Cutting Costs and Improving Image: Optimal Packaging Carton Inventory Control for Hospital Equipment Industry

Laila Nafisah(✉), Puryani, Apriani Soepardi, and Mochammad Chaeron

Department of Industrial Engineering, UPN Veteran Yogyakarta, Yogyakarta, Indonesia
laila@upnyk.ac.id

Abstract. The Indonesian hospital equipment industry, PT MAK, has the potential to contribute to the health sector and national economy. One of the strategies to achieve this is ensuring availability of products that customers want. However, the lack of packaging cartons often results in physical damage to products and delays in delivery. To address this issue, a study was conducted on proper packaging carton inventory control using stochastic joint replenishment orders for multi-item products with probabilistic demand. The aim was to minimize total inventory cost. The results showed that joint ordering every 25 days and order lot size proportional to demand rate per period reduced inventory cost by 27.72% per year. Implementing this policy could improve company image and loyalty of consumers.

Keywords: Inventory control · packaging carton · hospital equipment · cost minimization · probabilistic demand

1 Introduction

In the midst of the coronavirus (Covid-19) pandemic, the demand for medical devices has skyrocketed. The availability of around 3000 hospitals, 9000 health centers and private clinics in Indonesia today, shows the need and high market potential for medical devices in Indonesia. The Indonesian hospital equipment industry has great potential and opportunity to be able to develop itself and contribute to the health sector and the national economy. One of the strategies in order to continue to grow and be able to compete is its ability to respond to customer desires, especially in ensuring the availability of products that customers want.

PT Mega Andalan Kalasan (PT MAK) is one of the largest hospital equipment manufacturers in Indonesia. The company has successfully developed more than 10 types of product classes with a total of more than 600 types of hospital equipment products. Most of the components that make up PT MAK's products are produced by PT MAK's own subsidiaries. On average, the products are assembled using knock down method. PT MAK is in charge of assembling all components produced from its subsidiaries into ready-to-sell products (assembly to order). The production system used is make to order,

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but because almost all of its products have been registered in the company's catalog, most of the customer orders are repetitive orders. so the company continues to carry out production activities without waiting for requests to come.

The problem that occurs in the company is that it often happens that the packaging line is not operating due to the delivery of packing cardboard that is delayed. Delays in shipping packing boxes may occur due to various factors, including the large variety of items ordered, the number of orders, production capacity, delivery priorities and other causes faced by vendors. If packing boxes are not available, then the products will just be stacked and are very at risk of being scratched or experiencing other physical damage. If this happens, it will have an impact on rework, so it will definitely cause additional costs for the company. Another impact due to the unavailability of packing cadres is that the company cannot immediately carry out shipping activities to customers. Of course this will affect consumer assessment and loyalty to the company. So far, the order for packing boxes at PT MAK is done independently or individual orders, even though it is from the same vendor, which can result in waste.

If this condition continues and is allowed, it is feared that the greater the loss experienced by the company. Therefore, the purpose of this study is to determine the optimal packing cardboard inventory control policy that considers multi-item products and probabilistic demand in order to minimize the total inventory costs incurred. The expected benefit of this research is to produce an inventory control policy model that can be used as a reference for other relevant industries in determining the optimal time and number of orders so that stockouts can be avoided and the production process can run smoothly so that ultimately company losses can be minimized. The approach used to solve the problem is a stochastic joint replenishment system.

The organization of this paper is divided into four sections. The first part is introduction, the second part is literature review, the third part is model development, the fourth part is numerical example and the last part is the conclusion of the research results that have been done.

2 Literature Review

Inventory can be defined as stock of goods kept in a warehouse for future scale or using it in common day to day activities. A probabilistic (stochastic) inventory model is an inventory model in which one or more parameters cannot be known with certainty and must be described by a probability distribution. The existence of probabilistic phenomena in the inventory system makes its management more difficult than the deterministic inventory system [1]. A very important consideration in any probabilistic model is the possibility of inventory shortages or stockouts. Demand variation will cause a shortage especially during lead time when the retailer only has a limited amount of goods to cover the demand during lead time and the goods ordered have not been arrived yet [2]. The occurrence of inventory shortages will potentially cause losses to the company, such as stopping the production process, losing profit opportunities, losing the trust of consumers, incurring extra costs for reorders, or even losing consumers. The problem of inventory shortages can be avoided by forming inventory reserves or better known as safety stock. However, this will result in increased inventory costs, namely storage costs for inventory reserves. The greater the inventory reserve, the greater the storage costs. The main concern in probabilistic models is the analysis of inventory behavior during lead time. Based on that situation, there are three possibilities that can happen for the probabilistic inventory model. The first one is when demand during lead time is constant but the lead time itself varies. The second is when lead time is constant but demand during lead time is not, and the last possibility is when both lead time and demand during lead time are variable [2]. Therefore, inventory control must be carried out in such a way as to serve the needs of goods appropriately and at a low cost.

