Uncovering Coal Price Volatility: Comparing Parameter Estimation Approaches for Mean Reversion Modeling

Muhammad Adam Gana, Eko Wicaksono, Shofa Rijalul Haq, and Aldin Ardian(✉)

Mining Engineering Department, Universitas Pembangunan Nasional "Veteran" Yogyakarta, Jalan Padjajaran (Ring Road Utara) 104, Yogyakarta 55283, Indonesia
{shofa.haq,aldin.ardian}@upnyk.ac.id

Abstract. This scientific article examines the modeling of coal price volatility using a mean reversion model (MRM) and compares the performance of different parameter estimation approaches. The aim of the study is to identify which parameter estimation approach is best suited for modeling the volatility of coal prices. The study uses annual discrete time data from 2022 to 2031 to estimate the MRM parameters using three approaches: linear regression method (LRM), least square method (LSM), and moment method (MM). The results show that the MM approach produces the highest volatility, while the LRM has the lowest reversion value but higher volatility than the LSM. The findings suggest that the MM approach may be more suitable for modeling coal price volatility due to its ability to capture higher levels of volatility. These results have implications for understanding the dynamics of the coal market and can inform decisions related to pricing, risk management, and investment in the coal industry.

Keywords: coal price · mean reversion model · parameter estimation · simulation · stochastic process

1 Introduction

Volatility is a well-known phenomenon in mining industries. The volatility might happen due to the price, cost, and even grade uncertainty. Ardian and Kumral [1] show that price is a significant variable that affect the project valuation followed by cost, average grade, recovery, and interest rate. Given that the price volatility cannot be accurately forecasted, a simulation through stochastic process can be a solution. Stochastic processes has some properties such as (a) treat a random variable as a random process where only the latest observation determine the future outcomes or it can also be said as memoryless (Markov property), (b) exhibits non deterministic behavior, (c) the random variable evolves overtime at least in part random, and (d) it has discrete time increments [2, 3].

Stochastic process is considered as an alternative mathematical modeling in the uncertainty and risk analysis. Compared to statistical approach where the historical data is essential, a stochastic process does not necessarily need it. Somehow, historical data can still be benefited for parameter estimation for instance. Comparing to statistical

H. Ku et al. (Eds.): ICARSE 2022, ASSEHR 748, pp. 56–64, 2023.
https://doi.org/10.2991/978-2-38476-048-0_7

approaches where the model tries to minimize the error between model and the historical data, the stochastic process and its simulation are commonly used to capture the volatility. Capturing volatility is an important approach to model the nature of the mining industries. Thus, based on some literatures, stochastic process is suitable for price modeling, especially commodity price or stock price [2].

In this research, a stochastic process called mean reversion model (MRM) that is introduced by Uhlenbeck and Ornstein [4] was applied to model coal prices. The MRM assumes a price, at some point, reverts to the expected value at a specific and static speed and volatility. Having a reversion characteristic, the MRM is beneficial for the current situation where the coal price shot up and was expected to return to its mean or expected value. In addition, MRM has been implemented to model some commodities prices such as gold prices [5], oil and natural gas prices [6], primary aluminum, copper, nickel, lead, tin, and zinc prices [7, 8].

There is also an application of MRM for coal price modeling in China, but the paper more focuses on the parameter calibration through hybrid estimation method [9]. Deng [10] in his thesis examined the MRM with additional jump diffusion to the energy commodity prices including coal spot price. However, one of the important things in the MRM is the parameter estimation process. Different method might obtain different result. Thus, three methods to estimate the MRM parameter were observed in this paper. Those three methods were Linear Regression Method (LRM), Least Square Method (LSM), and Moment Method (MM). Further explanation of those three parameters can be found in the following section.

This paper consists of four sections. The Sect. 1 explained the natural uncertainty in the coal mining industry and introduce the MRM as an alternative model that can be used to overcome the problem. In Sect. 2, the methods used to model the coal prices and the parameter estimation approaches were presented. The results then discussed in the subsequent section. Finally, the conclusion can be drawn in the Sect. 4.

