



The Queueing Model on the Parking Area: A Case Study at Hanoi University of Science and Technology

Cuong Duong Manh^(✉)  and Mai Nguyen Thi

Department of Industrial Management, School of Economics and Management,
Hanoi University of Science and Technology, Hanoi, Vietnam
cuong.duongmanh@hust.edu.vn

Abstract. This research aims to determine the queue model in a motorcycle parking area during rush hours at Hanoi University of Science and Technology (HUST) and its performance measures. Queueing for parking at the D3 – D5 parking lot is considered a case study. The observation approach is used to collect data in this study. Data taken are the number of arrivals and service time. The one-sample Kolmogorov-Smirnov test with SPSS Statistics is used to test arrival distribution and service time distribution. From there, determine the model of the parking lot based on Kendall Notation. The results of the analysis show that the arrival distribution was the Poisson distribution, and the service time distribution was the Exponential distribution. Based on the results of data analysis in this research and Kendall Notation, the queue model obtained was $(M/M/4): (FIFO/\infty/\infty)$. Accordingly, the proposed queue model is consistent with the actual queue model at the considered parking lot.

Keywords: Queueing Theory · Parking Lots · Kendall Notation

1 Introduction

Queues frequently emerge when the number of persons in need of services exceeds the capacity or when the service facilities are insufficient. In other words, when demand for a service exceeds the available capacity, a queue forms [1]. We often encounter a lots of queueing systems to wait for service crowd-based and random.

The queueing theory is an important study in modern society [2], which is related to the mathematical modeling and analysis of systems that serve random demands. Queueing models are widely used in industry to improve customer services, such as in supermarkets, banks, and toll booths on highways [3]. Production systems, transportation and stocking systems, communication systems, and information processing systems are all important application areas of queueing models. In addition, queueing models are particularly beneficial for the design of these systems in terms of layout, capacities, and control [4, p. 7]. The purpose of using queueing theory is to design service facilities, cope with randomly fluctuating demand, and maintain a balance between service costs (facility opening) and customer waiting costs [5].

© The Author(s) 2023

P. Tra Lam and P. Quang Huy (Eds.): ICECH 2022, AEBMR 238, pp. 127–135, 2023.

https://doi.org/10.2991/978-94-6463-150-0_9

With the development of the economy, the number of vehicles in Vietnam has increased significantly, and the queuing phenomena has become very widespread in road traffic. Research studies supporting the parking-related queueing theory have been done abroad. In the existing literature, various queueing models have been studied for parking systems, with the $M/M/s$ queue being the most often utilized. However, “the queueing theory applied to the problematics of parking has not yet been consistently researched and presented to the public, but it has only been partially analyzed and treated” [6]. And there are few studies on the scale of parking spaces in education buildings [7].

Through qualitative observations of the parking areas at the Hanoi university of science and technology (HUST), it can be found that the parking behavior of the university has distinctive features from other areas. During the class start and drop-off time, there is an accumulation of vehicles in the parking area. In this paper, the study is mainly focused on the time frame class starts in the morning. This is a showcase of how the entire modeling process of the mechanism of queues is formed, from data collection to construction queues and analysis of the queueing model. Propose solutions, then, are raised to improve the service process at the parking lot.

2 Basic Knowledge

2.1 Queueing Theory

Queueing theory is mostly considered a branch of applied probability theory that studies queues or waiting lines mathematically. It primarily employs mathematical models to examine the relationship between arrival interval, service intensity, queueing time, queue length, and other parameters in the context of specific service regulations in the system [8, 9]. In a way of succinct definition, “queueing theory is expressed as it relatively pertains to a waiting line as the mathematical analysis” [10]. Queueing theory consists of three parts: the input process, the service agencies, the queue discipline [11, 12]:

The input process is also known as the arrival process. The input process is the process by which objects enter the system and claim to meet a specific requirement. The requirements input process to the system is a stream of random events that follow probability distributions such as the Poisson distribution and the Erlang distribution.

The service agencies determine service rules and statistical service time rules. The service time is expressed as the number of time units required to serve one unit, i.e., to provide a specific service. Typically, it is presumable that it is independent of the arrival process and one another and service times distribution of servers to be identically, which can be divided into exponential distribution (M), Erlang distribution (E_k), fixed-length distribution (D), general shifted distribution (G).

