On the Strong Rainbow Antimagic Coloring of Some Special Graph

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Abstract. Let $G(V(G), E(G))$ be a connected, undirected, and simple graph with vertex set $V(G)$ and edge set $E(G)$. For a bijective function $f : V(G) \longrightarrow \{1, 2, ..., |V(G)|\}$, the associated weight of an edge $uvE(G)$ under f is $w(uv) = f(u) + f(v)$. The function f is called an edge-antimagic vertex labeling if every edge has distinct weight. For two vertices u and v of G, a $u - v$ geodesic path in G is a $u - v$ path of length $d(u, v)$, where $d(u, v)$ is the distance between u and v (the length of a shortest $u - v$ path in G. A geodesic two edges uv, $u'v' \in E(P)$ it satisfied $w(uv) \neq w(u'v')$. If for every two vertices u and v of G, there exists a rainbow $u - v$ path, the f is called a strong rainbow antimagic labeling of G. When we assign each edge uv with the color of the edge weight $w(uv)$, thus we say the graph G admits a rainbow antimagic coloring. The strong rainbow antimagic connection number of G, denoted by $srac(G)$, is the smallest number of colors taken over all strong rainbow coloring induced by strong rainbow antimagic labelings of G. In this paper we have determined the connection number of strong rainbow antimagic coloring of some special graph.

Keywords: rainbow coloring · strong rainbow · strong rainbow antimagic coloring · connection number

1 Introduction

Graph G is a set of ordered pairs of vertices and edges, denoted by $G = (V(G), E(G))$ where $V(G)$ is a non-empty and finite set of vertices, and $E(G)$ are the set of (possibly empty) unordered pairs of distinct vertices in G called edges, $E = e_1, e_2, ..., e_n$ [1]. Graph coloring is the assignment of color to each vertex in graph G, so that a minimum path is obtained between two directly connected vertex that are not the same color [2]. One of the big concepts in graph theory is graph labeling [3]. Graph labeling is the activity of mapping a set of graph elements (vertices, edges, or both) into a set of numbers called labels [4]. Graph

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G is said to be antimagic if there is a bijection $f : E(G) \longrightarrow 1, 2, 3, ..., E(G)$ so that the resulting weight for each vertex is different. The weight of the vertex v below f, $wf(v)$, is the sum of the labels of the neighbor edge with v, that is $wf(v) = \sigma_{(uv \in E(G))} f(uv)$. In this case f is called antimagic labeling [5]. Rainbow coloring is the coloring of the edges of G which causes a rainbow-connected graph G with neighboring edges to have the same color [6]. The rainbow-connection number in graph G, denoted $rc(G)$ is the smallest positive integer k such that G has a rainbow-k coloring, with neighboring edges having the same color [7]. The path from $u - v$ is called a rainbow path if no two edges of the path have the same color. A graph G with edge coloring c is said to be rainbow connection if every $u, v \in G$ has a rainbow path [8]. Rainbow Antimagic Coloring is a combination of edge-rainbow coloring and edge-antimagic labeling. Antimagic labeling is called rainbow coloring if the weight of each edge is a rainbow coloring [9].

Definition 1. Edge-antimagic labeling is a bijective function that maps a set of vertex $V(G)$ to $1, 2, 3, ..., V(G)$, if the weight of each $e = uv$ then $wt(uv) = f(u) + f(v)$ is different [1].

Definition 2. Edge-rainbow coloring is edge coloring with rainbow coloring, if graph G is said to be rainbow connected if there is the least number of colors needed, denoted $rc(G)$.

Definition 3. Rainbow Antimagic Coloring (RAC) is a rainbow-connected simple graph with a bijective function $f : V(G) \rightarrow 1, 2, 3, ..., V(G)$, and weight $e = uv \in E(G)$ under f is $wf(uv) = f(u) + f(v)$. While $wf(uv) \neq wf(u'v')$. If u and v have $u - v$ paths, then $f = rac$ [10].

