Hierarchical Structure of Semi-double Track Railway Line Using Supervisory Control Petri Net

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Abstract. Previously we built a Max-Plus Algebraic model for the problem of the hierarchical structure of the Semi-Double Track railway line based on the Petri Net model. Then we also perform a delay simulation on the model obtained. Based on this research, we conclude that the Petri Net model does not reflect the actual event. Thus, this research improved the Petri Net model on the problem of the hierarchical structure of the Semi-Double Track railway line using Supervisory Control Petri Net. We have obtained a new Petri Net model for Semi-Double Track railroad issues that is more relevant to actual conditions as the result of this research.

Keywords: Petri Net · Supervisory Control · Semi-Double Track Railway Line

1 Introduction

Researchers in [1] develop a Petri Net model for the Semi-Double Track railroad by considering priorities to avoid deadlocks. They based the model's prioritization on observations of critical issues mentioned in the literature. However, we admit that the Petri Net model they made was based on their understanding and thinking at that time, which means that other versions of the model can be made. Furthermore, although the Petri Net model can overcome issues that might cause deadlocks, it may be less relevant to actual conditions because it requires trains to run alternately from opposite directions to be able to go through the railroad tracks continuously.

As an illustration, consider Fig. 1, two transitions are enabled, namely t_1 and t_7. Suppose we fire t_7, which means a train runs from right to left, then the token in place p_{15} is reduced by one, and the token in place p_6 increases by one. This condition makes the transition t_8 enable because the conditions of p_6, p_{14}, and p_{18} fulfill t_8. Next, suppose we fire t_8, then places p_6, p_{14}, and p_{18} have no more tokens, place p_7 has one token, and p_{15} again has three tokens. The absence of tokens on p_{14} and p_{18} causes t_8 never to fire again. It means no train can run from right to left again after a train departed previously.

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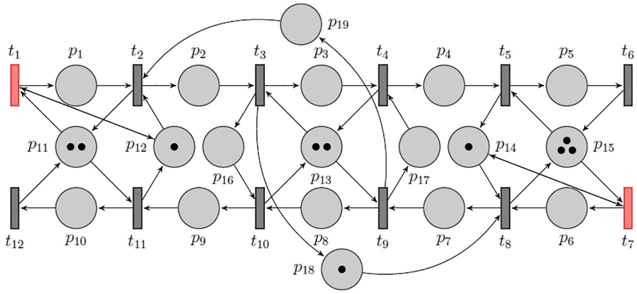

Fig. 1. The Petri Net Model of Semi-Double Track Problem Compiled by [1].

Even though place p_{14} and p_{18} will have another token after t_3 and t_5 are fired, which means waiting for a train running from left to right.

So, based on the explanation above, the authors conclude that the model constructed in [1] needs to be more relevant. Moreover, regarding the results of the delay simulation carried out in [2], it can conclude that based on the results of the delay simulation carried out on the Max-Plus Algebra model based on the Petri Net Model in [1] is robust, in this case, it means it is not good. For example, giving initial delays to several train arrivals makes directly following delays straight to zero. Then, based on [3, 4], and [5] regarding supervisory control, the author tries to use it to solve problems in [1] and obtain a more relevant Petri Net model.

2 Supervisory Control Petri Net

A Petri net is a directed bipartite graph. The structure of a Petri net is described as (P, T, D^+, D^-) where P and T are disjoint sets representing the vertex of the graph, known as places and transitions. In addition, D^+ and D^- are integer matrices with non-negative elements representing the flow relation between the two vertex types [4].

Let Z be the set of integers, n be the number of places, and m be the number of transitions. Then, the arcs connecting transitions to places are represented by the matrix $D^+ \in Z^{n\times m}$ and the arcs connecting places to transitions are represented by $D^- \in Z^{n\times m}$, with all D^+ and D^- elements greater than or equal to zero. In addition, the incidence matrix from a Petri Net is defined as $\mathrm{D} = \mathrm{D}^+ - \mathrm{D}^-$.

A place invariant is a set of places whose number of tokens remains constant for all possible markings, that is represented as an n-dimensional vector x integer, where n is the number of places in the Petri Net. Place invariant is defined as any integer vector $x \in Z^n$ that satisfies.

$$x^T \mu = x^T \mu_0 \tag{1}$$

where μ_0 is the initial marking, and μ represents the following marking.

The place invariant can be calculated by determining the integer solutions to the following Equation,

$$x^T D = 0 \tag{2}$$

where D is the $n \times m$ Petri Net size incidence matrix.

Given the firing by vector q, we get.

$$x^T \mu(k+1) = x^T(\mu(k) + Dq(k)) = x^T \mu(k).$$

The purpose of supervisory control is to limit the reachable marking places μ_p so that.

$$l^T \mu_p \leq b \tag{3}$$

where l is an integer weight vector, and b is an integer scalar.

