An Opinion Dynamics Model Considering the Decay of the Opinion Leader's Influence

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Abstract. This paper studies the influence of the opinion leader in the process of opinion dissemination. An extended HK model is proposed by considering the decay of the influence of the opinion leader, and then some simulation experiments are conducted in order to analyze the role of the opinion leader in the evolution of online opinion events and find the factors which affect the influence of the opinion leader. Based on these simulation results, some scientific suggestions are provided for solving online public opinion problems.

Keywords: opinion leader · HK model · decay · online public opinion

1 Introduction

With the development of Internet technology, social media such as micro-blogs have become the main channel for people to receive information and express their opinions. The rapid development of micro-blogs has led to the problems of uneven information quality, low quality, and more negative information on the Internet [1]. Opinion leaders play an important guiding role in the evolution of online opinion dissemination. Therefore, it is important to study the influence of opinion leaders on online public opinion.

Meanwhile, the rise of complex networks has inspired scholars to improve and extend the classical online opinion evolution models. The models are classified into discrete opinion models and continuous opinion models according to the discrete and continuous opinions of the models. Discrete opinion models generally use -1 and 1 to denote opposing and supporting opinions, respectively. The classical discrete opinion models are Ising model [2, 3], voter model [4], Galam model [5], Sznajd model [6], etc. Continuous opinion models represent the opinion with values in the interval [0, 1]. The classical continuous opinion models are DW (Deffuant-Weisbuch) model [7], HK (Hegselmann-Krause) model [8], DeGroot model [9] and so on. In addition, there is a CODA model for continuous opinions and discrete actions [10]. These classical models provide directions and methods for solving public opinion problems.

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In most studies on opinion leaders [11, 12], it is generally assumed that the influence of opinion leaders is constant, and their changes over time are rarely analyzed. In reality, the influence of opinion leaders decays over time [1]. Therefore, the main innovation of this paper is to quantify the influence of opinion leaders and introduce an opinion leader influence function.

2 Materials and Methods

2.1 Traditional HK Model

Suppose there are N agents, denoted by the labels 1, 2,..., N. Let $X_i(t) \in [0, 1]$ be the opinion value of agent i at time t. For agent I, if $\left|X_i(t) - X_j(t) \leq\right|\varepsilon$, then agent j is called the neighbor of agent i, and ε is said to be the trust level of agent i. In the HK model, each agent's opinion is influenced only by its own opinion and those of its neighbors. The fusion process of the opinions of agent i in the traditional HK model is as follows [8]

$$X_i(t+1) = \sum_{j \in M_i(t)} a_{ij} X_j(t), i = 1, 2, \cdots, N \tag{1}$$

where

$$M_i(t) = \left\{ j \middle| \left| X_i(t) - X_j(t) \right| \leq \varepsilon, j \in \{1, 2, \cdots, N\} \right\} \tag{2}$$

a_{ij} indicates the acceptance degree of agent i to its neighbor's (i.e. agent j) opinion, and $\sum\limits_{j \in M_i(t)} a_{ij} = 1$. Greater a_{ij} means that agent i is more likely to accept the opinions of agent j, and smaller a_{ij} means that agent i less likely to be influenced by agent j.

2.2 Extending the HK Model

Based on the traditional HK model, we construct an extended HK model by considering the influence of opinion leader. Moreover, the influence of opinion leader is time-varying.

It is assumed that the leader's opinion is constant. Let X_0 be the leader's opinion. The extend HK model is as follows,

$$\begin{aligned} X_i(t+1) = (1 - B_i(\mathrm{t})) \left[\sum_{j \in M_i(t)} a_{ij} X_j(t) \right] + B_i(t) X_0 \\ i = 1, \cdots, N \end{aligned} \tag{3}$$

