On Local (a, d)-Edge Antimagic Coloring of Some Specific Classes of Graphs

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Abstract. For any graph $G = (V, E)$, the order and size of G are p and q. Let $G(V, E)$ be a graph with the vertex set V and the edge set E, and let w be the edge weight of graph G. with $|V(G)| = m$ and $|E(G)| = n$. A labeling of a graph G is a bijection f from $V(G)$ to the set $\{1, 2, .., p|V(G)|\}$. The bijection f is called an edge antimagic labeling of graph if for any two vertex u and v in path $u - v$,$u \neq v$, where $\{w(uv) : w(uv) = f(u) + f(v), uv \in E\}$, are distinct. Any local edge antimagic labeling induces a proper edge coloring of G where the edge uv is assigned the color $w(uv)$. The local edge antimagic coloring of graph is said to be a local (a, d)-edge antimagic coloring of G if the set of their edge colors form an arithmetic sequence with initial value a and different d. The local (a, d)-edge antimagic chromatic number $\chi' le(G)$ is the minimum number of colors needed to color G such that a graph G admits the local (a, d)-edge antimagic coloring. Furthermore, In this paper, we will obtain the lower and upper bound of $\chi' le(G)$. The results of this research are the exact value of the local (a, d)-edge antimagic chromatic number of some graphs. In this paper we have studied local (a, d)-edge antimagic coloring on special graphs, namely centipede graphs, lotus graphs, caterpillar graphs, double star graphs, and double broom graphs

Keywords: local (a, d) antimagic coloring · edge antimagic coloring · spacial graph

1 Introduction

Graph $G = (V, E)$ defined $V(G)$ as the set of vertices and $E(G)$ is the set of edges [11]. A graph G is called antimagic if G has an antimagic labeling. An antimagic labeling is a bijection from the set of edges to $1, 2, ..., q$ such that all the vertex

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weights are distinct, where a weight of the vertex v is $w(u) = \sum_{(e \in E(u)} f(e)$, and $E(u)$ is the set of edges incident to u. The concept of antimagic graph labeling can be seen in [9,10]. Arumugam et al. [5] in 2017 defined the local vertex antimagic labeling. Agustin et al. [1] in 2017 introduce the local edge antimagic labeling.

The local edge antimagic labeling is a bijection from the set of vertices to $1, 2, 3, ..., p$ such that for any two adjacent edges are not received the same edge-weight (color), where the edge-weight $w(e = uv)$ is the sum of two end vertices labels, $w(uv) = f(u) + f(v), uv \in E(G)$. The local edge antimagic chromatic number $\chi' lea(G)$ is the least number of colors used in any local edge antimagic labeling of G. Local (a, d)-edge antimagic colouring of graph is motivated from the topic of local edge antimagic coloring [1,2,4].

A (a, d)-edge local antimagic labeling is a bijection l from the set of vertices to $\{1, 2, 3, ..., p\}$ such that for any two adjacent edges are not received the same edge-weight (color), where an edge-weight $w(e = uv)$ is the sum of two end vertices labels, $w(uv) = f(u) + f(v), uv \in E(G)$ and the set of all edge-weights are formed an arithmetic progression $\{a, a + d, a + 2d, ..., a + (c - 1)d\}$, for some integers $a, d > 0$ and c is the number of distinct colors are used in the proper coloring. A (a, d)-edge local antimagic labeling is denoted by (a, d)-ELA labeling [3,13]. The local antimagic labeling have been studied by [5–8,12].

The (a, d)-edge local antimagic coloring number of a graph G is the least number of colors used in any (a, d)-edge local antimagic labeling of G and is denoted by $\chi'_{(a,d)-ela}(G)$ by Sundaramoorthy et al. [13]

Proposition 1. [13]. If $\Delta(G)$ is maximum degree of G, we have $\chi'_{lea}(G) \geq \Delta(G)$.

Proposition 2. [13]. If the graph G admits an (a, d)-ELA labeling f , then $\chi'_{(a,d)-ela}(G) \geq \chi'_{lea}(G) \geq \chi'(G) \geq \Delta(G)$.

Proposition 3. [13]. If the graph G admits an (a, d)-ELA labeling with c-colors, then $d \leq \frac{2p-4}{c-1}$.

