On Local (*a, d*)-Edge Antimagic Coloring of Some Graphs

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Abstract. All graphs considered in this paper are simple, finite and connected graph. Let $G(V, E)$ be a graph with the vertex set V and the edge set E, and let w be the edge weight of graph G. with $|V(G)| = m$ and $|E(G)| = n$. A labeling of a graph G is a bijection f from $V(G)$ to the set $\{1, 2, ..., |V(G)|\}$. The bijection f is called an edge antimagic labeling of graph if for any two vertex u and v in path $x-y$,$u \neq v$, where $\{w(uv) : w(uv) = f(u) + f(v), uv \in E\}$, are distinct. Any local edge antimagic labeling induces a proper edge coloring of G where the edge uv is assigned the color $w(uv)$. The local edge antimagic coloring of graph is said to be a local (a, d)-edge antimagic coloring of G if the set of their edge colors form an arithmetic sequence with initial value a and different d. The local (a, d)-edge antimagic chromatic number $\chi_{la}(a, d)(G)$ is the minimum number of colors needed to color G such that a graph G admits the local (a, d)-edge antimagic coloring. Furthermore, In this paper, we will obtain the lower and upper bound of $\chi_{la}(a, d)(G)$. The results of this research are the exact value of the local (a, d)-edge antimagic chromatic number of some graphs.

Keywords: local (a, d) antimagic coloring · edge antimagic coloring · spacial graph

1 Introduction

$G = (V, E)$ is a graph where $V(G)$ is the set of vertices and $E(G)$ is the set of edges, the definition of graph can be see in [12,13]. With $|V(G)| = m$ is the number of vertices of G and $|E(G)| = n$ is the number of sides of G. labeling graph is a bijection mapping that assigns natural numbers to the vertices and edges of graph G. labeling is called vertex labeling or edge labeling depending on the domain located on the vertex or edge, if the labeling domain is a vertex

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then it is called vertex labeling, if the labeling domain is an edge then it is called side labeling. Labeling is called antimagic if all the side weights have different values [14].

The concept of antimagic graph labeling can be seen in [10,11]. The local antimagic labeling on graph G with $|V|$ vertices and $|E|$ edges are defined to $f : E \to \{1, 2, ..., k\}$ so that the weights of any two adjacent vertices are different, that is, $w(u) \neq w(v)$ where $w(u) = \sum_{e \epsilon E(u)} f(e)$ and $E(u)$ are the sets of edges that incident to u. Therefore, any local antimagic labeling induces a proper vertex coloring of G where vertex u is assigned the color $w(u)$. The local antimagic labeling have been studied by [5–8,15]. The local antimagic chromatic number is denoted by $\chi_{la}(G)$, the local antimagic chromatic number is the minimum number of colors taken from all stains induced by local antimagic labeling of G [15].

Furthermore, introduces the concept of local edge antimagic coloring of graphs. Defined as a bijection $f : V(G) \to \{1, 2, 3, ..., |V(G)|\}$, is called local edge antimagic labeling if any two adjaction edges e_1 and $e_2, w(e1) \neq w(e_2)$, where for $e = uv\epsilon G, w(e) = f(u) + f(v)$. Therefore, local edge antimagic labeling induces a proper edge coloring of G if any edge is assigned the color w(e).The color of each edge $e = uv$ is assigned by (e) which is determined by the sum of the two labels and the sum of vertices $f(u)$ and $f(v)$. The local edge antimagica chromatic number, denoted by $\chi_{lea}(G)$ is the minimum number of colors that are taken over by all staining induced by the local edge antimagic labeling of graph G. The concept of local edge antimagic coloring of graphs can be seen in [1,2,4].

A (a, d)-edge antimagic and super (a, d)-edge total antimagic we refer all definitions to [9]. Local (a, d)-antimagic coloring of a graph a bijection $f : V(G) \to \{1, 2, 3, ..., |V|\}$ is called an edge antimagic labeling of graph if the element of the edge weight set $w(uv) = f(u) + f(v)$, where $uv\epsilon E(G)$, are distinct. Edge antimagic labeling induces a local edge antimagic coloring of G if each edge of G is colored with a weight of $w(e)$. The antimagic coloring of a graph is said to be a local (a, d)-edge antimagic coloring of G if the set of edge colors forms an arithmetic sequence with initial values of a and different d. Furthermore, the local (a, d)-antimagic chromatic number $\chi_{le}(a, d)(G)$ is the minimum number of colors needed to color G such that a graph G admits the local (a, d)-antimagic coloring [3].