To determine and ensure the availability of the right inventory in the right quantity and time, in the probabilistic inventory system policy there are three decision variables to be determined, namely determining the size of the economic ordering lot, determining the reorder point, and determining the safety stock. Probabilistic inventory control methods are classified into 2 basic methods, namely the continuous review model and the periodic review model.

Periodic review model is an optimization model for determining time-based inventory control policies, namely by reviewing physical inventory periodically at certain time intervals equal to ordering as many orders as the maximum inventory level. The policy to be decided and the solution to be sought with the periodic review model is not much different from the continuous review model, which answers questions related to how many items to order, when to order, and how much safety reserve is needed. The difference is that the periodic review model determines the time period between orders (T) first, which is assumed to be fixed between cycles, and then can determine the lot size of the goods to be ordered.

Stochastic joint replenishment order is one of the developments of the periodic review model developed by [3]. Stochastic joint replenishment order is a model of ordering several items simultaneously to the same vendor where item demand is probabilistic.

Many researchers have studied the deterministic joint replenishment problem. To mention a few, Brown suggested a simple heuristic procedure, and Goyal [4] proposed a more systematic but lengthy procedure which results in the optimal solution. Silver achieved near optimal results with a simple procedure which was later modified by Goyal and Belton [5] and Kaspi and Rosenblatt. Although several papers have analyzed the coordinated replenishment policies, the optimal coordinated replenishment policy class for multi-product inventory systems is still an open question [6], and [7].

3 Methodology

This research was conducted at PT MAK which is located in Kalasan, Jarakan, Tirtomartani, Sleman, Yogyakarta. This research will focus on carton packaging. The data needed in this study include data on the types of raw materials needed, demand data, supplier data, lead times, data relating to storage and ordering of goods, and other data.

The approach used in this research is the stochastic joint replenishment order. This method is an optimization model to determine inventory control policies with probabilistic demand.

The parameter notations used in this study are as follows:

TC : expected total costs per period
OC : expected order cost per period
HC : expected storage cost per period
SC : expected shortage cost per period
π_i : shortage cost per unit
P_i : price of item i per unit
I : fraction of storage cost per unit per period
a : order cost per one-time order
T : cycle time (interval between orders)
D_i : average demand during one unit of time
L : lead time
R_i : *inventory maximum level* item i (unit)
N : stock out expectation (unit)
ss_i : *safety stock* (unit)
x : the demand during $(T + L)$ with probability density function $f(x)$
σ : standard deviation of demand during one unit of time
z : multiplier of σ (safety stock factor)
k : integer multiplier constant

The assumptions used in this research are:

1) Demand is stochastic and normally distributed.
2) The time interval between orders is fixed
3) There is no fluctuation in costs.
4) If a shortage occurs, It's will backorder to cover it.
5) All items are ordered simultaneously from the same source

The objective function of this research is to minimize expected total cost of inventory per period (TC), it consists of expected cost of ordering per period (OC), expected cost of holding per period (HC), and expected cost of shortage of inventory per period (SC). The decision variables are interval between orders (T), *inventory maximum level* (R_i), and safety stock (ss_i). The problem solving method in this paper uses the approach of [3].

The total expected cost per period is given by

$$\begin{aligned} TC &= OC + HC + SC \\ TC &= \tfrac{a}{T} + \tfrac{DTh}{2} + hz_\alpha\sigma\sqrt{T+L} + \tfrac{B}{T}\int_R^\infty [x-R]f(x)dx \end{aligned} \quad (1)$$

That is:

$$\begin{aligned} R &= D(T+L) + z_\alpha\sigma\sqrt{T+L} \\ x &= D_{T+L} \end{aligned} \quad (2)$$

$$\begin{aligned} z_\alpha &= \tfrac{R-D(T+L)}{\sigma\sqrt{T+L}} \\ ss &= z_\alpha\sigma\sqrt{(\mathrm{T}+\mathrm{L})} \end{aligned} \quad (3)$$

$$\overline{S}(x) = \int_R^\infty [x - R]f(x)dx$$
$$\overline{S}(x) = \sigma\sqrt{T+L}\big[f(z_\alpha) - (z_\alpha)\psi(Z\alpha)\big] \tag{4}$$

$$f(z_\alpha) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}$$
$$\psi(z_\alpha) = \frac{1}{\sqrt{2\pi}}\int_{z_\alpha}^{\infty} e^{-\frac{1}{2}z^2}dz = 1 - F(z) \tag{5}$$