If for every two vertices u and v of G, there exists a rainbow $u - v$ path, the f is called a strong rainbow antimagic labeling of G. When we assign each edge uv with the color of the edge weight $w(uv)$, thus we say the graph G admit a rainbow antimagic coloring. The strong rainbow antimagic connection number of G, denoted by $srac(G)$, is the smallest number of colors taken over all strong rainbow coloring induced by strong rainbow antimagic labelings of G. In this paper, we have determined the connection number of strong rainbow antimagic coloring of some special graph.

2 Results

In this paper, we discuss some new results of the strong rainbow antimagic coloring some graphs.

Lemma 1. Let G be any connected graph. Then, $srac(G) \geq max\{rc(G), \Delta(G)\}$.

Proof. Based on Definition 1, it is known that $rac(G) \geq rc(G)$. Let $x \in V(G)$ where $d(x) = \Delta(G)$ and $f : V(G) \longrightarrow 1, 2, ...|V(G)|$ are bijective functions such that $f(u) \neq f(x)$, for each $ux \in E(G)$ and for each $ux, vx \in E(G), w(ux) \neq w(vx)$. then, $rac(G) \geq \Delta(G)$. Based on the explanation above, it can be seen that $rac(G) \geq max\{rc(G), \Delta(G)\}$ [6].

Table 1. The geodesic of J_n

case	Vertex 1	Vertex 2	Rainbow Path
1	x	x_i	x, x_i
2	x	y_i	x, x_i, y_i
3	x_i	x_i	$x_i x, x_i$
4	y_i	y_i	y_i, x_i, x, x_i, y_i

Theorem 1. Let J_n be a Jahangir Graph, the strong rainbow antimagic connection number of J_n is $n \leq srac(Jn) \leq n+3$.

Proof. Graph J_n is a Jahangir Graph. The vertex set of J_n is $v(J_n) = \{x\} \cup \{x_i; 1 \leq i \leq n\} \cup \{y_i; 1 \leq i \leq n\}$ and the edge set of J_n is $E(J_n) = \{xx_i; 1 \leq i \leq n\} \cup \{x_i y_i; 1 \leq i \leq n\} \cup \{y_i x_{i+1}; 1 \leq i \leq n\}$. The cardinality of vertices is $|V(J_n)| = 2n+1$ and cardinality of edges of Jahangir graph is $|E(J_n)| = 3n$. To prove $n \leq srac(J_n) \leq n+3$, we will show $n \leq srac(J_n)$ and $srac(J_n \leq n+3$. Firstly we show $n \leq srac(J_n)$. Based on Lemma 1, thus we identify the maximum degree and strong rainbow coloring of J_n is with $src(J_n) = 4$ and $\Delta(J_n) = n$.

$$srac(G) \geq max\{src(G), \Delta(G)\}$$
$$srac(J_n) \geq max\{src(J_n), \Delta(J_n)\}$$
$$srac(J_n) \geq max\{4, n\}$$
$$srac(J_n) = n$$

Therefore we show $srac(J_n) \leq n+3$. Let $f : V(J_n) \rightarrow \{1, 2, ..., 2n+1\}$ defined as follows

$$f(x) = 1$$
$$f(x_i) = i+1, \text{ for } 1 \leq i \leq n+1$$
$$f(y_i) = 2n+1-i, \text{ for } 1 \leq i \leq n$$

For the vertex labeling above, then we have edge weights that induces edges coloring of the graph J_n by the following:

$$w(xx_i) = i+2, \text{ for } 1 \leq i \leq n$$
$$w(x_i y_i) = 2n+2, \text{ for } 1 \leq i \leq n-1$$
$$w(x_n y_n) = 3n+2$$
$$w(y_i x_i) = 2n+3, \text{ for } 1 \leq i \leq n-1$$
$$w(y_n x_1) = 2n+3$$

Based on edge weights above we know that $w(y_i xi+1) = w(y_n x1) = 2n+3$. Then we have the set of edge weight $W = 3, 4, 5, i+2, ..., n+2, 2n+2, 2n+3, 3n+2$, this edge weights induces edges coloring of the graph J_n, so we have the $srac(J_n) \leq n+3$.
Next we show the geodesic of J_n by Table 1.

In Table 1, we will prove that the Jahangir with n vertices J_n admits a strong rainbow antimagic coloring. We describe its strong rainbow antimagic connection number in the following Fig. 1.

