Decomposition Hamilton in Cayley Graphs with Certain Invers Generator of Dihedral-2*n* Group

Astri Kumala[1], Hery Susanto[1](✉), Desi Rahmadani[1], and Wahyu Henky Irawan[2]

[1] State University of Malang, Jl. Semarang 5, Malang, Indonesia
hery.susanto.fmipa@um.ac.id
[2] Universitas Islam Negeri Maulana Malik Ibrahim Malang, Jl. Gajayana 50, Malang, Indonesia

Abstract. The Hamilton decomposition of graph G is a partition of the edge set into a Hamilton cycle and 1-factor if the vertex degree is odd or a partition into a Hamilton cycle if the degree of the vertex is even. In 2020, the focus was on determining the Hamilton decomposition of a Cayley graph in the dihedral-2p group, where p is a single prime. In this paper, the Hamiltonian decomposition of the Cayley graph will be determined from the dihedral-2n group, with $n \geq 3$. This research aims to determine Hamiltonian decomposition, which focuses on generating $\{r^{n-1}, s\}$ from dihedral groups. The research method is to determine the dihedral-2n group, determine the set of vertices and the set of edges of the Cayley graph, construct the Cayley graph generated by $\{r^{n-1}, s\}$, and decompose the Cayley graph using Hamiltonian. The vertex degree of the Cayley graph, constructed by the dihedral-2n group and generated by $\{r^{n-1}, s\}$, , is an odd vertex. Graph decomposition results are a Hamilton cycle and 1-factor (perfect matching).

Keywords: Cayley graph · decomposition · dihedral group · Hamilton cycle · 1-factor

1 Introduction

Algebraic graph theory is the scientific study of the application of algebraic tools and procedures to graph theory. Every year, the advancement of graph theory results in the emergence of new graph types. The Cayley graph is a type of graph that is in development. In 1878, a scientist named Arthur Cayley discovered Cayley graphs in order to illustrate the concept of abstract groups described by generating sets. Among the numerous research on graphs created from dihedral groups, the Cayley graph cospectral over dihedral groups discusses non-isomorphic cospectral with D_{2p}, $p \geq 13$ dihedral [1]. A graph is said to be singular in a dihedral group if it is a singular matrix [2]. A dihedral group graph with the parameter Hosoya that is not connected [3]. There exists a dihedral group D_{2n} and a symmetric subset of D_{2n}, allowing the Kirchhoff index and the resistance distance between two vertices of the graph $Cay(D_{2n}, S)$ to be computed [4].

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Research on Cayley graphs is an intriguing topic for graph algebra theory research and study. Here are various studies on algebraic graphs, such as the primitive edge of the Cayley graph on the abelian group and the dihedral group, which describes an edge-primitive graph if the automorphism group operates primitively on the edge set [5]. Cayley graph illustrating the symmetry group and the number of isomorphic graph patterns produced [6]. The whole categorization of graph products from groups to abelian that compose Cayley graphs as described by the standard is planar [7]. Grouping Pfaffian qualities on a dihedral group, the Cayley graph was derived from a complete bipartition graph by applying the dihedral group to Pfaffian attributes [8]. In addition, research on the properties of Cayley graphs in the S_n group has various results, one of which is the bipartite transposition graph of S_n [9].

The Hamiltonian decomposition of the Cayley graph in the dihedral group has not been the subject of extensive research. Paths and circuits infinite groups explains the theory that for any connected Cayley graph in dihedral-$2p$ where p is prime, there exists a Hamilton cycle. [10]. Decomposition Hamilton path of a complete multipartite graph is decomposable into Hamiltonian path disjoint [11]. Hamiltonian decomposition of a Cayley graph with a single end-vertex [12]. Then in 2020 [13] continue the research of Holsztynski and Strube by analyzing the Hamilton decomposition of a Cayley graph in a dihedral group with D_{2p}, where p is a prime. However, Zhou's research has not examined the decomposition if every connected edges of Cayley graph in dihedral-$2n$ with $n \geq 3$ has a Hamilton decomposition, if the dihedral group is altered to a symmetric group, if the dihedral group is supplanted by a dihedral subgroup, or if Euler's method is used to find the decomposition. In the previously study, Zhou simply demonstrates that if p is a single prime, then connected graph in Cayley of dihedral-$2p(D_{2p})$ has a Hamilton decomposition. Zhou shows that if a graph $Cay(D_{2p})$ has an even vertex degree then Hamilton decomposition is Hamilton cycles and if a graph $Cay(D_{2p})$ has an odd vertex degree then the Hamilton decomposition is a Hamilton cycle and 1-factor (perfect matching). Therefore, in this paper will continue and expand on previous research that has been carried out by Zhou, which show that a Hamilton decomposition exists in the Cayley graph for dihedral-$2n$ (D_{2n}) where $n \geq 3$, with certain generator $H' = \langle r^{n-1}, s\rangle$.