Observation 2.1 [14]. For any graph G,

$$\chi_{le}(a, d)(G) \geq \chi_{lea}(G) \geq \Delta(G),$$

where $\chi_{lea}(G)$ is a chromatic number of local edge coloring of G and $\Delta(G)$ is maximum degrees of G

2 Main Result

In this paper, we will studied the existence of local (a, d)-edge antimagic coloring of a some graph, and determine the chromatic number of local (a, d)-edge antimagic coloring of some graph include centipede graph $CP_{n,1}$, Lotus graph Lo_n, Caterpillar graph $CP_{n,m}$, double star graphs $DS_{n,m}$, and the double Broom graph $DBr_{m,n,k}$. We also analyse the lower bound of the local (a, d)-edge antimagic coloring of the graphs.

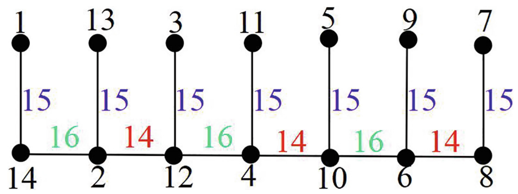

Fig. 1. The local $(a.d)$-edge antimagic coloring of $CP_{(7,1)}$

Theorem 2.1. Let $CP(n,1)$be a centipede graph, for $\forall n \geq 2$ then $(2n,1)$-edge local antimagic colouring number is $\chi'_{(2n,1)-ela}(CP(n,1)) = 3$.

Proof. Let $V(CP_{(n,1)}) = \{x_i, y_i, 1 \leq i \leq n\}$ and $E(CP_{n,1}) = \{x_i y_1; 1 \leq i \leq n; y_i y_{i+1}, 1 \leq i \leq n-1\}$ for $\forall n \geq 2$ then (2n,1). Then $|V(CP(n,1))| = 2n$ and $|E(CP(n,1))| = n + n - 1 = 2n - 1$. Now, define a bijection $f_1 : V(CP(n,1) \rightarrow \{1, 2, ..., n\}$by:

$$f(x_i) = \begin{cases} i, & for\ \imath\ even \\ 2n - i + 1, & for\ \imath\ odd \end{cases}$$

$$f(y_i) = \begin{cases} 2n - i - 1, & for\ i\ ganjil \\ i, & for\ i\ \ genap \end{cases}$$

the edge-weights of CP (n,1) are

$$w(x_i y_i) = 2n + 1$$

$$w(y_i y_{i+1}) = \begin{cases} 2n, & for\ ,n\ even, 1 \leq i \leq n-1 \\ 2n+1, & \quad for\ ,n\ odd, 1 \leq i \leq n-1 \end{cases}$$

It is easy to identify f_1 proves a proper edge coloring of $CP(n,1)$ and hence $\chi'_{(2n,1)-ela}(CP(n,1)) \leq 3$. Since $\chi'_{lea}(CP_{(n,1)} = 3$, it follow, we get $\chi'_{(2n,1)-ela}(CP_{(n,1)}) \geq 3$. Hence $\chi'_{(2n,1)-ela}(CP_{(n,1)}) = 3$ (Fig. 1)

Theorem 2.2. Let Lo_nbe a Lotus graph, for $\forall n \geq 1$ then $(n+3,1)$-edge local antimagic coloring number is $\chi'_{(n+3,1)-ela}(Lo_n) = n + 2$

Proof. Let $V(Lo_n) = \{x_i; 1 \leq i \leq n+1\} \cup \{y_i; 1 \leq i \leq n\} \cup a \cup b$ and $E(Lo_n) = \{ax_i; 1 \leq i \leq n+1\} \cup \{ab\} \cup \{x_i y_i; 1 \leq i \leq n\} \cup \{y_i x_{i+1}; 1 \leq i \leq n\}$ for $\forall n \geq 1$. Then $|V(Lo_n| = 2n + 3$ and $|E(Lo_n| = 3n + 2$.