Observation 1.1. [1]. For any graph G, $\chi_{lea}(G) \geq \chi(G)$, where $\chi(G)$ is a chromatic number of vertex coloring of G.

Observation 1.2. [3]. For any graph G, $\chi_{le}(a, d)(G) \geq \chi_{lea}(G) \geq \chi(G)$

Observation 1.3. [1]. For any graph G, $\chi_{lea}(G) \geq \Delta(G)$, where $\Delta(G)$ is maximum degrees of G

2 Main Result

In this paper, we will studied the existence of local (a,d)-edge antimagic coloring of a some graph, and determine the chromatic number of local (a,d)- edge antimagic coloring of some graph include broom graph $Br_{n,m}$, book graph B_n, firecraker $F_{n,3}$, complete graph Kn and the dragon graph Dg_n. We also analyse the lower bound of the local (a,d)-edge antimagic coloring of the graphs.

Observation 2.1. For any graph G,

$$\chi_{le}(a,d)(G) \geq \chi_{lea}(G) \geq \Delta(G),$$

where $\chi_{lea}(G)$ is a chromatic number of local edge coloring of G and $\Delta(G)$ is maximum degrees of G

Theorem 2.1. For $Br_{n,m}$ be a broom graph with $m \geq 3, n \geq 3$, $\chi_{le(n+1,1)}(Br_{n,m}) = m+1$.

Proof. Let $Br_{n,m}$ be a broom graph with $V(Br_{n,m}) = \{x_t; 1 \leq t \leq m\} \cup \{y_t; 1 \leq t \leq n\}$ and $E(Br_{n,m}) = \{x_t y_1; 1 \leq t \leq n-1\} \cup \{y_t y_{t+1}; 1 \leq t \leq n-1\}$. The cardinality of the vertices set of $|V(Br_{n,m}| = m+n$, and the cardinality of the edges set of $|E(Br_{n,m}| = m+n-1$. The local (a,d)-antimagic chromatic number of $Br_{n,m}$ is $\chi_{le(n+1,1)}(Br_{n,m}) = m+1$.

To prove $\chi_{le(n+1,1)}(Br_{n,m}) = m+1$ first, we will prove that $\Delta(Br_{n,m}) \geq m+1$. Based on Observation 1.3 we have $\chi_{le(a,d)}(Br_{n,m}) \geq \Delta(Br_{n,m})$. In Agustin et al. [1] If $\Delta(G)$ is maximum degrees of G, then we have $\chi_{lea}(G) \geq \Delta(G)$. Based on these results, it can be concluded that $\chi_{le(a,d)}(Br_{n,m}) \geq m+1$. To show $\chi_{le(a,d)}(Br_{n,m}) \leq m+1$, by defining the bijection $f : V(Br_{n,m}) \rightarrow \{1, 2, 3..., |V(Br_{n,m})|\}$

$$f(y_t) = \begin{cases} \frac{t+1}{2}, & for\ t \equiv 1(mod\ 2) \\ n+1-\frac{t}{2}, & for\ t \equiv 0(mod 2) \end{cases}$$

$$f(x_t) = \begin{cases} n+\frac{m+1}{2}+\frac{t+1}{2}-1, & for\ t,\ m \equiv 1(mod\ 2) \\ n+\frac{m+2}{2}+\frac{t+1}{2}-1, & for\ t \equiv 1(mod\ 2) \\ & and\ m \equiv\ 0(mod\ 2) \\ n+\frac{m+2}{2}+\frac{t+1}{2}, & for\ t \equiv 0(mod\ 2) \end{cases}$$

the following is a way to see that f is the local (a,d)-antimagic labeling of $Br_{n,m}$ and the edge weights

$$w(x_t y_t) = \begin{cases} n+\frac{m+1}{2}+\frac{t+1}{2}, & for\ m \equiv 1(mod\ 2) \\ n+\frac{m+2}{2}+\frac{t+1}{2}, & for\ m \equiv 0(mod\ 2) \end{cases}$$

$$w(y_t y_{t+1}) = \begin{cases} n+1, & for\ t \equiv 1(mod\ 2) \\ n+2, & for\ t \equiv 0(mod\ 2) \end{cases}$$

The set of edge weights based on the edge weights obtained is $W = \{n+1, n+2, n+3, ..., n+m+1\}$. We can determine the value of the smallest weight of the edge is $a = n-1$ and $d = 1$, then we will have $\chi_{le(n+1,1)}(Br_{n,m}) \leq m+1$. it can be concluded that $\chi_{le(n+1,1)}(Br_{n,m}) = m+1$ (Fig. 1).