Then Eq. (1) will become

$$TC = \frac{a}{T} + \frac{DTh}{2} + hz_\alpha\sigma\sqrt{T+L} + \frac{B}{T}\Big[\sigma\sqrt{T+L}(f(z_\alpha) - (z_\alpha)[1 - F(z_\alpha)])\Big] \tag{6}$$

To find probability of stockout at buyer**,** we can find z optimal, with to take the derivatives Eqs. (6) with respect to z and set them to zero.

$$\frac{\partial TC}{\partial z} = 0$$

$$\sigma h\sqrt{T+L} + \frac{B}{T}\sigma\sqrt{T+L}\left[\frac{df(z)}{dz} - [1 - F(z_\alpha)] + z_\alpha f(z_\alpha)\right] = 0$$

Given that demand is normally distributed $\frac{df(z)}{dz} = -zf(z)$ and we get probability of stockout:

$$1 - F[z(T)] = \alpha = \frac{h}{B}T \tag{7}$$

To ensure that the total cost is minimal, the second derivative of Eq. (6) needs to be such that it is more than zero.

$$\frac{\partial^2\text{TC}\,(T, z)}{\partial^2 z} = \frac{B}{T}\,\sigma\sqrt{T+L}f(\text{z}) > 0$$

By substituting Eq. (7) into Eq. (6), we get

$$\widehat{TC} = \frac{a}{T} + \frac{DTh}{2} + \frac{B}{T}\sigma\sqrt{T+L}f[z(T)] \tag{8}$$

In order to obtain an expression which is easy to solve, maka $\sqrt{T+L}f[z(T)]$ will be approximated using the first three terms of a Taylor series expansion [3].

Taking the derivative of both sides of Eqs. (7) with respect to T,

$$-\frac{\partial \text{F}[z(T)]}{\partial z(T)} \cdot \frac{\partial[z(T)]}{\partial \text{T}} = \frac{h}{B}$$

$$\frac{\partial \text{z}}{\partial \text{T}} = -\frac{h}{Bf[z(T)]}$$

Therefore,

$$\frac{\partial z(T)}{\partial \text{T}} = \frac{\partial f[z(T)]}{\partial \text{z(T)}} \cdot \frac{\partial \text{z(T)}}{\partial \text{T}} = -\text{z(T)}f[z(T)]\left\{-\frac{h}{Bf[z(T)]}\right\} = \text{z(T)} \cdot \frac{h}{B} \tag{9}$$

The first two derivatives of $\sqrt{T+L}f[z(T)]$ are

$$\left[\sqrt{T+L}f[z(T)]\right]' = \frac{f[z(T)]}{2\sqrt{T+L}} + \sqrt{T+L}\cdot \mathrm{z(T)}\frac{h}{B}$$

$$\left[\sqrt{T+L}f[z(T)]\right]'' = \frac{\mathrm{z(T)}}{\sqrt{T+L}}\frac{\mathrm{h}}{\mathrm{B}} - \frac{f[z(T)]}{4(T+L)^{\frac{3}{2}}} - \frac{\sqrt{T+L}}{f[z(T)]}\frac{h^2}{B^2}$$

Evaluation $\sqrt{T+L}f[z(T)]$ at $T_{det} = \sqrt{\frac{2a}{Dh}}$ which is a deterministic solution model, then obtained

$$\sqrt{T+L}f[z(T)] \approx b + b'(T - T_{det}) + \frac{b''(T - T_{det})^2}{2} \tag{10}$$

That is,

$$b = \sqrt{T_{det}+L}f[z(T_{det})]$$

$$b' = \frac{f[z(T_{det})]}{2\sqrt{T_{det}+L}} + z(T_{det})\frac{h}{B}\sqrt{T_{det}+L}$$

$$b'' = \frac{z(T_{det})}{\sqrt{T_{det}+L}}\frac{h}{B} - \frac{f[z(T_{det})]}{4(T_{det}+L)^{\frac{3}{2}}} - \frac{\sqrt{T_{det}+L}}{f[z(T_{det})]}\frac{h^2}{B^2}$$

Substituting Eq. (10) into Eq. (8), the equation will be obtained

$$TC(T) = \frac{u}{T} + \frac{vT}{2} + w \tag{11}$$

That is,

$$\begin{aligned} u &= a + B\sigma(b - b'T_{det} + \tfrac{b''T_{det}^2}{2}) \\ v &= Dh + B\sigma b'' \\ w &= B\sigma\left(b' - b''T_{det}\right) \end{aligned}$$