Now, define a bijection $f : V(Lo_n) \rightarrow \{1, 2, 3, ..., n\}$ by

$$f(a) = 1$$
$$f(b) = 2n + 3$$
$$f(x_i) = n + i + 1$$
$$f(y_i) = n - i + 2$$

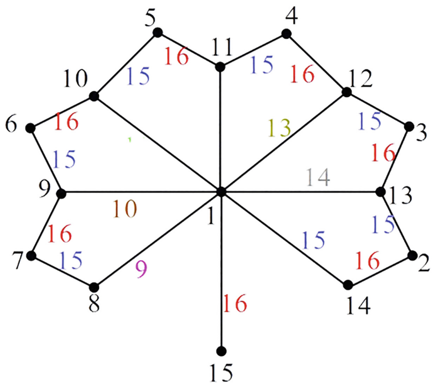

Fig. 2. The local $(a.d)$-edge antimagic coloring of $Lo_{(6)}$

the edge-weight of Lo_n are

$$
\begin{aligned}
w(ab) &= 2n+4\\
w(ax_i) &= n+i+2\\
w(x_iy_i) &= 2n+3\\
w(y_ix_{i+1}) &= 2n+4\\
w(ab) &= 2n+4
\end{aligned}
$$

based on the function of point labels and side weights, side weights are obtained that satisfy the arithmetic sequence with $a = n+3, d = 1$ $W = n+3, n+4, n+i+2, ..., 2n+2, 2n+3, 2n+4$.
From the set of side weights, it can be seen how many different side weights there are $n+2$.
Denotes the maximum degree (Delta) of the graph Lo_n shown in Fig. 2.
Based on the image can be determined $\triangle(Lo_n) = n+2, \chi_{le(a,d)}(Lo_n \geq n+2$

Theorem 2.3. Let $CP(n,m)$be a caterpillar graph, for $\forall n \geq 2$ then $(4n,1)$-edge local antimagic coloring number is $\chi'_{(4n,1)-ela}(CP(n,m)) = m+2$.

Proof. Let $V(CP_{(n,m)}) = \{x_i; 1 \leq i \leq n\} \cup \{(y_i)^j; 1 \leq i \leq n; 1 \leq j \leq m\}$ and $E(CP_{(n,m)}) = \{x_ix_{i+1}; 1 \leq i \leq n-1\} \cup \{x_iy_i{}^j; 1 \leq i \leq n, 1 \leq j \leq m\}$ for $\forall n \geq 2$. then $|V(CP(n,m)| = nm+n$, and $|E(CP(n,m)| = nm+n-1$ Now, define a bijection $f : V(CP(n,m)) \to \{1, 2, 3, ..., |V(CP(n,m)\}$by

$$
f(x_i^j) = \begin{cases} j+(m+1)(\frac{i+1}{2}-1), & for\ i\ even \\ j+(m+1)(n-\frac{i}{2}, & for\ i\ odd \end{cases}
$$

$$
f(x_i) = \begin{cases} (n-\frac{i-1}{2}(m+1), & for\ i\ even \\ \frac{i}{2}(m+1), & for\ i\ odd \end{cases}
$$

The edge-weights of CP (n,m) are

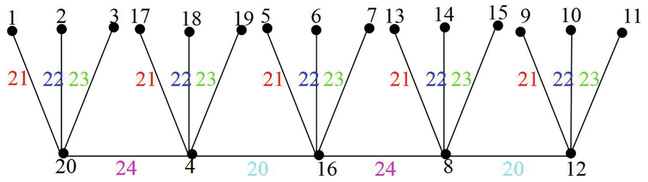

Fig. 3. The local $(a.d)$-edge antimagic coloring of $CP(3,5)$

$$w(x_i x_{i+1}) = \begin{cases} (n+1)(m+1), & for\ i\ even \\ n(m+1), & for\ i\ odd \end{cases}$$

$$w(x_i y_i^j) = n(m+1) + j$$

based on the function of point labels and side weights, side weights are obtained that satisfy the arithmetic sequence with $a = 4n, d = 1$
$W = 4n, 4n+1, ..., (n+1)(m+1$.
From the set of side weights, it can be seen how many different side weights there are $m+2$.
Denotes the maximum degree (Delta) of the graph $CP_{(n,m)}$ shown in Fig. 3.
Based on the image can be determined $\triangle(CP_{(n,m)}) = m + 2, \chi_{le(a,d)}(CP_{(n,m)} \geq m+2$

Theorem 2.4. Let $DS(n,m)$be a Double Star graph with $\forall n \geq 2$ for $n \geq m$ then $(n+3,1)$-edge local antimagic coloring number is $\chi'_{(n+3,1)-ela}(DS(m,n)) = n+1$ and if $n < m$ then (n+3,1)-edge local antimagic coloring number is $\chi'_{(n+3,1)-ela}(DS(m,n)) = m+1$.