Theorem 2. Let SJ_n be a Semi-Jahangir Graph, the strong rainbow antimagic connection number of SJ_n is $n+1 \leq srac(SJn) \leq n+3$.

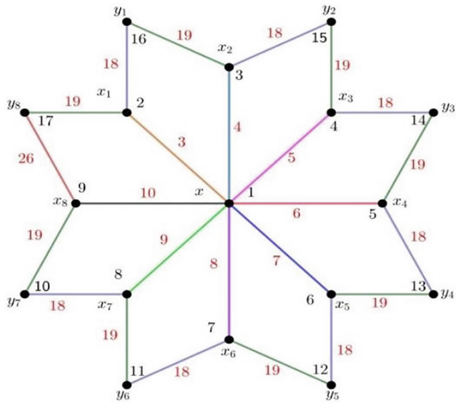

Fig. 1. A strong rainbow antimagic coloring of J_8.

Proof. Graph SJ_n is a Semi-Jahangir Graph. The vertex set of SJ_n is $v(SJ_n) = \{x\} \cup \{x_i; 1 \leq i \leq n+1\} \cup \{y_i; 1 \leq i \leq n\}$ and the edge set of SJ_n is $E(SJ_n) = \{xx_i; 1 \leq i \leq n+1\} \cup \{x_i y_i; 1 \leq i \leq n\} \cup \{y_i x_{i+1}; 1 \leq i \leq n\}$. The cardinality of vertices is $|V(SJ_n)| = 2n+2$ and cardinality of edges of Semi-Jahangir graph is $|E(SJ_n)| = 3n+2$. Based on $n+1 \leq srac(SJ_n)$ to identify the maximum degree and strong rainbow coloring of SJ_n, we can follows Lemma 1 to get the strong rainbow antimagic coloring number $srac(SJ_n) \leq n+1$, so it will be proved that $srac(SJ_n) \leq n+3$. Let $f: V(SJ_n) \rightarrow \{1, 2, 3, ..., 2n+2\}$ defined such that

$$f(x) = 1$$
$$f(x_i) = i+1, \text{ for } 1 \leq i \leq n+1$$
$$f(y_i) = 2n+3-i, \text{ for } 1 \leq i \leq n$$

From the labeling of vertex above we will get the edge weights, we can find the edge weights that cause edge coloring in the graph SJ_n with the following conditions

$$w(xx_i) = i+2, \text{ for } 1 \leq i \leq n$$
$$w(y_i x_i) = 2n+4, \text{ for } 1 \leq i \leq n$$
$$w(y_i x_{i+1}) = 2n+5, \text{ for } 1 \leq i \leq n$$

Based on the provisions of the edge weights above, it can be seen that the edge weights in a graph SJ_n are $W = 3, 4, 5, i+2, ..., n+2, 2n+4, 2n+5$. It is this edge weight that induces edge coloring on the SJ_n graph, so that $srac(SJn) \leq n+3$ is obtained.

For further explanation, here we present the SJ_n geodesic with Table 2.

It will be proved that the Semi Jahangir with n vertices SJn has a strong rainbow antimagic coloring, shown in Fig. 2.

Theorem 3. Let F_n be a Friendship Graph, the strong rainbow antimagic connection number of F_n is $2n \leq srac(F_n) \leq 2n+1$.

Proof. Graph F_n is a Friendship Graph. The vertex set of F_n is $V(F_n) = \{x\} \cup \{x_i; 1 \leq i \leq n\} \cup \{y_i; 1 \leq i \leq n\}$ and the edge set of F_n is $E(F_n) =$

Table 2. The geodesic of J_n

case	Vertex 1	Vertex 2	Rainbow Path
1	x	x_i	x, x_i
2	x	y_i	x, x_i, y_i
3	x_i	x_i	$x_i x, x_i$
4	y_i	y_i	y_i, x_i, x, x_i, y_i

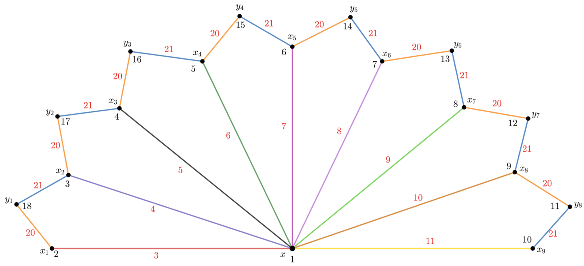

Fig. 2. The strong rainbow antimagic coloring of SJ_8.