The queue discipline explains the behavior of clients who arrive and discover that all servers are full. They can leave right away or wait in line. Customers who are queueing are summoned to service based on the characteristics of the customer. Customers are called to serve in a variety of ways: First in First Out/First Come First Served – FIFO/FCFS; Service in Random Order – SIRO; Last in First Out/Last Come First Served – LIFO/LCFS; Processor Sharing – PS; Priority service – PNP and so on.

The following is a fundamental queueing diagram [13, p. 23] (Fig. 1).

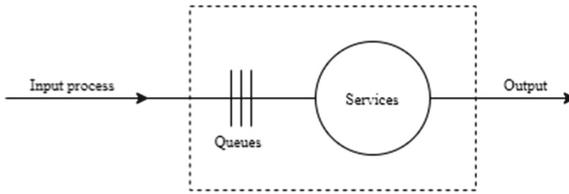


Fig. 1. Basic queuing model.

The initial proposed queuing model is (M/M/s): (FIFO/∞/∞). The M/M/s queue is provided with $s > 1$ identical server (also known as the channel) which can serve users in parallel. In this system, the arrival process distribution is Poisson with intensity $\lambda > 0$, the service time distribution is Exponential with parameter $\mu > 0$ at all servers. When a user comes and discovers that all servers are occupied (i.e., there are at least s users in the system), he or she queues up in the waiting line. The waiting users get served in the order of a FIFO (First in First Out) discipline. In the case an arriving user discovers that there are fewer than s users in the system (i.e., there are idle servers at the moment of arrival), he or she selects any server and begins service immediately. Final, the capacity queue (service system) there is no limit (∞) and unlimited (∞) input source [14, p. 56].

The measures of performance for the queuing system are the probability that the system will not work (P_o), utilization factor of the system (ρ), the average number of customers in the system (L_s), the average number - the customers waiting in line for service (L_q), the average time of a customer in the system (W_s), and the average time a customer waits in line to be served (W_q) as in Table 1.

In practice, it is common to use the serving station utilization factor ρ to evaluate the system with:

- $\rho < 0.4$: low, wasteful operation system
- $\rho \in [0,4;0,6]$: on medium, the system works efficiently
- $\rho > 0.6$: high, the system works slowly, needs to be improved.

Table 1. The performance metrics in the queue model (M/M/s): (FIFO/∞/∞).

Notations	Formulas
P_o	$P_o = \frac{1}{\left[\sum_{n=0}^{k-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n \right] + \frac{1}{k!} \left(\frac{\lambda}{\mu} \right)^k \frac{k\mu}{k\mu - \lambda}} \quad (k\mu > \lambda)$
ρ	$\rho = \frac{\lambda}{k \cdot \mu}$
L_s	$L_s = \frac{\lambda \mu (\lambda / \mu)^k}{(k-1)! (k\mu - \lambda)^2} P_o + \frac{\lambda}{\mu}$
L_q	$L_q = L_s - \frac{\lambda}{\mu}$
W_s	$W_s = \frac{\mu (\lambda / \mu)^k}{(k-1)! (k\mu - \lambda)^2} P_o + \frac{1}{\mu} = \frac{L_s}{\lambda}$
W_q	$W_q = W_s - \frac{1}{\mu} = \frac{L_q}{\lambda}$

2.2 Kolmogorov-Smirnov Test

In this study, we use a one-sample Kolmogorov - Smirnov test to assess the arrival process distribution and the service time distribution. To get the conclusion whether the expected distribution is consistent with the distribution of the observations, the hypothesis test for the Kolmogorov - Smirnov test is as follows:

H_0 = data follows the specified distribution.

H_1 = data does not follow the specified distribution.

With test criteria:

If the Asymp Sign value (2-tailed) > 0.05 then H_0 is accepted.

If the Asymp Sign value (2-tailed) < 0.05 then H_0 is rejected.

2.3 Kendall Notation

Kendall Notation is a standard system used to describe and classify queuing models to suit a queuing system [15]. It is needed for queue modeling. The Kendall notation is used to describe and model the queuing system under consideration. It is shown in (a/b/c): (d/e/f), where: a is the arrival distribution, b is the service distribution, c is the number of services, d is the service rule, e is the maximum allowable number in the queue system, f is the size of the calling source.