For example, suppose that the constraint $\mu_1 + \mu_2 \leq 1$ is applied, which means that at most one of the two places p_1 and p_2 can be marked with a token, or in other words both places cannot be marked at the same time, and neither place can be marked at the same time can have more than one token.

The inequality constraint in (3) can be transformed into an equation by including the non-negative slack variable μ_c. So that it becomes $\mu_1 + \mu_2 + \mu_c = 1$ or, in general, it can be written as

$$l^t \mu_p + \mu_c = b. \tag{4}$$

The slack variable represents the new set of p_c holding the additional tokens required to satisfy the equation. Compliance with the rules of the Petri state space ensures that μ_c is a vector with non-negative elements. So, if (4) can be imposed on controlled marking places, then the number of tokens in the place will always be less than or equal to b.

The delimited place is part of a separate net known as the controller net. The structure of the controller net is calculated based on the addition of the slack variable, which is a place invariant using (4) for the entire Petri Net system.

This control problem, in general can be stated as follows for all constraints in (3) can be grouped and written in matrix form as follows,

$$L\mu_p \leq b \tag{5}$$

where μ_p is the marking vector of the initial Petri Net, $L \in Z^{n_c \times n}$, $b \in Z^{n_c}$, and n_c is the number of constraints in (3). This Inequality is the relationship between the elements in the two vectors $L\mu_p$ and b can be thought of as logical conjunction of n_c separate inequalities. After knowing the slack variable, the constraints in (5) can be rewritten as

$$L\mu_p + \mu_c = b \tag{6}$$

with $\mu_c \in Z^{n_c}$ is integer vector representing the marking of the place controller.

The D_c matrix contains the arcs connecting the place controllers to the initial Petri Net transitions. The incidence matrix $D \in Z^{(n+n_c)\times m}$ is a closed loop system defined as

$$D = \begin{bmatrix} D_p \\ D_c \end{bmatrix} \tag{7}$$

and the marking vector $\mu \in Z^{n+n_c}$ and the initial marking is as follows,

$$\mu = \begin{bmatrix} \mu_p \\ \mu_c \end{bmatrix}, \mu_0 = \begin{bmatrix} \mu_{p0} \\ \mu_{c0} \end{bmatrix}. \tag{8}$$

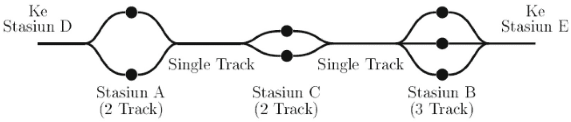

Fig. 2. Illustration of Semi-Double Track Railway Case on [1].

Note that (6) is a form of (1), so the invariant determined by Eq. (6) on the system ((7), (8)) must satisfy (2) with x replaced by X, a matrix formed by n_c different invariant.

$$X^T D = \begin{bmatrix} L \ I \end{bmatrix} \begin{bmatrix} D_p \\ D_c \end{bmatrix} = 0$$

$$LD_p + D_c = 0 \tag{9}$$

where $I \in Z^{n_c \times n_c}$ is the identity matrix if the coefficient on the slack variable in (6) equals 1.

Theorem 1. *(Controller Synthesis* [4]*). If*

$$b - L\mu_{p_0} \geq 0 \tag{10}$$

then the Petri Net Controller $D_c \in Z^{n_c \times m}$ with initial marking μ_{c_0} is

$$D_c = -LD_p \tag{11}$$

$$\mu_{c_0} = b - L\mu_{p_0}. \tag{12}$$

By applying constraints (5) on the system (7) with marking (8), assuming that transition with input arcs from D_c can be controlled. If Inequality (10) is invalid, then the constraint cannot be applied because the initial conditions are outside the range specified by the constraint.

3 Results and Discussions

The Fig. 2 is given to facilitate the re-explanation of the concern of problem in [1].

Based on Fig. 2, there are two tracks at Station A, two at Station B, and three at Station C. A single-track railway connects the three stations. Each track segment on the railroad can only be used by one train at the k time. The dot in the image is the pinpoint where the train can stop. For each line at the station, trains can use it differently, provided that the line is not used by other trains.

Generally, trains can run from Station A to Station B or vice versa. This situation can be modeled in Petri Net, as shown in Fig. 3. Then related to the limitation that only two tracks are available at Station A, two tracks at Station B, and three tracks at Station B, a simulation can be carried out using supervisory control. In this case the constraints given are $p_1 + p_{10} \leq 2, p_2 + p_9 \leq 1, p_3 + p_8 \leq 2, p_4 + p_7 \leq 1$, and $p_5 + p_6 \leq 3$. The Petri Net

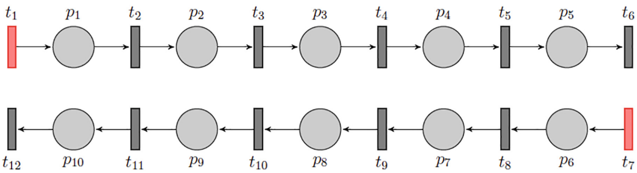

Fig. 3. The Petri Net Model from The Case before Applying Supervisory Control.