Proof. $V(DS_{(n,m)}) = \{x, y, x_i, y_j, z_i; 1 \leq i \leq n, 1 \leq j \leq m\}$ and $E(DS_{n,m}) = \{xy\} \cup \{x_i x; 1 \leq i \leq n\} \cup \{y_j y; 1 \leq j \leq m\}$, for $\forall n, m \geq 2$ .Then $|V(DS(n,m))| = n+m+2$ and $|E(DS(n,m))| = n+m+1$.

Case 1. For $m = n$

To prove $\chi_{le(m+3,1)}(DS_{(m,n)}) = m+1$ first, we will prove that $\Delta(DS_{m,n}) \geq m+1$. Based on Observation we have $\chi_{le(a,d)}(DS_{m,n}) \geq \Delta(DS_{m,n})$. in Agustin et al. [1] If $\Delta(G)$ is maximum degrees of G, then we have $\chi_{lea}(G) \geq \Delta(G)$. Based on these results, it can be concluded that $\chi_{le(a,d)}(DS_{m,n}) \geq m+1$. To show $\chi_{le(a,d)}(DS_{m,n}) \leq m+1$, by defining the bijection $f : V(DS_{m,n}) \rightarrow \{1, 2, 3..., |V(DS_{m,n})|\}$

$$f(y) = 1$$
$$f(x_i) = i+1, 1 \leq i \leq n$$
$$f(z) = n+2$$
$$f(y_j) = m+j+2, 1 \leq j \leq m$$

the edge-weights of $DS(m,n)$ are

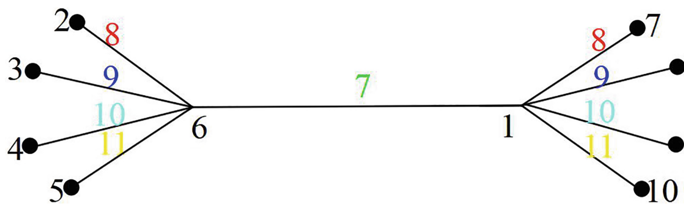

Fig. 4. The local $(a.d)$-edge antimagic coloring of $DS(4,4)$

$$w(xx_i) = m + 3 + i, 1 \leq i \leq m$$
$$w(yy_j) = n + 3 + j, 1 \leq j \leq n$$
$$w(xy) = n + 3$$

based on the function of point labels and side weights, side weights are obtained that satisfy the arithmetic sequence with $a = m + 3, d = 1$
$W = n + 3, n + 4, ..., n + i + 3$.
From the set of side weights, it can be seen how many different side weights there are $m + 1$.
Denotes the maximum degree (Delta) of the graph $DS_{(m,n)}$ shown in Fig. 4.

Based on the image can be determined $\triangle(DS_{(m,n)}) = m + 1, \chi_{le(a,d)}(DS_{(m,n)}) \geq m + 1$

Case 2. For $n > m$

To prove $\chi_{le(n+3,1)}(DS_{(m,n)}) = n + 1$ first, we will prove that $\Delta(DS_{m,n}) \geq n + 1$. Based on Observation we have $\chi_{le(a,d)}(DS_{m,n}) \geq \Delta(DS_{m,n})$. in Agustin et al. [1] If $\Delta(G)$ is maximum degrees of G, then we have $\chi_{lea}(G) \geq \Delta(G)$. Based on these results, it can be concluded that $\chi_{le(a,d)}(DS_{m,n}) \geq n + 1$. To show $\chi_{le(a,d)}(DS_{m,n}) \leq n + 1$, by defining the bijection $f : V(DS_{m,n}) \rightarrow \{1, 2, 3..., |V(DS_{m,n})|\}$

$$f(y) = 1$$
$$f(x_i) = i + 1, 1 \leq i \leq n$$
$$f(z) = n + 2$$
$$f(y_j) = n + j + 2, 1 \leq j \leq m$$

the edge-weights of $DS(m,n)$ are

$$w(xx_i) = n + 3 + i, 1 \leq i \leq n$$
$$w(yy_j) = n + 3 + j, 1 \leq j \leq m$$
$$w(xy) = n + 3$$