3 Research Methods

The type of research used in this study is quantitative research. The purpose of this study is to determine the queuing model applicable to the D3 HUST motorcycle parking area and measure the performance of the model and find out if the application of the model is effective.

The data collection method used is to directly observe the queues occurring. In this study, the data collected is the data on the number of vehicles arriving and the service time. Observations were made from the time the parking staff started serving students until the start of the first kip study in the morning (6:45 am).

Since parking area is a complicated system comprised of adequate system components and their interdependence; thus, defining the parking model is required prior to analyzing and planning parking capacities [6]. Based on the observed and collected results, a test is implemented to check the appropriate distribution to determine the arrival process distribution and the service time distribution by the one-sample Kolmogorov-Smirnov test. Statistics SPSS 20.0 is used. The next step is to define a queuing model based on Kendall Notation after finding a suitable distribution for incoming delivery, service provision, and queuing system analysis. Once, the queuing pattern was found, the performance measure was calculated in the queuing model based on the formula and using the POM-QM software for Windows.

4 Results and Discussion

The study was conducted over three months starting from April 4, 2022, to July 1, 2022, on weekdays from Monday to Friday. Observations were made in the morning at the time slots from the start of the parking queue (usually 6:20 am) until the first class started (6 h 45 min). Due to personal schedules, the total number of observation days are 21 days. In the parking lot, there is no limit regarding the size of the queue, the queue is counted when at least 2 objects are queuing to get the service [5]. Based on the research results, the parking queue schedule is shown in Table 2.

During the 21 days of observation, four Mondays, four Tuesdays, five Wednesdays, three Thursdays, and five Fridays were observed (observe Fig. 2).

The expected distribution was tested to check the consistent of the arrival process distribution and the service time distribution applicable to the D3parking area. The process of hypothesis testing was performed using SPSS 20.0 software with the hypothesis for one sample Kolmogorov-Smirnov test as follows:

4.1 Arrival Distribution Test

H_0 = the arrival population follows the Poisson distribution.

H_1 = the arrival population does not follow the Poisson distribution.

Decision - making is based on probability values (Asymp. Sig. (2 tailed)) with a value of $\alpha = 0.05$. If the Asymp Sign value (2-tailed) > 0.05 then H_0 is accepted, conversely if the Asymp Sign value (2-tailed) < 0.05 then H_0 is rejected (Table 3).

4.2 Service Distribution Test

H_0 = the service process is exponentially distribution.

H_1 = the service process is not exponentially distribution.

Table 2. Queuing schedule at the parking lot

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Queue	✓	✓	✓	✓	✓

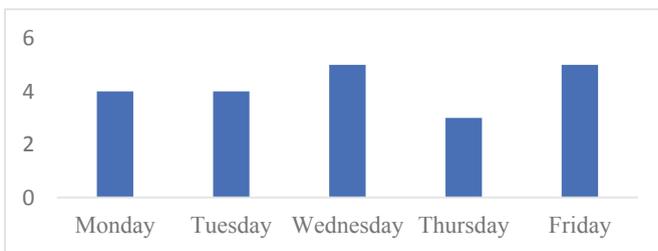


Fig. 2. The observation times

Table 3. Arrival Distribution Test

Day		Asymp. Sig. (2 tailed)	Result
Mon	1	0.231	H_0 accepted
	2	0.754	H_0 accepted
	3	0.367	H_0 accepted
	4	0.548	H_0 accepted
Tue	1	0.552	H_0 accepted
	2	0.873	H_0 accepted
	3	0.986	H_0 accepted
	4	0.635	H_0 accepted
Wed	1	0.378	H_0 accepted
	2	0.137	H_0 accepted
	3	0.846	H_0 accepted
	4	0.672	H_0 accepted
	5	0.811	H_0 accepted
Thu	1	0.840	H_0 accepted
	2	0.290	H_0 accepted
	3	0.179	H_0 accepted
Fri	1	0.204	H_0 accepted
	2	0.684	H_0 accepted
	3	0.967	H_0 accepted
	4	0.783	H_0 accepted
	5	0.860	H_0 accepted

Decision - making is based on probability values (Asymp. Sig. (2 tailed)) with a value of $\alpha = 0.05$. If the Asymp Sign value (2-tailed) > 0.05 then H_0 is accepted, conversely if the Asymp Sign value (2-tailed) < 0.05 then H_0 is rejected.