$\{xx_i; 1 \leq i \leq n\} \cup \{y_i; 1 \leq i \leq n\} \cup \{x_i y_i l 1 \leq i \leq n\}$. The cardinality of vertices is $V(F_n) = 2n + 1$ and cardinality of edges of Friendship graph is $E(F_n) = 3n$. According to Theorem 3, $srac(F_n) \geq 2n$. Here we identify the maximum degree and the strongest rainbow color of the F_n. With $src(F_n) = 2, \Delta(F_n) = 2n$ and use Lemma 1 to prove the lower bound we get strong rainbow antimagic coloring number $srac(F_n) \geq 2n$.
Next we will prove the upper bound, let us define a vertex labeling $f : V(F_n) \rightarrow \{1, 2, 3, ..., 2n + 1\}$ as follows

$f(x) \;= 1$
$f(x_i) = i + 1$, for $1 \leq i \leq n$
$f(y_i) = 2n + 2 - i$, for $1 \leq i \leq n$

Therefore, the vertex weight are
$w(xx_i \;= i + 2$, for $1 \leq i \leq n$
$w(xy_i) \;= 2n + 3 - i$, for $1 \leq i \leq n$
$w(x_i y_i) = 2n + 3$, for $1 \leq i \leq n$

The edge weight that induces edge coloring on the F_n graph is $W = 3, 4, 5, i + 2, ..., n + 2, n + 3, 2n + 1$ so $srac(F_n) \leq 2n + 1$ is obtained.
The shortest distance (geodesic) of the F_n graph is presented in Table 3.

The Friendship graph with n nodes F_n has a strong rainbow antimagic coloring, the proof of which can be seen in Fig. 3.

Theorem 4. Let F_n be a Friendship Graph, the strong rainbow antimagic connection number of F_n is

$$srac(F_n) = \begin{cases} 4, & \text{for } n = 3 \\ n, & \text{for } n \geq 4 \end{cases}$$

Table 3. The geodesic of F_n

case	Vertex 1	Vertex 2	Rainbow Path
1	x	x_i	x, x_i
2	x	y_i	x, x_i, y_i
3	x_i	x_i	$x_i x, x_i$
4	y_i	y_i	y_i, x, y_i

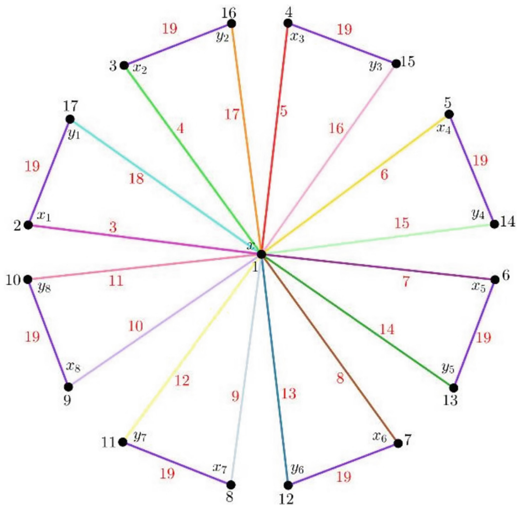

Fig. 3. The strong rainbow antimagic coloring of F_8

Proof. Let F_n be a Fan Graph with a vertex set $V(F_n) = \{z\} \cup \{y_i : 1 \leq i \leq n\}$ and edge set is $E(F_n) = \{zy_i; 1 \leq n\} \cup \{y_i y_{i+1}; 1 \leq i \leq n-1\}$. It's clear that $|V(F_n)| = n+1$ and $E|V(F_n)| = 2n-1$. To prove the strong rainbow antimagic coloring of F_n, we have to show the lower bound $srac(F_n) \geq n$, we have two cases, odd and even, which are $srac(F_n) = 4$ for $n = 3$ and $srac(F_n) = n$ for $n \geq 3$. Here we will show the lower bound. Fan Graph has a maximum degree $\Delta(F_n) = n$ and strong rainbow coloring number $src(F_n) = 2$. We use Lemma 1 get strong rainbow antimagic coloring number $srac(F_n) \geq n$. Then Let us define a vertex labeling $f : V(F_n) \rightarrow \{1, 2, 3, ..., n+1\}$ we will prove the upper bound, as follows