based on the function of point labels and side weights, side weights are obtained that satisfy the arithmetic sequence with $a = n + 3, d = 1$

$$W = \{n + 3, n + 4, ..., n + i + 3\}.$$

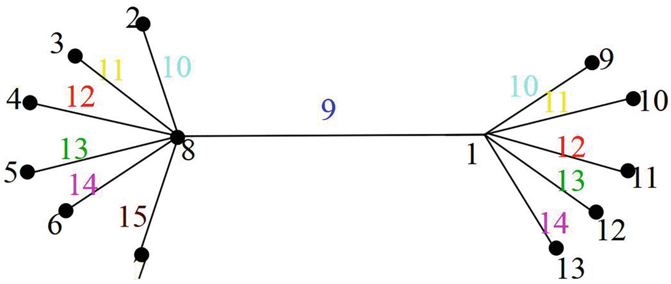

Fig. 5. The local $(a.d)$-edge antimagic coloring of $DS(5,6)$

From the set of side weights, it can be seen how many different side weights there are $n+1$. Denotes the maximum degree (Delta) of the graph $DS_{(m,n)}$ shown in Fig. 5.

Based on the image can be determined $\triangle(DS_{(m,n)}) = n + 1, \chi_{le(a,d)}(DS_{(m,n)}) \geq n+1$

Case 3. For $m > n$

To prove $\chi_{le(n+3,1)}(DS_{(m,n)}) = m+1$ first, we will prove that $\Delta(DS_{m,n}) \geq m+1$. Based on Observation we have $\chi_{le(a,d)}(DS_{m,n}) \geq \Delta(DS_{m,n})$. in Agustin et al. [1] If $\Delta(G)$ is maximum degrees of G, then we have $\chi_{lea}(G) \geq \Delta(G)$. Based on these results, it can be concluded that $\chi_{le(a,d)}(DS_{m,n}) \geq m+1$. To show $\chi_{le(a,d)}(DS_{m,n}) \leq m+1$, by defining the bijection $f : V(DS_{m,n}) \rightarrow \{1, 2, 3..., |V(DS_{m,n})|\}$

$$f(y) = 1$$
$$f(x_i) = i+1, 1 \leq i \leq n$$
$$f(z) = n+2$$
$$f(y_j) = n+j+2, 1 \leq j \leq m$$

the edge-weights of $DS(m,n)$ are

$$w(xx_i) = n+3+i, 1 \leq i \leq n$$
$$w(yy_j) = n+3+j, 1 \leq j \leq m$$
$$w(xy) = n+3$$

based on the function of point labels and side weights, side weights are obtained that satisfy the arithmetic sequence with $a = n+3, d = 1$

$$W = \{m+2, m+3, m+4, ..., n+j+3\}.$$

From the set of side weights, it can be seen how many different side weights there are $m+1$. Denotes the maximum degree (Delta) of the graph $DS_{(m,n)}$ shown in Fig. 6.

Based on the image can be determined $\triangle(DS_{(m,n)}) = m + 1, \chi_{le(a,d)}(DS_{(m,n)}) \geq m+1$

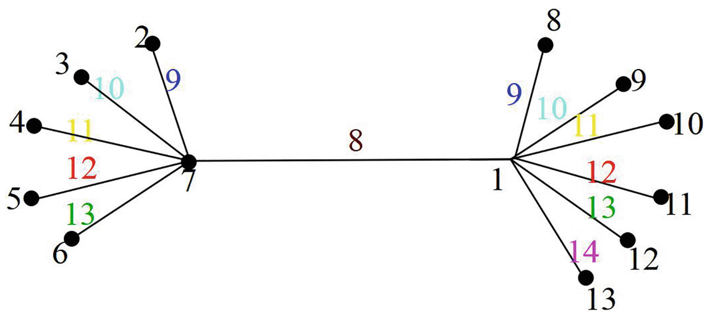

Fig. 6. The local $(a.d)$-edge antimagic coloring of $DS(6,5)$

Theorem 2.5. Let $DBr_{m,n,k}$ be a Double Broom graph with $m,n,k \geq 2$, if $n \geq k$ for m even then $(m+n+1,1)$-edge local antimagic colouring number is $\chi'_{ela(m+n+1,1)}(DBr_{(m,n,k)}) = n+1$.and if $n < k$ for m even then (m+k,1)-edge local antimagic colouring number $\chi'_{ela(m+k,1)}(DBr_{(m,n,k)}) = k+1$.