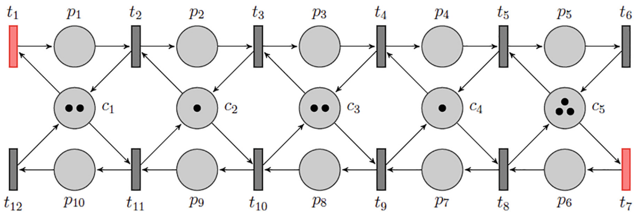

Fig. 4. Model Petri Net with Supervisory Control Stage I.

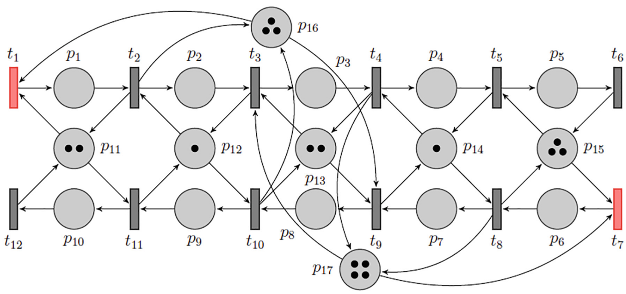

Fig. 5. Model Petri Net with Supervisory Control Stage II.

model is obtained using the Scilab Software with the constraints for supervisory control, as shown in Fig. 4. There are additionally several places, namely $p_{11}, p_{12}, p_{13}, p_{14}$, and p_{15} which are place controls. This Petri Net model is the same as the Petri Net model without priority in [1].

Of course, this model still needs to be used as the final result. As explained in [1], this model requires priority considerations to avoid deadlocks. By providing additional limits on the designed supervisory control, namely $p_2 + p_8 \leq 3$ and $p_3 + p_6 \leq 4$, a Petri Net model is obtained, as shown in Fig. 5. These constraints are needed to provide at least one lane at a station so that it can be used by trains crossing in the opposite direction (under critical conditions). In the most vital conditions possible, having one track at a station guarantees that the train can still run and occupy the one provided path.

In Fig. 5, there are several other places, namely $p_{11}, p_{12}, p_{13}, p_{14}, p_{15}, p_{16}$, and p_{17} which are place controls. Therefore, the Petri Net model is expected to solve problems that allow deadlocks to occur. It's just that after conducting a simulation using the help of Pipe, the model can still experience deadlocks. It is because no additional rules control

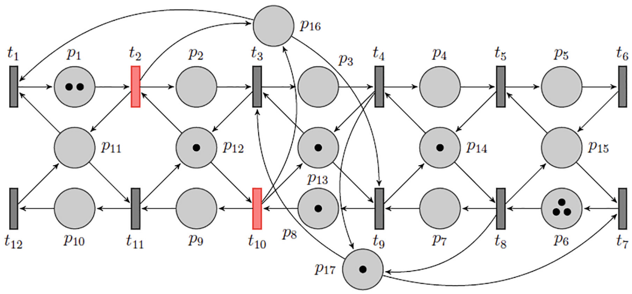

Fig. 6. Example of Critical Case State

trains from the right or left-hand trains that depart first. As an illustration, it can be seen in Fig. 6. Based on this figure, it is interpreted that two trains at Station A run towards Station C, and one train at Station C goes towards Station A. Therefore, they can go first between Station A or C trains. This condition is not a problem if the train at Station A takes precedence. However, it is different if the train at Station C takes precedence, a deadlock will occur.

From these problems, a priority can be added, namely by giving priority to transition t_2 with a priority level of 5, t_3 with a priority level of 4, t_{10} with a priority level of 3, and t_{11} with a priority level of 2. In addition, priority is also given to transition t_4 with a priority level of 5, t_5 with a priority level of 4, t_8 with a priority level of 3, and t_9 with a priority level of 2. This can provide a solution to avoiding deadlocks.

4 Conclusion

From the information above, we can use supervisory control Petri nets to construct Petri Net models for Semi-Double Track railroad issues. In this research, a new version of the Petri Net model for Semi-Double Track railroad issues is more relevant to actual conditions compared to the model obtained in [1]. Because in this model, trains can run from left to right flexibly and do not have to wait for other trains running from right to left or vice versa, unlike the model in [1] which requires trains to run in such a way.

5 Further Research

For further research, we will build a max plus model under the Petri net model that has been obtained. And for the primary analysis, namely Controller on max-plus Algebraic system for semi-double track Railway line problems. This research is intended to determine the optimal departure time required for a train to arrive at its destination under the just-in-time criteria. This study suggests that departures be made as late as possible to obtain the most optimal time needed by the train to complete the journey. Bearing in mind that in the max-plus model, there are multiple synchronizations due to the use of the same resource, which causes some delays.

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