where

$$B_i(t) = \frac{\mathrm{A}_i \lambda_i}{e^{\theta(t - t_1)}} \tag{4}$$

- θ denotes the decay coefficient and t_1 represents the time when the leader first expresses his opinion. $B_i(t)$ is called the influence function of the opinion leader. This function reflects the decay of the influence of the opinion leader over time, the ability of ordinary agents to receive the opinions of the leader, and the proportion of ordinary agents in the network who can receive the leader's opinion.
- The following points are made for Eqs. (3) and (4).
- It is assumed that the opinion leader have high self-confidence and its opinion does not change with time, i.e., X_0 is a constant.
- A_i indicates whether agent i in the network has received an opinion leader's opinion, $A_i = 1$ indicates that agent i has received an opinion leader's opinion, and $A_i = 0$ indicates that an agent has not received an opinion leader's opinion. In the case of $A_i = 0$, the update of agent's opinion is only influenced by the neighbor's opinion, and the formula for the update of opinion is Eq. (1). In this paper, we assume that the agents who can receive information from the opinion leader are fixed, i.e., they do not change over time. The proportion of agents who can receive the leader's information is called the reception proportion.
- λ_i denotes the trust degree of agents i toward opinion leader, and $\lambda_i \in [0, 1]$. The smaller the λ_i, the less agent i trusts the opinion leader and the less the influence of the opinion leader's opinion on the update of agent i's opinion; the larger the λ_i, the more agent i trusts the opinion leader and the more agent i's opinion is easily influenced by the opinion leader's opinion. In this paper, λ_i does not change over time.
- θ is the decay coefficient. Life cycle theory was first used to study the development of agents and different families, and later it was widely used in products, enterprises, etc. Similarly, micro-blogs also have a unique life cycle [1]. Therefore, the influence effect of the opinion leader on agents in micro-blogs is also changing, and the acceptance of the opinion leader by agents will gradually decrease over time. In this paper, we use θ to denote the decay coefficient [13].
- t_i is the time when the opinion leader first express his opinion.

3 Results and Discussion

In this paper, simulation experiments are conducted in Python. First, a network with $N = 99$ agents and one leader is created, and the number of agents and the distribution of initial opinions do not change during the subsequent simulations. a_{ij} and λ_i are randomly assigned, and the initial opinions of 99 ordinary agents are evenly distributed in [0, 1]. The average value of the initial opinions of all ordinary agents in the network is 0.5132 (the average values of opinions in this paper are accurate to 4 decimal places).

The opinion evolution of 100 agents are analyzed by considering three factors, that is, the decay coefficient, the proportion of agents which can receive the leader's opinion, and the time when the leader first express his opinion.

3.1 Effect of the Decay Coefficient θ on the Evolutionary Process

Different values of decay coefficients are selected for the experiments, and if no special instructions are given, the values of opinion leader's opinions are taken as 1. Other specific parameters are shown in Table 1.

Table 1. Description of parameters of experiment a

Parameter Name	Parameter values
Trust level ε	0.5
Receiving proportion	0.3
Time for the opinion leader to express their opinions t_1	0
Decay coefficient θ (Parameters analyzed in this group of experiments)	0.02/0.2/2

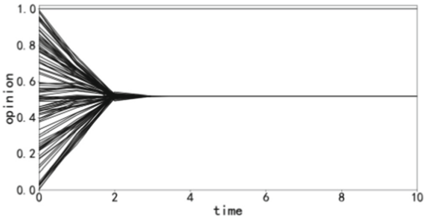

Fig. 1. No opinion leader

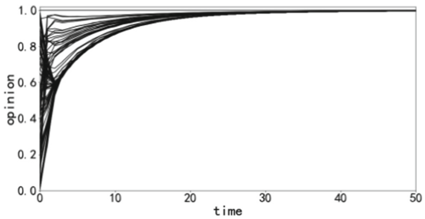

Fig. 2. $\theta = 0.02$

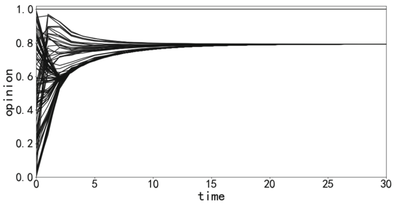

Fig. 3. $\theta = 0.2$

In the absence of opinion leader, the evolution of 99 agents' opinions is shown in Fig. 1. In the presence of the opinion leader, the decay coefficients are 0.02, 0.2, and 2, respectively, and the evolution of 100 agents' opinions is shown in Fig. 2–4, and the comparison of the evolution is shown in Table 2.