Proof. $V(DBr_{(m,n,k)}) = \{z_i, 1 \leq i \leq k\} \cup \{x_j, 1 \leq j \leq m\} \cup \{y_l, 1 \leq l \leq n\}$ and $E(DBr_{m,n,k}) = \{xz_i, 1 \leq i \leq k\} \cup \{x_j x_{j+1}; 1 \leq j \leq m-1\} \cup \{x_m y_l; 1 \leq l \leq n\}$, for $\forall m,n,k \geq 2$ .Then $|V(DBr(m,n,k))| = m+n+k$ and $|E(DBr(m,n,k))| = k+m+n-1$

Case 1. For $n = k$
Now, define a bijection $f : V(DBr(m,n,k) \to \{1,2,...,\}$by:

$$f(z_i) = m+n+i, 1 \leq i \leq k$$

$$f(x_j) = \begin{cases} \frac{1+j}{2}, & for\ \jmath\ odd; 1 \leq j \leq m \\ m+n+1-\frac{j}{2}, & for\ \jmath\ even; i \leq j \leq m \end{cases}$$

$$f(y_l) = \frac{m}{2} + l, 1 \leq l \leq n$$

the edge-weights of $DB_r(m,n,k)$ are

$$w(x_m y_l) = m+n+l+1.$$

$$w(x_1 z_i) = m+n+i+1.$$

$$w(x_j x_{j+1}) = \begin{cases} m+n+1, & for\ \jmath\ odd; 1 \leq j \leq m \\ m+n+2, & for\ \jmath\ even; i \leq j \leq m \end{cases}$$

based on the function of point labels and side weights, side weights are obtained that satisfy the arithmetic sequence with $a = m+n+1, d = 1$ if $n \geq k$ and $a = m+k, d = 1$ if $n < k$

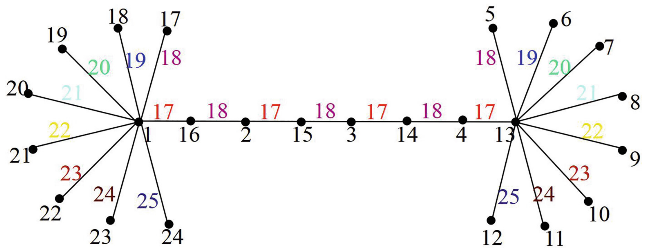

Fig. 7. The local $(a.d)$-edge antimagic coloring of $DB_r(8, 8, 8)$

$W = m + n + 1, m + n + 2, ..., m + n + i.$
From the set of side weights, it can be seen how many different side weights there are $n + 1$ if $n \geq k$ and $k + 1$ if $n < k$. Denotes the maximum degree (Delta) of the graph $DBr_{(m,n,k)}$ shown in Fig. 7.

Based on the image can be determined $\triangle(DBr_{(m,n,k)}) = n + 1, \chi_{le(a,d)}(DBr_{(m,n,k)}) \geq n + 1$ if $n \geq k$ and $\triangle(DBr_{(m,n,k)}) = k + 1, \chi_{le(a,d)}(DBr_{(m,n,k)}) \geq k + 1$ if$n < k$.

Case 2. For $k > n$

Now, define a bijection $f : V(DBr(m, n, k)) \rightarrow \{1, 2, 3, ..., |V(DBr(m, n, k)\}$by

$$f(z_i) = k + n + i, 1 \leq i \leq k$$

$$f(x_j) = \begin{cases} \frac{1+j}{2}, & for\ \jmath\ odd; 1 \leq j \leq m \\ m + n + 1 - \frac{j}{2}, & for\ \jmath\ even; i \leq j \leq m \end{cases}$$

$$f(y_l) = \frac{m}{2} + l, 1 \leq l \leq n$$

the edge-weights of $DB_r(m, n, k)$ are

$$w(x_m y_l) = m + n + l + 1.$$

$$w(x_1 z_i) = m + n + i + 1.$$

$$w(x_j x_{j+1}) = \begin{cases} m + n + 1, & for\ \jmath\ odd; 1 \leq j \leq m \\ m + n + 2, & for\ \jmath\ even; i \leq j \leq m \end{cases}$$

based on the function of point labels and side weights, side weights are obtained that satisfy the arithmetic sequence with $a = m + n + 1, d = 1$ if $n \geq k$ and $a = m + k, d = 1$ if $n < k$.
$W = m + n + 1, m + n + 2, ..., m + n + i.$
From the set of side weights, it can be seen how many different side weights there are $n + 1$ if $n \geq k$ and $k + 1$ if $n < k$ .Denotes the maximum degree (Delta) of the graph $DBr_{(m,n,k)}$ shown in Fig. 8.