2 Methods

Mean Reversion Model

The stochastic process model used in this study was MRM. There are three important parameters in the model, the κ, μ, and σ. The MRM model is shown in the Eq. (1).

$$dx_t = \kappa(\mu - x_t)dt + \sigma\sqrt{x_t}\varepsilon\sqrt{\Delta t} \tag{1}$$

where κ is the mean reversion rate ($\kappa \geq 0$), x_t is the initial value, μ is the mean value, σ is the standard deviation ($\sigma > 0$), and $\varepsilon\sqrt{\Delta t}$ is the random component by standard Brownian motion [11]. The MRM model consists of two components, the drift term ($\kappa(\mu - x_t)dt$) and the random component ($\sigma\sqrt{x_t}\varepsilon\sqrt{\Delta t}$). The drift term brings the simulation to a specific point at a specific rate, the random component fluctuates the simulation arbitrarily with a specific volatility.

Even though the stochastic process models the random variable randomly, a relatively subjective adjustment is still needed. Therefore, a critical step using the model is the parameters estimation. In this study, the MRM parameters were estimated in three ways, namely by linear regression methods, least square methods, and moment methods. Then, those three parameters were compared and studied.

Linear Regression Method (LRM)
The MRM parameters can be estimated through LRM statistical model. The model is presented in Eq. (2) where y is the dependent variable, α is the intercept, β is the slope, x is the dependent variable, and ε is the error.

$$y = \alpha + \beta x + \varepsilon_t \tag{2}$$

Then, the Mean reversion rate (κ), mean (μ), and standard deviation (σ) parameters are determined by applying Eqs. (3), (4), and (5) where RSE is the return standard error estimated by Eq. (6) [11].

$$\kappa = -\beta \tag{3}$$

$$\mu = \frac{\alpha}{\kappa} \tag{4}$$

$$\sigma = \frac{RSE}{\mu} \tag{5}$$

$$RSE = \sqrt{\frac{1}{(n-2)}\left[\sum (y-\bar{y})^2 - \frac{\left[\sum (x-\bar{x})(y-\bar{y})\right]^2}{\sum (x-\bar{x})^2}\right]} \tag{6}$$

Least Square Method (LSM)
The LSM models the trend of the random variable by minimizing the squared error that includes time series analysis. The LSM might assume more than one variable affecting the price, so it can be referred to as Multiple Linear Regression. Parameter estimation by LSM is similar with the LRM, where μ and σ are obtained through Eqs. (4) and (5). The β is then estimated by using the F-Test function which aims to determine the influence of all free variables together on all the independent variables [12]. The F-test formula can be seen in Eq. (7).

$$F = \frac{SSR_R - SSR_{UR}/q}{SSR_{UR}/(n-k-1)} \sim F_{q,\ n-k-1} \tag{7}$$

where the F is the F-test value, SSR_R is the restricted quadratic squared error (a model that assumes the coefficient of the independent variable = 0), SSR_{UR} is the unrestricted quadratic squared error (a model that assumes the coefficient of the independent variable as it is), q is the number of the restricted parameter, n is the sample size, k is the number of independent variable, and $n - k - 1$ is the degree of freedom [13].

Moment Method (MM)
The Moment Method (MM) is basically a linear regression model that can be modeled through Eq. (2), but with a further analysis of the time series random variable return [14].

The time series random variable in this case is the commodity price. The idea applying MM is because the price changes are volatile, so it is necessary to know the return value of the commodity price. The price return formula is defined in Eq. (8) [14].

$$R_t = \ln\left(\frac{P_{t+1}}{P_t}\right) \tag{8}$$

where,

R_t: commodity price return at time t

P_t: commodity price at time t

P_{t+1}: commodity price at time $t + 1$

Furthermore, the κ is estimated through Eq. (9) [15].

$$\kappa = -\frac{\ln(\varphi)}{dt} \tag{9}$$

The φ was obtained through Eq. (10).

$$\varphi = \frac{(\mathrm{Nx}\sum_{t=1}^{N} P_{t-1}P_t) - (\sum_{t=1}^{N} P_{t-1}\mathrm{x}\sum_{t=1}^{N} P_t)}{(\mathrm{Nx}\sum_{t=1}^{N} P^2{}_{t-1}) - (\sum_{t=1}^{N} P_{t-1}{}^2)} \tag{10}$$

where,

κ: rate of return on commodity prices

d_t: time lapse

P_{t-1}: commodity prices at time t-1

P_t: commodity prices at time t

N: amount of commodity price data

Then, the μ is defined in Eq. (11) [16].