$$f(z) = 2$$
$$f(y_1) = n+1$$
$$f(y_2) = 1$$
$$f(y_i) = \begin{cases} \frac{i-2}{2} + 2, & 3 \leq i \leq n, \text{ for even} \\ \frac{-i+1}{2} + n + 1, & 3 \leq i \leq n, \text{ for odd} \end{cases}$$

Table 4. The geodesic of F_n

case	Vertex 1	Vertex 2	Rainbow Path
1	z	x_i	z, x_i
2	x_i	x_n	x_i, z, x_n

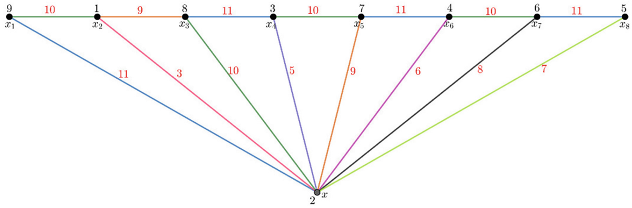

Fig. 4. The strong rainbow antimagic coloring of F_8

It can be seen that the edge weights that induce edge coloring on a Fan graph with n vertices F_n we get $srac(F_n) = 4$, for $n = 3$ and $srac(Fn) = n$, for $n \geq 3$. The geodesic of the Fn is presented in Table 4.

The Friendship graph with n nodes F_n has a strong rainbow antimagic coloring, the proof of which can be seen in Fig. 4.

Theorem 5. Let S_n be a Star Graph, the strong rainbow antimagic connection number of S_n is $srac(S_n) \leq n$

Proof. Graph S_n is a Star Graph. The vertex set of S_n is $V(S_n) = \{y\} \cup \{x_i; 1 \leq\leq n\}$ and the edge set of S_n is $E(S_n) = \{yx_i; 1 \leq i \leq n\}$. The cardinality of vertices is $|V(Sn)| = n + 1$ and cardinality of edges of Star graph is $|E(Sn)| = n$. Here we show n $srac(n)$, then we identify the lower bound to get a strong rainbow coloring of Sn by the maximum degree of Star graph . Star graph has a strong rainbow coloring number $src(Sn) = 2$ and $\Delta(Sn) = n$. Based on Lemma 1, we get strong rainbow antimagic coloring number $srac(Sn) \geq n$. Then next section, we will prove the upper bound, let us define a vertex labeling $f : V(S_n) \rightarrow \{1, 2, 3, ..., n + 1\}$ as follows

$f(y) \ = 1$
$f(x_i) = i + 1$, for $1 \leq i \leq n$
And the vertex weight from S_n graph are $w(yx_i) = i + 2$, for $1 \leq i \leq n$

The edge weight that induces edge coloring on the Sn graph is $W = \{3, 4, 5, i + 2, ..., n\}$, so $srac(S_n) \geq n$ is obtained.

The shortest distance (geodesic) of the S_n graph is presented in Table 5 and the Star graph with n nodes S_n has a strong rainbow antimagic coloring, the proof of which can be seen in Fig. 5.

Table 5. The geodesic of S_n graph

case	Vertex 1	Vertex 2	Rainbow Path
1	y	x_i	y, x_i
2	x_i	x_n	y_i, z, y_i

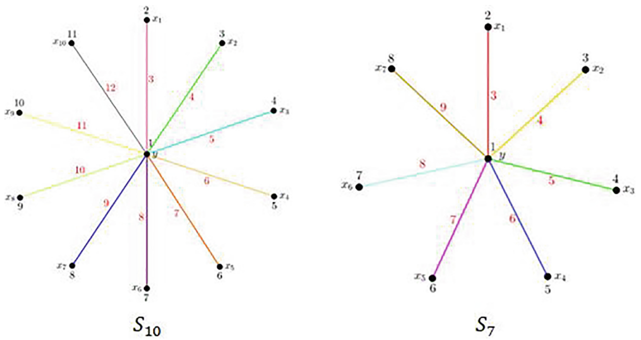

Fig. 5. (left) The strong rainbow antimagic coloring of S_{10} and (right) the strong rainbow antimagic coloring of S_7.