The service time data is processed in more detail. There are 700 related observations collected regarding service time. With the number of samples obtained ($m = 300$ samples), the test of fitness of the exponential distribution to the service time is done with the Asymp results. Sig. (2-tailed) = 0.216 > 0.05 , so the hypothesis H_0 is accepted or service time obeys the law of exponential distribution with density μ .

Based on the test results related to the arrival process distribution and the service delivery obtained above along with the results of the observations, the queuing pattern using Kendall Notation is determined: (a/b/c): (d/e/f) with: $a =$ Poisson distribution (M) with an average vehicle generation density of λ_i ($i = 1 \dots 22$) (vehicle/min), $b =$ exponential distribution (M) with the density of students going out of the ticket counters (finished service): $\mu = 7$ (vehicle/min), $c =$ four (4), $d =$ first in first out (FIFO), $e =$

unlimited (∞) and $f = \text{unlimited } (\infty)$. From this result, we come up with a queuing model at parking lot D3 as $(M/M/4): (FIFO/\infty/\infty)$.

The parking area service system performance measurements in the queue model $(M/M/4): (FIFO/\infty/\infty)$ were calculated through the software POM-QM for Windows 5 (observe Table 4).

From the results in Table 4, there is only one case on Friday, June 24, 2022, the system works effectively with the system usage coefficient $\rho = 0.57$. Most observations on the other days of the week are considered to have a high system utilization coefficient $\rho > 0.6$. The system, theoretically, operates economically, and slowly. The system, therefore, should be improved. However, most observations L_q is less than or equal to 1, which means that there is almost no queue in the parking area. But based on actual observations, the number of students arriving at each time is not the same, there are times when the system is overloaded, and students have to wait in long queues to wait for parking. The number can be up to several dozen people.

Table 4. Performance Size in the Queue Model $(M/M/4): (FIFO/\infty/\infty)$

Day		P_o (%)	ρ (%)	L_s (vehicles)	L_q (vehicles)	W_s (minutes)	W_q (minutes)
Mon	11	6	68	4	1	0.19	0.04
	22	5	71	4	1	0.2	0.06
	33	3	79	5	2	0.24	0.1
	44	5	71	4	1	0.2	0.06
Tue	11	6	68	4	1	0.19	0.04
	22	7	64	3	1	0.18	0.03
	33	7	64	3	1	0.18	0.03
	44	7	64	3	1	0.18	0.03
Wed	11	3	79	5	2	0.24	0.1
	22	6	68	4	1	0.19	0.04
	33	5	71	4	1	0.2	0.06
	44	7	64	3	1	0.18	0.03
	55	7	64	3	1	0.18	0.03
Thu	11	7	64	3	1	0.18	0.03
	22	6	68	4	1	0.19	0.04
	33	5	71	4	1	0.2	0.06
Fri	11	6	68	4	1	0.19	0.04
	22	7	64	3	1	0.18	0.03
	33	7	64	3	1	0.18	0.03
	44	9	57	3	0	0.16	0.02
	55	8	61	3	0	0.17	0.03

If the efficiency of the system is taken into consideration, one more serving station should be added, from 4 to 5. The largest number of students came and came on Monday, June 6, 2022, and Wednesday, June 15, 2022. Therefore, 6:20 – 6:45 am on 6/6/2022 and 15/6/2022 are used as samples to test the 5-stations model. The value of the arrival rate is 22 vehicles/min and other parameters of the system are still the same as the 4-stations model. The results show that, the system utilization coefficient $\rho > 0.6$, in theory the system operates inefficiently, however, with the utilization factor not too high ($\rho = 0.63$), it can be implied that the system operates economically. The value of $L_s = 4$, $L_q = 0$, $W_s = 0.16$, $W_q = 0.02$, so it can be concluded that there will be no queue at the parking area with the 5-stations system. However, in terms of cost, the system manager will have to deal with a huge waste of operating costs, not to mention the huge cost of opening an additional service station.