$$\mu = \frac{1}{N}\sum_{t=1}^{N} R_t \tag{11}$$

where,

μ: drift return commodity prices

N: amount of commodity price return data

R_t: commodity price return data at time t

Finally, where the volatility measure the degree of the uncertainty regarding the future movement of commodity prices, the formula of volatility is defined in Eq. (12) [16].

$$\sigma = \sqrt{\frac{1}{(N-1)}\sum_{t=1}^{N} (R_t - R)^2} \tag{12}$$

where,

σ: commodity price return volatility

N: amount of commodity price return data

R_t: commodity price return data at time t

R: data on average commodity price returns

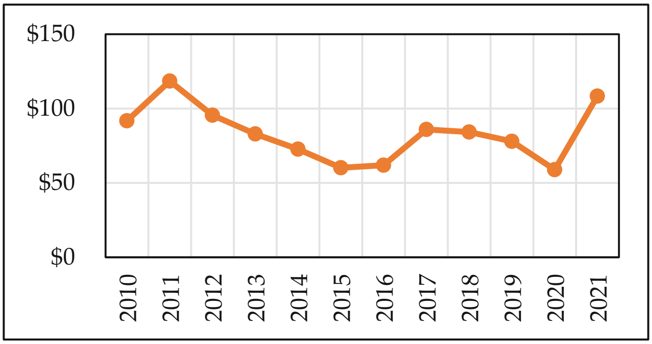

Fig. 1. Annual HBA from 2010 to 2021

3 Results

Data Collection

The data used in this research was the monthly Reference Coal Price (HBA) where it was collected from the official website of Ministry of Energy and Mineral Resources of Indonesia. The HBAs collected were from the period from January 2010 to December 2021 [17]. The monthly HBA was then modified to annual price by simply taking the average price of the 12-monthly price that can be seen in Fig. 1.

Figure 1 shows that the HBAs fluctuated and tended to decrease until 2020, after that experienced a significant improvement in 2021. These monthly data was used to estimate the parameters of the MRM.

Linear Regression Model (LRM)

In the LRM, the dependent variable (y) was the difference of HBA ($HBA_t - HBA_{t-1}$) and the independent variable (x) was the HBA_{t-1} [18]. The annual HBA (average), changes (y), , and the previous HBA (x) are presented in Table 1.

Based on the Eqs. (2) and (6), the values of α, β, and RSE were obtained 59.04, -0.71, and 19.67, respectively. Furthermore, the three parameters of the MRM can be estimated using Eqs. (3), (4), and (5). As a result, the values of $\kappa = 0.71$, $\mu = 83.01$, and $\sigma = 0.23$.

Least Square Method (LSM)

In LSM, the estimation of μ and σ are similar in the LRM, but the β. Thus, the $\alpha = 59.04$ and $RSE = 19.67$. The value of $\beta = -0.46$ was obtained through F-test. Furthermore, the parameters of the MRM can be estimated using Eqs. (3), (4), and (5). As a result, the values of $\kappa = 0.46$, $\mu = 128.03$, and $\sigma = 0.15$.

Moment Method (MM)

Based on Eq. (8) the prices returns were obtained as presented in Table 2.

Table 1. Annual, changes, and the previous HBA from 2010 to 2021

Year	Avrg. HBA	Changes	Previous
2010	$ 91.74	–	–
2011	$ 118.40	$ 26.66	$ 91.74
2012	$ 95.48	$ –22.92	$ 118.40
2013	$ 82.92	$ –12.56	$ 95.48
2014	$ 72.62	$ –10.30	$ 82.92
2015	$ 60.13	$ –12.49	$ 72.62
2016	$ 61.84	$ 1.71	$ 60.13
2017	$ 85.92	$ 24.08	$ 61.84
2018	$ 84.21	$ –1.71	$ 85.92
2019	$ 77.89	$ –6.32	$ 84.21
2020	$ 58.80	$ –19.09	$ 77.89
2021	$ 108.29	$ 49.49	$ 58.80

Table 2. Annual HBA Returns from 2010 to 2021

Year	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Returns	–	0.26	−0.22	−0.14	−0.13	−0.19	0.03	0.33	−0.02	−0.08	−0.28	0.61

Furthermore, before obtaining the κ by Eq. (9), the φ was first calculated based on the Eqs. (10). Accordingly, the φ and κ were obtained 0.01 and 0.47, respectively. Then, the μ based on the Eq. (11) was 124.17, and the σ based on the Eq. (12) was 0.27.