The optimal T value, T^*, can be determined by deriving Eq. (11) with respect to T and equaling 0

$$T^* = \sqrt{\frac{2u}{v}} \tag{12}$$

Furthermore, T^* can be substituted into Eq. (7) to determine the value of z and inputted into Eq. (11), then it will be obtained

$$TC^* = \sqrt{2uv} + w \tag{13}$$

The above equation is a periodic review model for single item cases. As for multi-items where the purchase/order of all items is carried out simultaneously, the problem solving approach uses the Joint Replenishment Order (JRO). So that the total cost equation in Eq. (8) is adjusted by giving index i which indicates the-i item in each variable $D_i, k_i, h_i, a_i, z_i, \sigma_i, u_i, v_i, w_i$. Then the total cost function for n products can be written as

$$\widehat{TC} = \frac{A}{T} + \frac{\sum_{i=1}^{n} \frac{a_i}{k_i}}{T} + \sum_{i=1}^{n}\left[\frac{D_i k_i T h_i}{2} + \frac{B_i}{k_i T}\sigma_i\sqrt{k_i T + L} f[z_i(k_i T)]\right] \tag{14}$$

Can be simplified to

$$\widehat{TC} = \frac{A}{T} + \frac{\sum_{i=1}^{n} \frac{u_i}{k_i}}{T} + \frac{T}{2}\sum_{i=1}^{n} k_i v_i + \sum_{i=1}^{n} w_i \tag{15}$$

The optimal T value, T^*, is determined by deriving Eq. (15) with respect to T and equaling 0

$$T^* = \sqrt{\frac{2\left(A + \sum_{i=1}^{n} \frac{u_i}{k_i}\right)}{\sum_{i=1}^{n} k_i v_i}} \tag{16}$$

The heuristic procedure to determine the interval between orders (T^*), *inventory maximum level* (R_i), and safety stock (ss_i) with the JRO approach can be done with the following steps:

1. Determine the value $T_{det} = \sqrt{\frac{2a}{Dh}}$
2. Calculating the value of α using the Eq. (7)
3. Find the values of $z_\alpha, f[z(T_{det})]$, and $\psi(z_\alpha)$ using Table 5–3 (Tersine, 1994, pp: 220–221) or Table B (Bahagia, 2006, pp: 256–257).
4. Find the values of b, b', and b''
5. Calculating the value u_i, v_i, and w_i
6. Determine the optimal ordering interval, T^* with JRO with the following steps:

   a) Calculate the ordering interval for each item i with the Eq. (12)

   $$T_i = \sqrt{\frac{2u_i}{v_i}}$$

   b) Identifying the smallest to largest T_i. Value based on the calculation in step 1), and the smallest T_i value is denoted as item 1 with a value of $k_i = 1$.
   c) rt the value of T_s' obtained from the smallest T_j' value calculated for each item using the equation

   $$T_j' = \sqrt{\frac{2\left(A + \sum_{i=1}^{j} u_i\right)}{\sum_{i=1}^{j} v_i}} < T_{j+1}$$

d) Determine the value of k *item* by trial-error in such a way that if $k = q$, then a value of k is obtained that satisfies the equation

$$\sqrt{(q-1)q} \leq \frac{T_i}{T_s'} \leq \sqrt{(q+1)q}$$
$$i = s+1, \ldots .n$$

e) Calculate the optimal T value (T^*) by using the equation

$$T^* = \sqrt{\frac{2\left(A + \sum_{i=1}^{n} \frac{u_i}{k_i}\right)}{\sum_{i=1}^{n} k_i v_i}}$$

f) Calculating the total combined cost $\widehat{TC}$ using the equation

$$\widehat{TC} = \frac{A}{T} + \frac{\sum_{i=1}^{n} \frac{u_i}{k_i}}{T} + \frac{T}{2}\sum_{i=1}^{n} k_i v_i + \sum_{i=1}^{n} w_i$$

g) Determining the maximum inventory level R^* at $T = T^*$ using the equation

$$R_i^* = D_i\left(T^* + L\right) + z_{\alpha i}\sigma_i\sqrt{T^* + L}$$

h) Determine the stockout expectation for each product, using the equation

$$\overline{S}_i(x) = \sigma_i\sqrt{T^* + L}\left[f(z_{\alpha i}) - (z_{\alpha i})\psi(z_{\alpha i})\right]$$

i) Determine the safety stock for each product, using the equation

$$ss_{\mathrm{i}} = z_{\alpha i}\sigma_i\sqrt{T^* + L}$$

7. Comparing the calculation results with the proposed model and the calculation with the company's method

4 Conclusion

The production system used by PT MAK is make to order, however, because most of the product types have repetitive orders, the company continues to carry out production activities without considering consumer demand or taking a make to stock system approach.

The problem that occurs in the company is that the packaging line is often stopped due to unavailable packing boxes.

This research is motivated by a condition where there are obstacles in the finished product packaging process due to the unavailability of raw materials in the warehouse. This scarcity of goods occurs due to the difficulty of determining the inventory needs that must be prepared due to many unpredictable factors. In production activities, the unavailability of materials will greatly hamper production activities. From this, this research was conducted to obtain the best policy in fulfilling inventory.

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