carry out plans well. But unfortunately students are not able to do the same thing with other questions. This happens because according to students this material is the easiest.

3.4 Checking the Results

3.4.1 Students with High Problem Solving Ability Scores

Students with high problem-solving ability scores re-examine the steps they have taken. It was proven that most of the questions were done correctly, although there were questions that had not been answered correctly because they were not careful when checking again.

3.4.2 Students with Moderate Problem-Solving Ability Scores

Students with moderate problem-solving ability scores only had time to re-examine some of the questions. On the other hand, they are also still thinking about unanswered questions. So that there is an inaccuracy as in Fig. 4.

3.4.3 Students with Low Problem-Solving Ability Scores

Students with low problem-solving ability scores were able to re-examine one question correctly but were unable to check other questions because they did not understand and did not work on the problem even though there was still a lot of time left.

4 Conclusion

The results showed that students with high scores were able to understand the problem well, make and implement plans properly and re-examine the steps that have been taken. Students with average scores are understanding most of the problems but there are still things they don't understand. Students with low scores do not understand the problem so they are unable to make and carry out plans or check back.

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