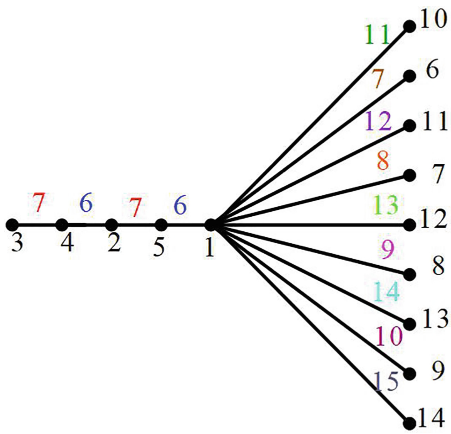

Fig. 1. The local $(a.d)$-edge antimagic coloring of $Br_{11,5}$

Theorem 2.2. For B_n be a Book graph with $n \geq 2$, $for\ n \equiv 1(mod\ 2)$ $\chi_{le(n+3,1)}(B_n) = n+2$, $for\ n \equiv 0(mod\ 2)$ $\chi_{le(3+\frac{3n}{2},1)}(B_n) = n+1$.

Proof. Let B_n be a Book graph with $V(W_n) = \{xy\} \cup \{x_t; 1 \leq t \leq n\} \cup \{y_t; 1 \leq t \leq n\}$ and $E(W_n) = \{yx\} \cup \{xx_t; 1 \leq t \leq n\} \cup \{yy_t; 1 \leq t \leq n\} \cup \{x_t y_t; 1 \leq t \leq n\}$. The cardinality of the vertices set of $|V(B_n| = 2n+2$, and the cardinality of the edges set of $|E(B_n| = 3n+1$. The local (a,d)-antimagic chromatic number of B_n is $\chi_{le(n+3,1)}(B_n) = n+2\ for\ n \equiv 1(mod\ 2)$, $\chi_{le(3+\frac{3n}{2},1)}(B_n) = n+1\ for\ n \equiv 0(mod\ 2)$

Case 1. For $n \equiv 1(mod\ 2)$

To prove $\chi_{le(n+3,1)}(B_n) = n+2$ first, we will prove that $\Delta(B_n) \geq n+1$. Based on Observation 1.3 we have $\chi_{le(a,d)}(B_n) \geq \Delta(B_n)$. in Agustin et al. [1] If $\Delta(G)$ is maximum degrees of G, then we have $\chi_{lea}(G) \geq \Delta(G)$. Based on these results, it can be concluded that $\chi_{le(a,d)}(B_n) \geq n+2$. To show $\chi_{le(a,d)}(B_n) \leq n+2$, by defining the bijection $f : V(B_n) \rightarrow \{1, 2, 3..., |V(B_n)|\}$

$$f(x_t) = t+1$$
$$f(y_t) = 2n+3-t$$
$$f(x) = n+2$$
$$f(y) = 1$$

the following is a way to see that f is the local (a,d)-antimagic labeling of B_n and the edge weights

$w(xy) = n+3$
$w(xx_t) = n+3+t$
$w(yy_t) = 2n+4-t$
$w(x_ty_t) = 2n+4$

The set of edge weights based on the edge weights obtained is $W = \{2n+1, 2n, 2n-1, ..., 2n+4\}$. We can determine the value of the smallest weight of the edge is $a = n+3$ and $d = 1$, then we will have $\chi_{le(n+3,1)}(B_n) \leq n+2$. it can be concluded that $\chi_{le(n+3,1)}(B_n) = n+2 \ for \ n \equiv 1(mod\ 2)$.