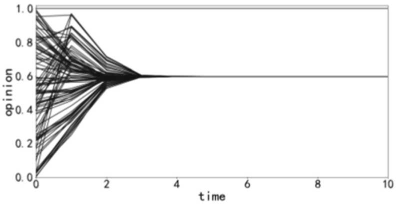

Fig. 4. $\theta = 2$

Table 2. Comparison of the evolution of experiment A

decay coefficient θ	Final opinion average	Time of reaching consensus
No opinion leaders	0.5178	3
0.02	0.9968	30
0.2	0.7924	15
2	0.5980	4

Comparing Figs. 1–4, it can be found that the opinion leader causes the trend of opinions in the network to converge to the opinion leader's opinion. By Table 2, it can be seen that there is a slight change from 0.5132 to 0.5178 in the opinion mean when no leader is added. After adding a leader with opinion 1, the final average opinion in the network increases to a certain extent compared with the initial opinion average in the three cases of $\theta = 0.02$, $\theta = 0.2$, and $\theta = 2$, and the network can reach consensus faster with the increase of θ. However, the final opinion average in the network gradually decreases with the increase of θ, i.e., the increase effect of the opinion leader on the final opinion average decreases.

3.2 Influence of the Receiving Proportion the Evolutionary Process

Therefore, in order to make opinion leader's opinion plays a greater role in public opinion and reach consensus more quickly in the network, the decay of opinion leader's influence should be reasonably controlled, and should not decay too fast or too slow. Internet users who receive opinion leader's opinions can be called upon to actively forward their comments so that more people can receive the information to slow down the decay of opinion leader's influence.

Different receiving proportions are selected for the experiments, and the specific parameters are shown in Table 3.

Initially, a randomly selected proportion of 0.3 ordinary agents are able to receive the opinion leader's opinions, and on top of that, 0.2 proportion of ordinary agents are added to increase the receiving proportion to 0.5, and thsi proportion 0.5 agents was fixed without change, and then 0.2 proportion of ordinary agents are added to increase the receiving proportion to 0.7. The evolutionary process corresponding to the receiving proportions 0.3, 0.5, and 0.7, are shown in Figs. 5–7, and the comparison of the evolutionary situation is shown in Table 4.

Table 3. Description of parameters of experiment B

Parameter Name	Parameter values
Trust level ε	0.5
Receiving proportion (parameter analyzed in this group of experiments)	0.3/0.5/0.7
Time for the opinion leader to express their opinions t_1	0
Decay coefficient θ	0.2

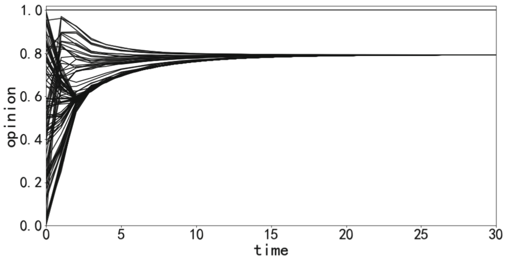

Fig. 5. Receiving proportion of 0.3

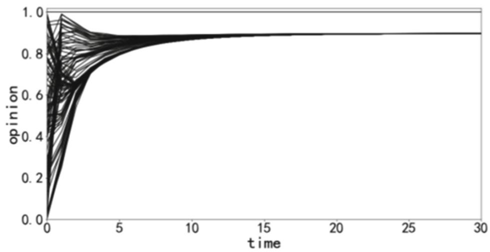

Fig. 6. Receiving proportion of 0.5

From Figs. 5–7, we can see that the receiving proportion increases from 0.3 to 0.7, and the final opinion average also increases from 0.7924 to 0.9416, and as the receiving proportion keeps increasing, the time to reach consensus in the network gradually shortens. It can be concluded that increasing the receiving proportion can make the average of final opinions in the network closer to the opinion leader's opinions and all agents can reach consensus earlier, which is also consistent with our common sense.

Therefore, the opinion leader can be made to express their opinions in multiple channels so that more people can receive the information.

3.3 The Effect of the Timing of the Opinion Leader's Opinions on the Evolutionary Process

Different publication times are selected for the experiments, and the specific parameters are shown in Table 5.