Theorem 6. Let P_n be a Path Graph, the strong rainbow antimagic connection number of P_n is $srac(P_n) \leq n-1$

Proof. Graph P_n is a Graph. The vertex set of P_n is $V(P_n) = \{x_i; 1 \leq i \leq n\}$ and the set of P_n is $E(P_n) = \{x_i x_{i+1}; 1 \leq i \leq n-1\}$. The cardinality of vertices is $|V(P_n)| = n$ and cardinality of edges of path graph is $|E(P_n)| = n-1$. Then we identify the maximum degree and strong rainbow coloring of P_n by following Lemma 1. Path graph has a strong rainbow coloring number $src(P_n) = n-1$ and maximum degree $\Delta(P_n) = 2$. So that we get $src(P_n) \geq n-1$.
Next section we will prove the upper bound, let us define a vertex labeling $f : V(P_n) \rightarrow 1, 2, 3, ..., n$ as follows

$$f(y) = 1$$

$$f(x_i) = \begin{cases} \frac{i+1}{2}, & \text{for even} \\ |\frac{n}{2}| + \frac{i}{2}, & \text{for odd} \end{cases}$$

Therefore, the vertex weight from P_n graph are $w(x_i x_{i+1} = |\frac{n}{2}|+i+1$, for $1 \leq i \leq n-1$

The edge weight that induces edge coloring on the P_n graph is $W = \{3, 4, 5, ..., n-1\}$, so $srac(P_n) \geq n$is obtained.
The shortest distance (geodesic) of the P_n graph is presented in Table 6 and the Path graph with n nodes P_n has a strong rainbow antimagic coloring, the proof of which can be seen in Fig. 6.

Table 6. The geodesic of S_n graph

case	Vertex 1	Vertex 2	Rainbow Path
1	x_i	x_i	x_i, x_i
2	x_i	y_n	$x_i, x_{i+1}, x_{i+2}, ..., x_n$

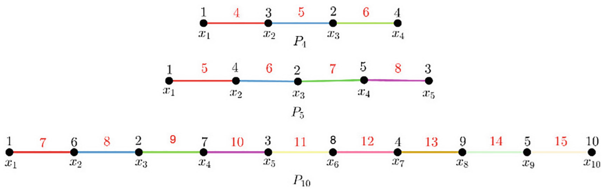

Fig. 6. The strong rainbow antimagic coloring of P_4, P_5, P_{10}.

Theorem 7. Let C_n be a Cycle Graph, the strong rainbow antimagic connection number of C_n is

$$Srac(C_n) = \begin{cases} |\frac{n}{2}|, & n \equiv 1{,}2 \pmod 4 \\ |\frac{n}{2}|, & n \equiv 0{,}3 \pmod 4 \end{cases}$$

Case 1. If $n \equiv 1 \bmod 4$, then define a vertex labeling as follows.

$$f(x_i) = \begin{cases} 2i-1, & 1 \le i \le \frac{n-1}{2}, \text{ for } i \text{ even} \\ 2i, & 1 \le i \le \frac{n-1}{2}, \text{ for } i \text{ odd} \end{cases}$$

$$f(x_i) = \begin{cases} 2i-n+1, & \frac{n+1}{2} \le i \le n, \text{ for } i \text{ even} \\ 2i-n, & \frac{n+1}{2} \le i \le n, \text{ for } i \text{ odd} \end{cases}$$

For the edges weights, we have

$$W(x_i x_{i+1}) = \begin{cases} 4i+1, & \text{for } 1 \le i \le \frac{n-1}{2} \\ 2i+2, & \text{for } i = \frac{n-1}{2} \\ 4i-2n+3, & \text{for } \frac{n-1}{2} < i \le n \end{cases}$$

$W(x_i x_{i+1}) = n+1$

Case 2. If $n \equiv 2 \bmod 4$, then define a vertex labeling as follows.