Case 2. For $n \equiv 0(mod\ 2)$

To prove $\chi_{le(3+\frac{3n}{2},1)}(B_n) = n+1$ first, we will prove that $\Delta(B_n) \geq n+1$. Based on Observation 1.3 we have $\chi_{le(a,d)}(B_n) \geq \Delta(B_n)$. in Agustin et al. [1] If $\Delta(G)$ is maximum degrees of G, then we have $\chi_{lea}(G) \geq \Delta(G)$. Based on these results, it can be concluded that $\chi_{le(a,d)}(B_n) \geq n+1$. To show $\chi_{le(a,d)}(B_n) \leq n+1$, by defining the bijection $f : V(B_n) \to \{1, 2, 3..., |V(B_n)|\}$.

$$f(x) = 2 + \frac{3n}{2}$$

$$f(y) = 1 + \frac{n}{2}$$

$$f(x_t) = \begin{cases} t, & for\ 1 \leq t \leq \frac{n}{2} \\ 1+t, & for\ \frac{n}{2}+1 \leq t \leq n \end{cases}$$

$$f(y_t) = \begin{cases} 2n+3-t, & for\ 1 \leq t \leq \frac{n}{2} \\ 2n+2-t, & for\ \frac{n}{2}+1 \leq t \leq n \end{cases}$$

the following is a way to see that f is the local (a,d)-antimagic labeling of B_n and the edge weights

$$w(x) = 2n+3$$

$$w(xx_t) = \begin{cases} 2+\frac{3n}{2}+t, & for\ 1 \leq t \leq \frac{n}{2} \\ 3+\frac{3n}{2}+t, & for\ \frac{n}{2}+1 \leq t \leq n \end{cases}$$

$$w(yy_t) = \begin{cases} 4+\frac{5n}{2}-t, & for\ 1 \leq t \leq \frac{n}{2} \\ 3+\frac{5n}{2}-t, & for\ \frac{n}{2}+1 \leq t \leq n \end{cases}$$

$$w(x_tyt) = 2n+3$$

The set of edge weights based on the edge weights obtained is $W = \{\frac{3n}{2}+3, \frac{3n}{2}+4, \frac{3n}{2}+5, ..., 3+\frac{5n}{2}\}$. We can determine the value of the smallest weight of the edge is $a = 3+\frac{3n}{2}$ and $d = 1$, then we will have $\chi_{le(3+\frac{3n}{2},1)}(B_n) \leq n+1$. it can be concluded that $\chi_{le(3+\frac{3n}{2},1)}(B_n) = n+1 \ for \ n \equiv 0(mod\ 2)$ (Figs. 2 and 3).

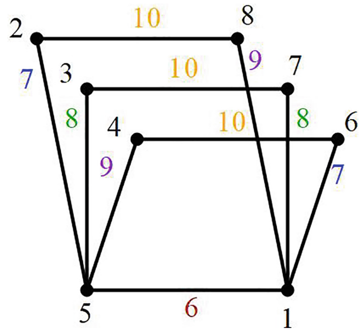

Fig. 2. The local $(a.d)$-edge antimagic coloring of B_3

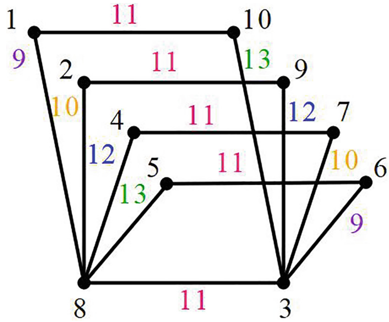

Fig. 3. The local $(a.d)$-edge antimagic coloring of B_4

Theorem 2.3. For $F_{m,3}$, be a firecraker graph with $n \geq 3$, $\chi_{le(3m,1)}(F_{m,3}) = 4$.

Proof. Let $F_{m,3}$ be a firecraker graph with $V(F_{m,3}) = \{x_t, y_t, z_t; 1 \leq t \leq m\}$ and $E(F_{m,3}) = \{x_t x_{t+1}; 1 \leq t \leq m-1\} \cup \{x_t y_t; 1 \leq t \leq m\} \cup \{y_t z_t; 1 \leq t \leq m\}$. The cardinality of the vertices set of $|V(F_{m,3}| = 3m$, and the cardinality of the edges set of $|E(F_{m,3}| = 3m - 1$. The local (a, d)-antimagic chromatic number of $F_{m,3}$ is $\chi_{le(3m,1)}(F_{m,3}) = 4$.