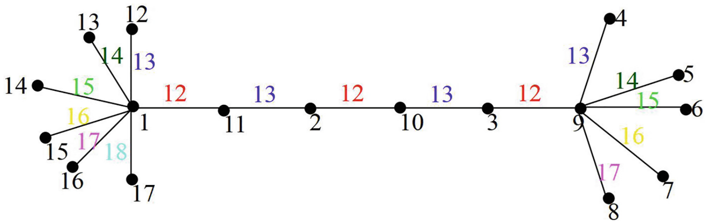

Fig. 8. The local $(a.d)$-edge antimagic coloring of $DB_r(6,5,6)$

Based on the image can be determined $\triangle(DBr_{(m,n,k)}) = k + 1, \chi_{le(a,d)}(DBr_{(m,n,k)}) \geq k+1$ if $n < k$.

Case 3. For $k > n$

Now, define a bijection $f : V(DBr(m,n,k)) \rightarrow \{1,2,3,...,$ $|V(DBr(m,n,k)\}$by

$$f(z_i) = m + n + i, 1 \leq i \leq k$$

$$f(x_j) = \begin{cases} \frac{1+j}{2}, & for \text{ȷ} \; odd; 1 \leq j \leq m \\ m + n + 1 - \frac{j}{2}, & for \text{ȷ} \; even; i \leq j \leq m \end{cases}$$

$$f(y_l) = \frac{m}{2} + l, 1 \leq l \leq n$$

the edge-weights of $DB_r(m,n,k)$ are

$$w(x_m y_l) = m + n + l + 1.$$

$$w(x_1 z_i) = m + n + i + 1.$$

$$w(x_j x_{j+1}) = \begin{cases} m + n + 1, & for \text{ȷ} \; odd; 1 \leq j \leq m \\ m + n + 2, & for \text{ȷ} \; even; i \leq j \leq m \end{cases}$$

based on the function of point labels and side weights, side weights are obtained that satisfy the arithmetic sequence with $a = m + n + 1, d = 1$ if $n \geq k$ and $a = m + k, d = 1$ if $n < k$

$W = m + n + 1, m + n + 2, ..., m + n + i.$

From the set of side weights, it can be seen how many different side weights there are $n+1$ if $n \geq k$ and $k+1$ if $n < k$. Denotes the maximum degree (Delta) of the graph $DBr_{(m,n,k)}$ shown in Fig. 9.

Based on the image can be determined $\triangle(DBr_{(m,n,k)}) = n + 1, \chi_{le(a,d)}(DBr_{(m,n,k)}) \geq n+1$ if $n \geq k$.

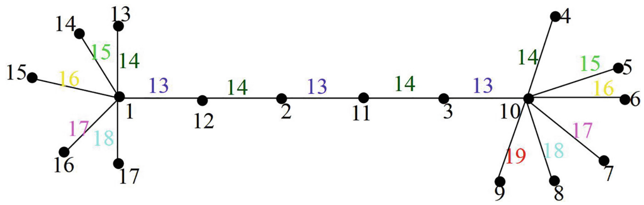

Fig. 9. The local $(a.d)$-edge antimagic coloring of $DB_r(6, 6, 5)$

3 Concluding Remarks

In this paper we have studied local (a, d)-edge anti-magic coloring on special graphs, namely centipede graphs, lotus graphs, caterpillar graphs, double star graphs, and double broom graphs. We have found that most of the local (a, d)-edge chromatic numbers reach the best lower bound. But because there is still little that researchers do in research related to the topic of local (a,d)-edge antimagic coloring. In further research, the problem of determining the (a,d)-ELA coloring number can be done on other graphs that are still open.

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