$$f(x_i) = \begin{cases} 2i-1, & 1 \le i \le \frac{n}{2}, \text{ for } i \text{ even} \\ 2i, & 1 \le i \le \frac{n}{2},, \text{ for } i \text{ odd} \end{cases}$$

$$f(x_i) = \begin{cases} 2i-n+1, & \frac{n}{2} \le i \le n, \text{ for } i \text{ even} \\ 2i-n, & \frac{n}{2} \le i \le n, \text{ for } i \text{ odd} \end{cases}$$

For the edges weights, we have

Table 7. The geodesic of F_n

case	Vertex 1	Vertex 2	Rainbow Path
1	z	x_i	z, x_i
2	x_i	x_n	x_i, z, x_n

$$W(x_ix_{i+1}) = \begin{cases} 4i+1, & \text{for } 1 \le i \le \frac{n}{2} \\ 2i+1, & \text{for } i = \frac{n}{2} \\ 4i-2n+1 & , \text{ for } \frac{n}{2} < i \le n \end{cases}$$

$W(x_ix_n) = n+1$

Case 3. If $n \equiv 0 \bmod 4$, then define a vertex labeling as follows.

$$f(x_i) = \begin{cases} 2i-1, & 1 \le i \le \frac{n}{2}, \text{ for } i \text{ even} \\ 2i, & 1 \le i \le \frac{n}{2}, \text{ for } i \text{ odd} \end{cases}$$

$$f(x_i) = \begin{cases} 2i-n, & \frac{n}{2} \le i \le n, \text{ for } i \text{ even} \\ 2i-(n+1), & \frac{n}{2} \le i \le n, \text{ for } i \text{ odd} \end{cases}$$

For the edges weights, we have

$$W(x_ix_{i+1}) = \begin{cases} 4i+1, & \text{for } 1 \le i \le \frac{n}{2} \\ 2i+2, & \text{for } i = \frac{n}{2} \\ 4i-2n+1, & \text{for } \frac{n}{2} < i \le n \end{cases}$$

$W(x_ix_n) = n$

Case 4. If $n \equiv 3 \bmod 4$, then define a vertex labeling as follows.

$$f(x_i) = \begin{cases} 2i-1, & 1 \le i \le \frac{n-1}{2}, \text{ for } i \text{ even} \\ 2i, & 1 \le i \le \frac{n-1}{2}, \text{ for } i \text{ odd} \end{cases}$$

$$f(x_i) = \begin{cases} 2i-n, & \frac{n-1}{2} \le i \le n, \text{ for } i \text{ even} \\ 2i-n, & \frac{n-1}{2} \le i \le n, \text{ for } i \text{ odd} \end{cases}$$

For the edges weights, we have

$$W(x_ix_{i+1}) = \begin{cases} 4i+1, & \text{for } 1 \le i \le \frac{n-1}{2} \\ 2i+1, & \text{for } i = \frac{n-1}{2} \\ 2i-2n+3, & \text{for } \frac{n-1}{2} < i \le n \end{cases}$$

$W(x_ix_n) = n+1$

Next we show the geodesic of C_n by Table 7.

The Friendship graph with n nodes C_n has a strong rainbow antimagic coloring, the proof of which can be seen in Fig. 7.

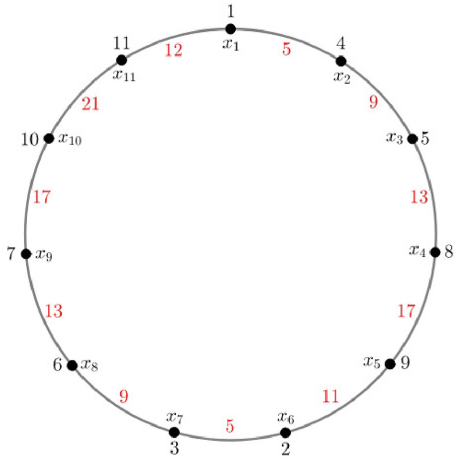

Fig. 7. The strong rainbow antimagic coloring of C_{11}.

3 Conclusions

We have obtained the exact value of strong rainbow antimagic coloring is obtained from several graphs, namely: Jahangir graph, Jahangir semi graph, friendship graph, fan graph, star graph, and path graph.

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