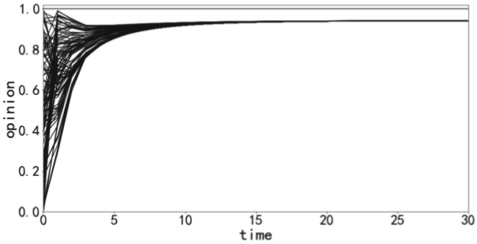

Fig. 7. Receiving proportion of 0.7

Table 4. Comparison of the evolution of experiment B

Receiving proportion	Final opinion average	Time of reaching consensus
0.3	0.7924	15
0.5	0.8963	13
0.7	0.9416	10

Table 5. Description of parameters of experiment C

Parameter Name	Parameter values
Trust level ε	0.5
Receiving proportion	0.3
Time for the opinion leader to express their opinions t_1 (Parameters analyzed in this group of experiments)	1/2/5
decay coefficient t_1	0.2

Table 6. Comparison of the evolution of experiment C

Time to publish your opinion	Final opinion average	Time of reaching consensus
1	0.7966	16
2	0.7963	17
5	0.7950	20

From Fig. 1, it can be seen that the consensus has been reached at time 4 without adding opinion leader. Therefore, timess 1 and 2 before the consensus of opinions and time 5 after the consensus are selected for analysis in this experiment. The evolutionary processes in the case of $t1 = 1$, $t1 = 2$, $t1 = 5$ are shown in Figs. 8–10, and the evolutionary comparison is shown in Table 6.

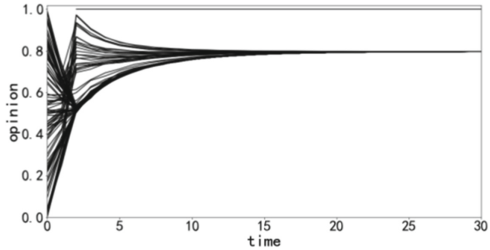

Fig. 8. $t1 = 1$.

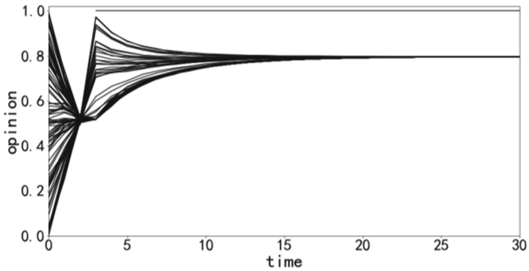

Fig. 9. $t1 = 2$.

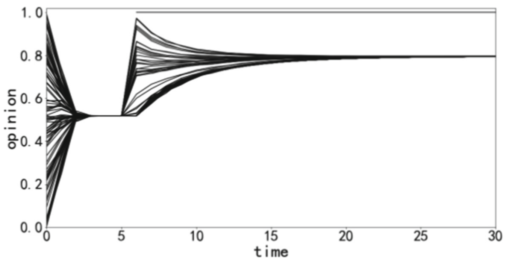

Fig. 10. $t1 = 5$.

According to Figs. 8–10, it can be seen that the timing of the opinion leader's publication does not have an excessive effect on the average of final opinions, but the opinion leader will influence ordinary agents, and the ordinary agents will follow the leader's opinions. Hence, the earlier the opinion leader publish their opinions, the earlier the final opinions of the network can reach consensus. Therefore, in order to reach consensus on public opinion events earlier, the leader need to express their opinions as early as possible at the early stage of public opinion events so that citizens can get relatively official and scientific information about the events earlier.

4 Conclusions

Based on the existing research, this paper quantifies the influence of opinion leader in the evolution of public opinion, proposes an opinion leader influence function, and analyzes the role of the opinion leader in the evolution of public opinion by using Python simulation experiments.

By the experiment results, in order to reach a consensus earlier and the final opinion is closer to the opinion leader's opinion, the opinion leader should express his or her opinion as early as possible, and at the same time call on people to actively forward comments after receiving the opinion leader's opinion so that more people can receive it. In addition, the opinion leader can express his or her opinion several times in order to slow down the decay of his or her influence.

This paper aims to provide analysis and help for online public opinion events, but there are some problems in our analysis. For example, insufficient factors are considered, and the experimental data in this paper are simulated data. In the future, we try to use real data to analyze the evolution of public opinion in different social networks, which will be helpful for solving practical problems.

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