To prove $\chi_{le(3m,1)}(F_{m,3}) = 4$ first, we will prove that $\chi_{lea}(P_n \triangleright P_m) \geq 4$. Based on Observation 1.2 we have $\chi_{le(a,d)}(F_{m,3}) \geq \chi_{lea}(P_n \triangleright P_m)$. in Agustin et al. [2] For $n, m \geq 3$, the local edge antimagic chromatic number of $P_n \triangleright P_m$ with grafting pendant vertex $x \epsilon V(P_m)$ is $\chi lea(P_n \triangleright P_m) = 4$. then we have $\chi_{le}(a,d)(F_{m,3}) \geq \chi_{lea}(Pn \triangleright Pm)$. Based on these results, it can be concluded that $\chi_{le(a,d)}(F_{m,3}) \geq 4$. To show $\chi_{le(a,d)}(F_{m,3}) \leq 4$, by defining the bijection $f : V(F_{m,3}) \rightarrow \{1, 2, 3..., |V(F_{m,3})|\}$

$$f(x_t) = \begin{cases} 2, & for\ t\ \equiv 1 \\ \frac{3t+3}{2} - 2, & for\ t\ \ \equiv 3(mod\ 2) \\ 3m - \frac{3t}{2} + 2, & for\ t \equiv 0(mod\ 2) \end{cases}$$

$$f(y_t) = \begin{cases} 3m+3-\frac{3t+3}{2}, & for\ t\ \equiv 1(mod\ 2) \\ \frac{3t}{2}, & for\ t\ \equiv 0(mod\ 2) \end{cases}$$

$$f(z_t) = \begin{cases} 1, & for\ t\ \equiv 1 \\ \frac{3t+3}{2}-1, & for\ t\ \equiv 3(mod\ 2) \\ 3m+1-\frac{3t}{2}, & for\ t \equiv 0(mod\ 2) \end{cases}$$

the following is a way to see that f is the local (a, d)-antimagic labeling of $F_{m,3}$ and the edge weights

$$w(x_t x_{t+1}) = \begin{cases} 3m+1, & for\ t\ \equiv 1 \\ 3m, & for\ t\ \equiv 3(mod\ 2) \\ 3m+3, & for\ t \equiv 0(mod\ 2) \end{cases}$$

$$w(x_t y_t) = \begin{cases} 3m+2,\ for\ t\ \equiv 1 \\ 3m+1,\ for\ t\ \equiv 3(mod\ 2) \\ 3m+2,\ for\ t \equiv 0(mod\ 2) \end{cases}$$

$$w(y_t z_t) = \begin{cases} 3m+1, & for\ t\ \equiv 1 \\ 3m+2, & for\ t\ \equiv 3(mod\ 2) \\ 3m+1, & for\ t \equiv 0(mod\ 2) \end{cases}$$

The set of edge weights based on the edge weights obtained is $W = \{3m, 3m+1, 3m+2, 3m+4\}$. We can determine the value of the smallest weight of the edge is $a = 3m$ and $d = 1$, then we will have $\chi_{le(3m,1)}(F_{m,3}) \leq 4$. it can be concluded that $\chi_{le(3m,1)}(F_{m,3}) = 4$ (Fig. 4).

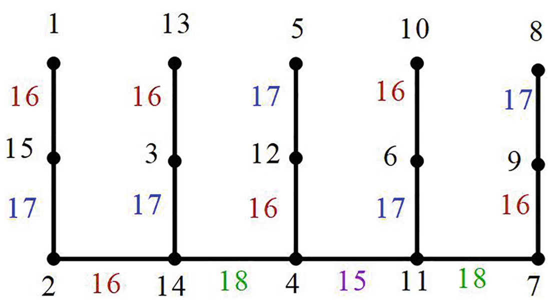

Fig. 4. The local $(a.d)$-edge antimagic coloring of $F_{5,3}$

Proof. Let K_n be a complete graph with $E(K_n) = \{x_t x_s; 1 \leq t \leq n, 1 \leq s \leq n, t \neq s\}$ and $V(K_n) = \{x_t; 1 \leq t \leq n\}$. The cardinality of the vertices set of $|E(K_n| = n^2 - n$, and the cardinality of the edges set of $|V(K_n| = n$. The local (a,d)-antimagic chromatic number of K_n is $\chi_{le(3,1)}(K_n) = 2n - 3$.