2 Results and Discussion

Following is a discussion on how to create the D_{2n} group and the inverse of the generator, which is the set of vertices and edges of the graph. $Cay(D_{2n}, H')$, $H' = \{r^{n-1}, s\}$.

The dihedral group is represented by D_{2n} for every $n \in \mathbb{Z}^+$ and $n \geq 3$ as the collection of symmetries in the regular n-side [14]. For instance, the dihedral group-8 (D_8) with the composition operation ($\circ$) applied to a regular quadrilateral containing all rotations and reflections can be expressed as follows: $D_8 = \left(1, r, r^2, r^3, s, sr, sr^2, sr^3\right)$. Then it is determined that the collection of vertex on D_{2n} equals to $\{s^a r^b | a = 0$ or $1; b = 0, 1, \ldots, n-1\}$.

The vertex set V used to generate the Cayley graph is a member of the dihedral-2n group once the group D_{2n} has been determined. This is a set of vertex to $V(G) = \{s^a r^b | a = 0$ or $1; b = 0, 1, \ldots, n-1\}$. In the meantime, the edge set of a Cayley graph constructed from a dihedral group consists of a pair of connected vertices between the members of the group set with the composition results of each member of the group set and the generator of the dihedral group. The edge set $E(G) = \{(g, gh); g \in D_{2n}, h \in \{r^{n-1}, s\}\}$. In accordance with the definition of Hamilton decomposition, if the vertex degree has an odd number, the Hamilton decomposition is Hamilton cycle and a 1-factor (perfect matching), however if the vertex degree is even, the Hamilton decomposition is two Hamilton cycle [15].

To demonstrate the inverse of a generator that can produce a dihedral-2n group, the entry below is made

Lemma 2.1 [16]
D_{2n} Is generated by the generator $\langle r^k, s\rangle$ if and only if k is relatively prime to n.

As a result of the preceding entry, $\{r^k, s\}$ is a generator on D_{2n}, since k is prime relative to n. Since the inverse of n is $n - 1$, we may say that $\{r^{n-1}, s\}$ constructs D_{2n}.

Theorem 2.2
If a graph $Cay\left(D_{2n}, H^{'}\right)$, $H^{'} = \{r^{n-1}, s\}$ as generator and $n \geq 3$, the graph $Cay\left(D_{2n}, H^{'}\right)$ has Hamilton decomposition. The decomposition Hamilton of graph $Cay(D_{2n}, H^{'})$ are Hamilton cycle $\{(r^a, r^{a+n-1}), (sr^b, sr^{b+n-1}), (sr^{n-2}, r^2) | a = 0, \ldots, n-1, b = 0, \ldots, n-2\}$ and 1-factor
$\{(1, s), (r^b, sr^{n-b}), (r^2, r^{2+n-1}) | b = 0, 3, 4 \ldots, n-1\}$.

Let graph $Cay(D_{2n}, H^{'})$, $H^{'} = \{r^{n-1}, s\}$, $n \geq 3$.

$\{(r^a, r^{a+n-1}), (sr^b, sr^{b+n-1}), (sr^{n-2}, r^2) | a = 0, \ldots, n-1, b = 0, \ldots, n-2\} + \{(1, s), (r^b, sr^{n-b}), (r^2, r^{2+n-1}) | b = 0, 3, 4 \ldots, n-1\}$ is the set of a graph edges $Cay(D_{2n}, H^{'})$.

***Proof*.** The existence of a Hamilton decomposition on the graph $Cay(D_{2n}, H^{'})$ with $H^{'}$ as generator will be demonstrated.

First, we shall demonstrate that each edge of the graph is a subgraph $Cay(D_{2n}, H^{'})$.

Consider the set of edges that make up the following Hamilton cycle.

$$\{\left(r^a, r^{a+n-1}\right), \left(sr^b, sr^{b+n-1}\right), \left(sr^{n-2}, r^2\right) | a = 0, \ldots, n-1, b = 0, \ldots, n-2\}$$

According to the definition of the edge set of a Cayley graph, each edge can be characterized as follows:

- The edge of $(r^a, r^{a+n-1}) = (r^a, r^a \circ r^{n-1})$; $r^a \in D_{2n}$, $r^{n-1} \in H^{'}$.
- The edge of $(sr^b, sr^{b+n-1}) = (sr^b, sr^b \circ r^{n-1})$; $sr^b \in D_{2n}$, $r^{n-1} \in H^{'}$.
- The edge of $(sr^{n-2}, r^2) = (sr^{n-2}, sr^{n-2} \circ s)$; $sr^{n-2} \in D_{2n}$, $s \in H^{'}$.