HBA Simulation Through the MRM with LRM Estimator

The MRM was then applied for the HBA simulation for the next 10 years (from 2022 to 2031) with the number of simulations as many as 1,000 times. The MRM by the LRM estimator can be seen in Fig. 2.

Figure 2 shows that the MRM by LRM estimator was able to capture the price volatility from $74.12 to $91.53 from 2022 to 2031. The LRM generated a uniform trend over time, which ranges from $76.30 to $91.36 in 2022 ($15.06 difference), then in 2031, it ranges from $74.69 to $90.13 ($15.44 difference).

HBA Simulation Through the MRM with LSM Estimator

Secondly, the MRM by LSM estimator can be seen in Fig. 3.

Figure 3 shows that the MRM by LSM estimator was able to capture the price volatility from $99.76 to $134.88 from 2022 to 2031. The LSM generated a trend that tends

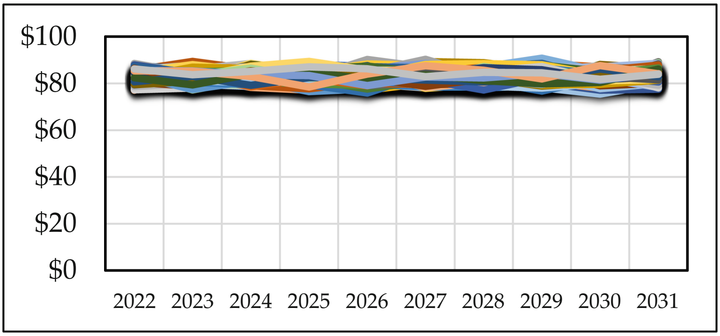

Fig. 2. HBA Simulation through MRM by LRM estimator

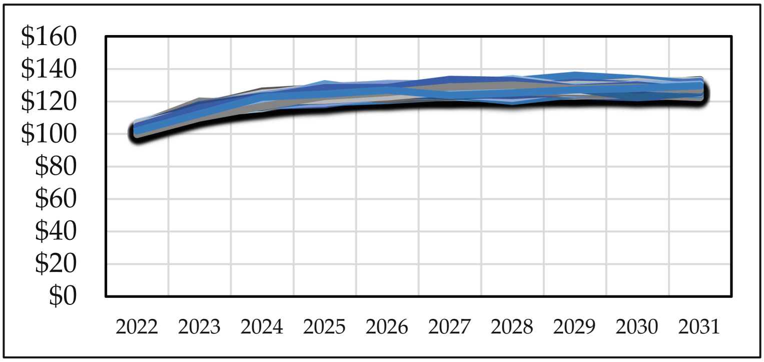

Fig. 3. HBA Simulation through MRM by LSM estimator

to increase over time, which ranges from $99.76 to $108.45 in 2022 ($8.69 difference), then in 2031, it ranges from $122.29 to $134.23 ($11.94 difference).

HBA Simulation Through the MRM with MM Estimator
Thirdly, the MRM by LSM estimator can be seen in Fig. 4.

Figure 4 shows that MRM by MM estimator was able to capture the price volatility from $94.97 to $140.42 from 2022 to 2031. The MM generated a trend that tends to increase over time, which ranges from $94.97 to $111.88 in 2022 ($16.91 difference), then in 2031, it ranges from $116.35 to $138.45 ($18.10 difference).

Based on the three simulations through MRM by three different estimators, the MM shows more volatility than the others. The LRM, on the other hand, has the lowest μ and higher volatility than the LSM.