To prove $\chi_{le(3,1)}(K_n) = 2n-3$ first, we will prove that $\chi_{lea}(K_n) \geq 2n-3$. Based on Observation 1.2 we have $\chi_{le(a,d)}(K_n) \geq \chi_{lea}(K_n)$. in Agustin et al. [1] For $n \geq 3$, the local edge antimagic chromatic number of K_n is $\chi_{lea}(K_n) = 2n-3$. Based on these results, it can be concluded that K_n is $\chi_{le(a,d)}(K_n) \geq 2n - 3$. To show $\chi_{le(a,d)}(K_n) \leq 2n-3$, by defining the bijection $f : V(K_n) \rightarrow \{1, 2, 3..., |V(K_n)|\}$

$$f(x_t) = \begin{cases} \frac{t+1}{2}, & for\ t \equiv 1 (mod\ 2) \\ n - \frac{t}{2} + 1, & for\ t \equiv 0 (mod\ 2) \end{cases}$$

the following is a way to see that f is the local (a,d)-antimagic labeling of K_n and the edge weights

$$w(x_t x_s; 1 \leq t \leq n, 1 \leq s \leq n, t \neq s) = \{3, 4, 5, ..., 2n - 1\}.$$

The set of edge weights based on the edge weights obtained is $W = \{3, 4, 5, ..., 2n - 1\}$. We can determine the value of the smallest weight of the edge is $a = 3$ and $d = 1$, then we will have $\chi_{le(3,1)}(W_n) \leq n + 2$. it can be concluded that $\chi_{le(3,1)}(K_n) = 2n - 3$ (Figs. 5 and 6).

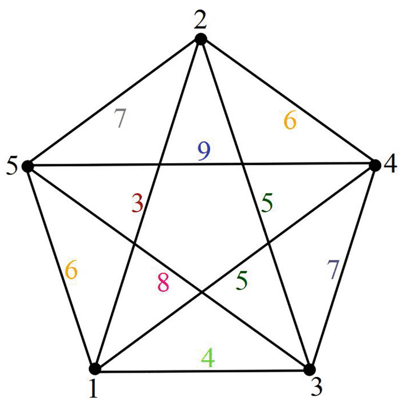

Fig. 5. The local $(a.d)$-edge antimagic coloring of K_5

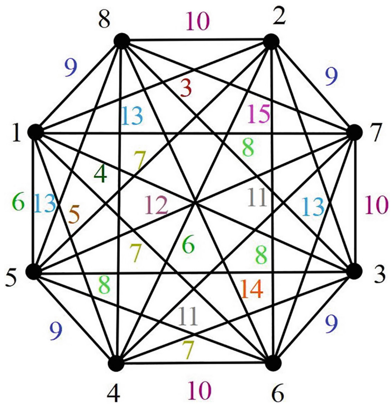

Fig. 6. The local $(a.d)$-edge antimagic coloring of K_8

Theorem 2.4. For Dg_n be a dragon graph with $n \geq 2$, $\chi_{le(3,1)}(Dg_n) = 2n+6$.

Proof. Let Dg_n be a dragon graph with $V(Dg_n) = \{x_t; 1 \leq t \leq n+2\} \cup \{y_t; 1 \leq t \leq n+2\} \cup \{z_t; 1 \leq t \leq n\}$ and $E(Dg_n) = \{x_t x_{t+1}; 1 \leq t \leq n+1\} \cup \{y_t y_{t+1}; 1 \leq t \leq n+1\} \cup \{z_t z_{t+1}; 1 \leq t \leq n-1\} \cup \{z_t y_t; 1 \leq t \leq n+2\} \cup \{z_t x_t; 1 \leq t \leq n+2\}$. The cardinality of the vertices set of $|V(Dg_n| = 3n+4$, and the cardinality of the edges set of $|E(Dg_n| = 2n+6$. The local (a,d)-antimagic chromatic number of Dg_n is $\chi_{le(3,1)}(Dg_n) = 2n+6$.