In the form of a Hamilton cycle, it is shown that the set is a subgraph of the graph $Cay(D_{2n}.H^{'})$.

Considering the set of edges of the graph $Cay(D_{2n}, H^{'})$ that remains after subtracting the edges forming the Hamilton cycle.

$$\left\{(1, s), \left(r^{b}, sr^{n-b}\right), \left(r^{2}, r^{2+n-1}\right)|b = 0, 3, 4 \ldots, n-1\right\}$$

It will be demonstrated that the last edges is a subgraph of the graph $Cay(D_{2n}, H^{'})$ that forms 1-factor or perfect matching.

- The edge of $(1, s) = (1, 1 \circ s)$; $1 \in D_{2n}, s \in H^{'}$.
- The edge of $\left(r^{b}, sr^{n-b}\right) = \left(r^{b}.r^{b} \circ s\right)$; $r^{b} \in D_{2n}, s \in H^{'}$.
- The edge of $\left(r^{2}, r^{2+n-1}\right) = \left(r^{2}, r^{2} \circ r^{n-1}\right)$; $r^{2} \in D_{2n}, r^{n-1} \in H^{'}$.

The set is shown to be a subgraph of the graph $Cay(D_{2n}, H^{'})$, which forms 1-factor or perfect matching.

It is demonstrated that Hamilton can decompose the graph $Cay(D_{2n}, H^{'})$. Since the number of vertex on the graph $Cay\left(D_{2n}, H^{'}\right)$ is odd. ■

The result of the Hamilton decomposition is the Hamilton cycle and the 1-factor.

Example 2.3
Following is an illustration of a graph $Cay(D_{2n}, H^{'})$ created and decomposed in a Hamilton man.

Known graph $Cay(D_{2n}, H^{'})$, $H^{'} = \langle r^{n-1}, s\rangle$ and $n \geq 3$.

Take $n = 4$, $D_{2n} = D_{8}$. Graf $Cay(D_{8}, H^{'})$ generated by a generator $H^{'} = \langle r^{3}, s\rangle$ the following applies.

$1 \in D_{8} \to \left(1, 1 \circ r^{3}\right) = (1, r^{3})$ and $(1, 1 \circ s) = (1, s)$.

$r \in D_{8} \to \left(r, r \circ r^{3}\right) = (1, r)$ and $(r, r \circ s) = (r, sr^{3})$.

$r^{2} \in D_{8} \to \left(r^{2}, r^{2} \circ r^{3}\right) = (r^{2}, r)$ and $\left(r^{2}, r^{2} \circ s\right) = (r^{2}, sr^{2})$.

$r^{3} \in D_{8} \to \left(r^{3}, r^{3} \circ r^{3}\right) = (r^{3}, r^{2})$ and $\left(r^{3}, r^{3} \circ s\right) = (r^{3}, sr)$.

$s \in D_{8} \to \left(s, s \circ r^{3}\right) = (s, sr^{3})$ and $(s, s \circ s) = (s, 1)$.

$sr \in D_{8} \to \left(sr, sr \circ r^{3}\right) = (sr, s)$ and $(sr, sr \circ s) = (sr, r^{3})$.

$sr^{2} \in D_{8} \to \left(sr^{2}, sr^{2} \circ r^{3}\right) = (sr^{2}, sr)$ and $\left(sr^{2}, sr^{2} \circ s\right) = (sr^{2}, r^{2})$. $sr^{3} \in D_{8} \to \left(sr^{3}, sr^{3} \circ r^{3}\right) = \left(sr^{3}, sr^{2}\right)$ and $\left(sr^{3}, sr^{3} \circ s\right) = (sr^{3}, r)$.

Figure 1 is a graph $Cay(D_{8}, H^{'})$ which can be decomposed by Hamiltonian. As a result of the vertex degree on the graph $Cay(D_{8}, H^{'})$ is odd, because the vertex degree is odd, the Hamilton decomposition consists of a Hamilton cycle and 1-factor.