5 Conclusion

From the analysis and calculation of the results of research in the pathway in of the motorcycle parking area of student at the D3 building at Hanoi University of Science and Technology on Monday to Friday, it can be stated that a queuing model at the parking lot is (M/M/4): (FIFO/ ∞ / ∞). Based on the calculation of performance measures of the service system in the queuing model, it is implied that the system is an inefficient operation. In terms of rationality, with the current model of 4 serving stations at peak hours, the system is operating at full capacity, opening all 4/4 serving stations. In case some students want to get out of the parking lot, it is completely difficult because there is no exit station but only an entrance station. Therefore, organizing such a system requires more operational solutions.

References

1. Soon, W. M. A., and Ang, K. C.: Introducing queuing theory through simulations. *The Electronic Journal of Mathematics and Technology*, 9(2), 152–165, (2015).
2. Aronu, C., Okoh, J., Onyeka, E., and Ikemefuna, S.A.: The Assessment of Bank Service Performance in Delta State, Nigeria: A Queuing Theory Approach. *Further Applied Mathematics* 1(1), 10–25 (2021).
3. Gorunescu, F., McClean, S. I., and Millard, P. H.: A queueing model for bed-occupancy management and planning of hospitals. *Journal of the Operational Research Society* 53, 19–24 (2002).
4. Adan, I., and Resing, J.: *Queueing Theory*. Eindhoven University of Technology, (2002).
5. Thoha, U. K., Setawani, S., and Dafik.: *Aplikasi Teori Antrian Model Multi Channel Single Phase Dalam Optimalisasi Layanan Pembayaran Pelanggan Pada Senyum Media Stationery Jember*. *Uswatun dkk : Aplikasi Teori Antrian Model Multi*, (2014).
6. Thoha, U. K., Setawani, S., and Dafik.: *The Application Of Multi Channel Single Phase Model Of Queuing Theory In Optimizing Customer Payment Service In Senyum Media Stationery Jember*. University of Jember, (2014).
7. Maršanić, R., Zenzerović, Z., and Mrnjavac, E.: Application of the Queuing Theory in the Planning of Optimal Number of Servers (Ramps) In Closed Parking Systems. *Economic Research/Ekonoska Istraživanja*, 24 (2), 26–43, (2011).

8. Zhao, Y., Zhou, Z., Pan, Q., and Zhou, T.: *G/M/N Queueing Model-Based Research on the Parking Spaces for Primary and Secondary School*. *Discrete Dynamics in Nature and Society*, (3), 1–7, (2020).
9. Willig, A.: *A Short Introduction to Queueing Theory*. Technical University Berlin, Telecommunication Networks Group p. 41, (1999).
10. Sundarapandian, V.: *Probability, Statistics and Queueing Theory*. PHI Learning Pvt. Ltd., p. 686, (2009)
11. Afolalu, S. A., Babaremu, K. O., Ongbali, S. O., Abioye, A. A., Abdulkareem, A., and Adejuyigbe, S. B.: *Overview Impact Of Application Of Queueing Theory Model On Productivity Performance In A Banking Sector*. *Journal of Physics Conference Series*, 1378 (3), 1–9, (2019).
12. Cooper, R. B.: *Chapter 10 Queueing theory*. *Handbooks in Operations Research and Management Science*, 2, 469–518 (1990).
13. Bhat, U. N.: *An Introduction to Queueing Theory: Modeling and Analysis in Applications*. Birkhäuser, 1–14, (2015).
14. Adan, I., and Resing, J.: *Queueing Systems*. Eindhoven University of Technology, 182, (2015).
15. Breuer, L., and Baum, D.: *An Introduction to Queueing Theory: and Matrix-Analytic Methods*. Springer Science & Business Media, (2005).
16. Thang, L. Q., and Khang, P. N.: *The Queueing theory and Telecommunication systems application*. Can Tho University, (2013).

Open Access This chapter is licensed under the terms of the Creative Commons Attribution-NonCommercial 4.0 International License (<http://creativecommons.org/licenses/by-nc/4.0/>), which permits any noncommercial use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.