To prove $\chi_{le(3,1)}(Dg_n) = 2n+6$ first, we will prove that $\Delta(Dg_n) \geq 2n+6$. Based on Observation 1.3 we have $\chi_{le(a,d)}(Dg_n) \geq \Delta(Dg_n)$. in Agustin et al. [1] If $\Delta(G)$ is maximum degrees of G, then we have $\chi_{lea}(G) \geq \Delta(G)$. Based on these results, it can be concluded that $\chi_{le(a,d)}(Dg_n) \geq 2n+6$. To show $\chi_{le(a,d)}(Dg_n) \leq 2n+6$, by defining the bijection $f : V(Dg_n) \rightarrow \{1, 2, 3..., |V(Dg_n)|\}$

$$f(x_t) = \begin{cases} n+3+\frac{t+1}{2}, & for\ t \equiv 1 (mod\ 2) \\ n+1-\frac{t-2}{2}+1, & for\ t \equiv 0 (mod\ 2) \end{cases}$$

$$f(y_t) = \begin{cases} 1+\frac{t+1}{2}, & for\ t \equiv 1 (mod\ 2) \\ 6+\frac{t}{2}+2n & for\ t \equiv 0 (mod\ 2) \end{cases}$$

$$f(z_t) = \begin{cases} 1, & for\ i \equiv\ 2 \\ 6+2n, & for\ i \equiv\ 1 \\ 7+2n, & for\ i \equiv\ 3 \end{cases}$$

the following is a way to see that f is the local (a,d)-antimagic labeling of Dg_n and the edge weights

$$w(x_t x_{t+1}) = \begin{cases} 2n+7, & for\ t \equiv 1 (mod\ 2) \\ 2n+8, & for\ t \equiv 0 (mod\ 2) \end{cases}$$

$$w(y_t y_{t+1}) = \begin{cases} 7+2n, & for\ t \equiv 1(mod\ 2) \\ 8+2n, & for\ t \equiv 0(mod\ 2) \end{cases}$$

$$w(z_t z_{t+1}) = \begin{cases} 2n+7, & for\ t \equiv 1(mod\ 2) \\ 2n+8, & for\ t \equiv 0(mod\ 2) \end{cases}$$

$$w(z_t y_t) = \begin{cases} 2+\frac{t+1}{2}, & for\ t \equiv 1(mod\ 2) \\ 2n+7-\frac{t}{2}, & for\ t \equiv 0(mod\ 2) \end{cases}$$

$$w(z_t x_t) = \begin{cases} 4+n+\frac{t+1}{2}, & for\ t \equiv 1(mod\ 2) \\ n+5-\frac{t}{2}, & for\ t \equiv 0(mod\ 2) \end{cases}$$

The set of edge weights based on the edge weights obtained is $W = \{3, 4, 5, ..., 2n+8\}$. We can determine the value of the smallest weight of the edge is $a = 3$ and $d = 1$, then we will have $\chi_{le(3,1)}(Dg_n) \leq 2n+6$. it can be concluded that $\chi_{le(3,1)}(Dg_n) = 2n+6$ (Fig. 7).

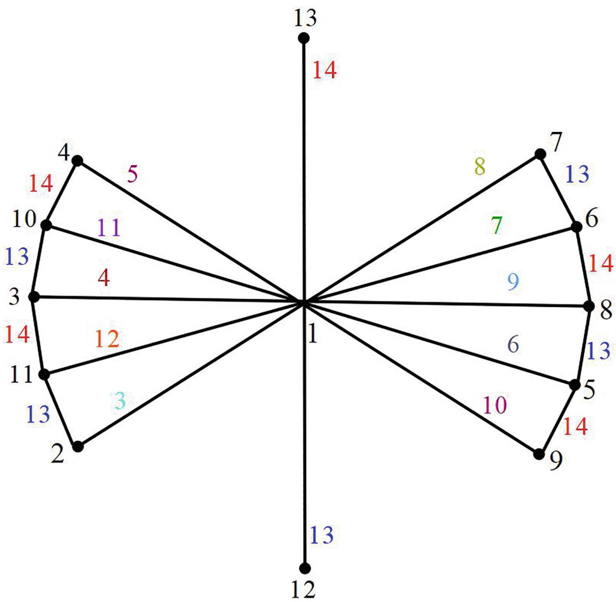

Fig. 7. The local $(a.d)$-edge antimagic coloring of D_3

3 Concluding Remarks

In this paper, we have studied the local (a, d)-edge antimagic coloring of special graph, namely broom graph, book graph, firecracker graph, complete graph, and dragon graph. We have found that most of the local (a, d)-edge chromatic numbers attain the best lower bound. However, due to there is still little research related to the topic local (a, d)-edge antimagic coloring. So we propose open problem.

Open Problem 3.1. Determine the exact value of the local (a, d)-edge antimagical chromatic number of all types of graphs regardless of what has been researched.

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