Cycle Hamilton is $\{(r, 1), \left(r, sr^{3}\right), \left(s, sr^{3}\right),$
$(sr, s), \left(sr^{2}, sr\right), \left(sr^{3}, sr^{2}\right), \left(r^{3}, r^{2}\right), (1, r^{3})\}$ and
1-factor or perfect matching is $\{(1, s), \left(r^{2}, r\right), \left(r^{3}, sr\right),$
$(sr^{3}, sr^{2})\}$.

The existence of a Hamilton decomposition on graph is demonstrated $Cay(D_{2n}, H^{'})$.

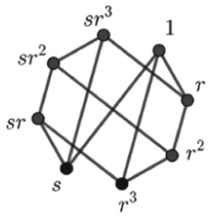

Fig. 1. Graph $Cay(D_8, H^{'})$

The pattern of graphed Hamiltonian decomposition results is depicted in Table 1 $Cay(D_{2n}, H^{'})$.

Figure 2 is a graphic of graph $Cay(D_{2n}, H^{'})$ and the result for Hamilton decomposition is Fig. 3 and Fig. 4. Figure 3 is Hamilton cycle of graph $Cay(D_{2n}, H^{'})$ and Fig. 4 is a 1-factor of graph $Cay(D_{2n}, H^{'})$.

Table 1. Hamilton decomposition of graph $Cay(D_{2n}, H^{'})$

Dihedral-2*n*	**Cycle Hamilton**	**1-Factor**
D_6	$(r,1), (r, sr^2), (s, sr^2), (sr, s), (sr, r^2), (1, r^2)$	$\{(1,s), (r^2, r), (sr^2, sr)\}$
D_8	$(r,1), (r, sr^3), (s, sr^3), (sr, s), (sr^2, sr), (sr^2, r^2), (r^3, r^2), (1, r^3)$	$\{(1,s), (r^2, r), (r^3, sr), (sr^3, sr^2)\}$
D_{10}	$(r,1), (r, sr^4), (s, sr^4), (sr, s), (sr^2, sr), (sr^3, sr^2)$ $(sr^3, r^2), (r^3, r^2), (r^4, r^3), (1, r^4)$	$\{(1,s), (r^2, r), (r^3, sr^2), (r^4, sr), (sr^4, sr^3)\}$
D_{12}	$(r,1), (r, sr^5), (s, sr^5), (sr, s), (sr^2, sr), (sr^3, sr^2)$ $(sr^4, sr^3), (sr^4, r^2), (r^3, r^2), (r^4, r^3), (r^5, r^4), (1, r^5)$	$\{(1,s), (r^2, r), (r^3, sr^3), (r^4, sr^2), (r^5, sr), (sr^5, sr^4)\}$

(*continued*)

Table 1. (*continued*)

Dihedral-2*n*	**Cycle Hamilton**	**1-Factor**
D_{14}	$(r,1),\left(r,sr^6\right),\left(s,sr^6\right),$ $(sr,s),\left(sr^2,sr\right),\left(sr^3,sr^2\right)$ $\left(sr^4,sr^3\right),\left(sr^5,sr^4\right),\left(sr^5,r^2\right),$ $\left(r^3,r^2\right),\left(r^4,r^3\right),\left(r^5,r^4\right),$ $\left(r^6,r^5\right),\left(1,r^6\right)$	$\left\{\begin{array}{c}(1,s),\left(r^2,r\right),\left(r^3,sr^4\right),\\ \left(r^4,sr^3\right),\left(r^5,sr^2\right),\left(r^6,sr\right),\\ \left(sr^6,sr^5\right)\end{array}\right\}$
D_{16}	$(r,1),\left(r,sr^7\right),\left(s,sr^7\right),$ $(sr,s),\left(sr^2,sr\right),\left(sr^3,sr^2\right)$ $\left(sr^4,sr^3\right),\left(sr^5,sr^4\right),(sr^6,sr^5)$ $\left(sr^6,r^2\right),\left(r^3,r^2\right),\left(r^4,r^3\right),$ $\left(r^5,r^4\right),\left(r^6,r^5\right),\left(r^7,r^6\right),$ $\left(1,r^7\right)$	$\left\{\begin{array}{c}(1,s),\left(r^2,r\right),\left(r^3,sr^5\right),\\ \left(r^4,sr^4\right),\left(r^5,sr^3\right),\left(r^6,sr^2\right),\\ \left(r^7,sr\right),\left(sr^7,sr^6\right)\end{array}\right\}$
D_{18}	$(r,1),\left(r,sr^8\right),\left(s,sr^8\right),$ $(sr,s),\left(sr^2,sr\right),\left(sr^3,sr^2\right)$ $\left(sr^4,sr^3\right),\left(sr^5,sr^4\right),(sr^6,sr^5)$ $\left(sr^7,sr^6\right),\left(sr^7,r^2\right),\left(r^3,r^2\right),$ $\left(r^4,r^3\right),\left(r^5,r^4\right),\left(r^6,r^5\right),$ $\left(r^7,r^6\right),\left(r^8,r^7\right),\left(1,r^8\right)$	$\left\{\begin{array}{c}(1,s),\left(r^2,r\right),\left(r^3,sr^6\right),\\ \left(r^4,sr^5\right),\left(r^5,sr^4\right),\left(r^6,sr^3\right),\\ \left(r^7,sr^2\right),\left(r^8,sr\right),\left(sr^8,sr^7\right)\end{array}\right\}$