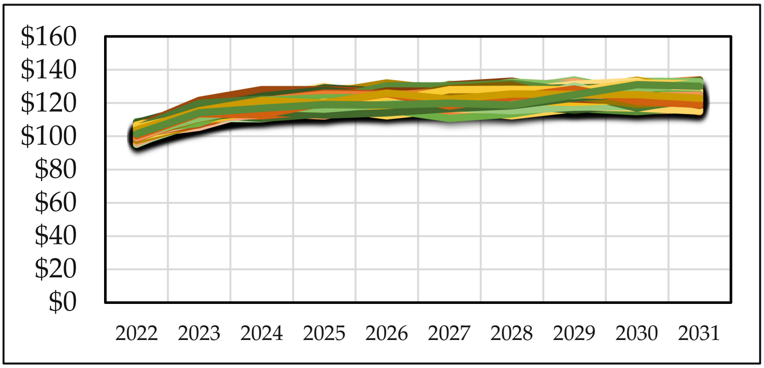

Fig. 4. HBA Simulation through MRM by MM estimator

4 Conclusion

In this research, the MRM was implemented to model the HBA stochastically. The three crucial parameters, μ, σ, and κ, were estimated through three different statistical estimators which were LRM, LSM, and MM. As a result, the MM exhibited more volatility than the others. Comparing the volatility, the MM used the price returns, not the RSE. The RSE tends to be more robust due to the average value of the random variables.

Acknowledgment. The authors would like to thank to the Institute of Research and Community Services (LPPM) Universitas Pembangunan Nasional "Veteran" Yogyakarta for providing financial support under Internal Grant Awards Number: B/129/UN.62/PT/V/2022.

References

1. Ardian, A. and M. Kumral, *Enhancing mine risk assessment through more accurate reproduction of correlations and interactions between uncertain variables* Mineral Economics, 2020.
2. Dixit, A.K. and R.S. Pindyck, *Investment under uncertainty*. 1994, Princeton, New Jersey: Princeton university press.
3. Hassler, U., *Stochastic Processes and Calculus: An Elementary Introduction with Applications*. 2016, Switzerland: Springer.
4. Uhlenbeck, G.E. and L.S. Ornstein, *On the theory of Brownian motion*. Phys Rev, 1930. **36**.
5. Haque, M.A., E. Topal, and E. Lilford, *Estimation of mining project values through real option valuation using a combination of hedging strategy and a mean reversion commodity price*. Natural Resources Research, 2016. **25**(4): p. 459–471.
6. Geman, H., *Mean reversion versus random walk in oil and natural gas prices*, in *Advances in Mathematical finance*. 2007, Springer. p. 219–228.
7. Wang, C., *Expected mean reversion in base metal prices: Evidence from the LME*. 인하대학교 정석물류통상연구원 학술대회, 2009: p. 54–59.
8. Ardian, A. and M. Kumral, *Incorporating stochastic correlations into mining project evaluation using the Jacobi process*. Resources Policy, 2020. **65**: p. 101558.

9. Sun, Q., W. Xu, and W. Xiao, *An empirical estimation for mean-reverting coal prices with long memory.* Economic Modelling, 2013. **33**: p. 174–181.
10. Deng, S., *Stochastic models of energy commodity prices and their applications: Mean-reversion with jumps and spikes*. 2000, University of California Energy Institute Berkeley.
11. Iglesias, E.M., *Testing of the mean reversion parameter in continuous time models.* Economics Letters, 2014. **122**(2): p. 187–189.
12. Ghozali, I., *Multivariate analysis application with IBM SPSS 23 program.* Semarang: Diponegoro University Publishing Agency, 2016. **4**: p. 352.
13. Hristu-Varsakelis, D. and C. Kyrtsou, *Testing for Granger causality in the presence of chaotic dynamics.* Brussels economic review, 2010. **53**(2): p. 323–327.
14. Tsay, R.S., *Analysis of financial time series*. 2005: John wiley & sons.
15. Smith, W., *On the simulation and estimation of the mean-reverting Ornstein-Uhlenbeck process.* Commodities Markets and Modelling, 2010.
16. Fouque, J.-P., G. Papanicolaou, and K.R. Sircar, *Mean-reverting stochastic volatility.* International Journal of theoretical and applied finance, 2000. **3**(01): p. 101–142.
17. Kementerian-ESDM, *Harga batubara acuan.* 2022.
18. Blanco, C. and D. Soronow, *Mean reverting processes-energy price processes used for derivatives pricing & risk management.* Commodities Now, 2001. **5**(2): p. 68–72.