(*continued*)

Table 1. (*continued*)

Dihedral-2*n*	**Cycle Hamilton**	**1-Factor**
D_{20}	$(r, 1), \left(r, sr^9\right), \left(s, sr^9\right),$ $(sr, s), \left(sr^2, sr\right), \left(sr^3, sr^2\right)$ $\left(sr^4, sr^3\right), \left(sr^5, sr^4\right), (sr^6, sr^5)$ $\left(sr^7, sr^6\right), \left(sr^8, sr^7\right), \left(sr^8, r^2\right),$ $\left(r^3, r^2\right), \left(r^4, r^3\right), \left(r^5, r^4\right),$ $\left(r^6, r^5\right), \left(r^7, r^6\right), \left(r^8, r^7\right),$ $\left(r^9, r^8\right), \left(1, r^9\right)$	$\left\{ \begin{array}{c} (1, s), \left(r^2, r\right), \left(r^3, sr^7\right), \\ \left(r^4, sr^6\right), \left(r^5, sr^5\right), \left(r^6, sr^4\right), \\ \left(r^7, sr^3\right), \left(r^8, sr^2\right), \left(r^9, sr\right), \\ \left(sr^9, sr^8\right) \end{array} \right\}$
⋮	⋮	⋮
D_{2n}	$\left(r^a, r^{a+n-1}\right), \left(sr^b, sr^{b+n-1}\right),$ $\left(sr^{n-2}, r^2\right);$ For $a = 0, \ldots, n-1,$ $b = 0, \ldots, n-2$	$\left\{ \begin{array}{c} (1, s), \left(r^b, sr^{n-b}\right), \\ \left(r^2, r^{2+n-1}\right) \end{array} \right\};$ For $b = 0, 3, 4, \ldots, n-1$

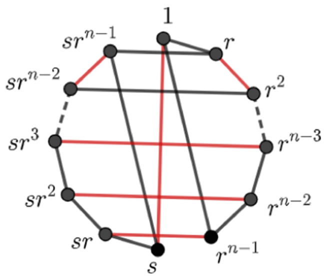

Fig. 2. Graph $Cay(D_{2n}, H^{'})$

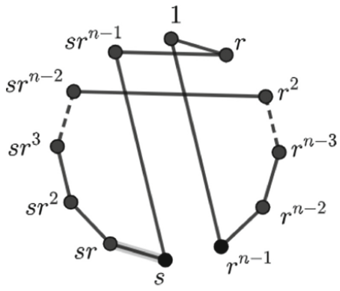

Fig. 3. Hamilton cycle of graph $Cay(D_{2n}, H^{'})$

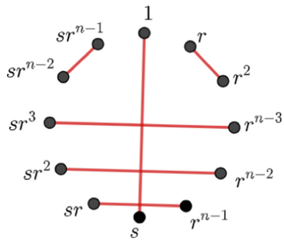

Fig. 4. 1-factor of graph $Cay(D_{2n}, H^{'})$

3 Conclusion

This article concludes that graphs are subject to a Hamilton decomposition $Cay\left(D_{2n}, H^{'}\right)$, $n \geq 3$, using a generator $H^{'} = \left\{r^{n-1}, s\right\}$ is a Hamilton cycle and a 1-factor (perfect matching). Derived from graph $Cay\left(D_{2n}, H^{'}\right)$ is $\left(r^a, r^{a+n-1}\right)$, $\left(sr^b, sr^{b+n-1}\right)$, $\left(sr^{n-2}, r^2\right)$ by $a = 0, \ldots, n-1$, $b = 0, \ldots, n-2$ and Graph 1-factor $Cay(D_{2n}, H^{'})$ is $(1, s)$, $\left(r^b, sr^{n-b}\right)$, $\left(r^2, r^{2+n-1}\right)$ by $b = 0, 3, 4, \ldots, n-1$.

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