The Effect of Helical Gears Parameters on Specific Sliding

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Abstract. Specific sliding is an essential factor in terms of surface wear of involute gears (involute gear teeth). At the same time, their values affect the heat generation in gearing. Since the values of the specific sliding reach its maximum at the gear tips and roots, the wear of these parts is the most significant, especially in the pinion's root. Thus, this specific place suffers pitting the most, as the values of specific sliding are the highest there. Therefore, it is beneficial for the designer to know how the geometrical and technological parameters can directly affect the value of specific sliding during the design of the gears. However, disadvantage is that the various parameters interact with each other, which can be difficult for the designer due to various design constraints. This article discusses the effect of changing geometrical parameters on the specific sliding magnitude. Furthermore, the interdependence of these parameters concerning each other is also discussed.

Keywords: Helical gear · Helix angle · Specific sliding · Pressure angle

1 Introduction

Geometrical parameters that directly influence the shape of the tooth involute and thus the size of the measured sliding are the pressure angle, the helix angle, and the individual shift coefficients of the wheel and pinion. In order to find the dependency of geometrical parameters on specific sliding and other parameters, one specified parameter must always remain constant. The aim of this study is to provide clear summarisation of interdependencies of geometrical parameters and their influences on specific sliding conditions.

2 Materials and Methods

2.1 Pressure Angle

The objective of the following problem is to find the dependence between the pressure angle and other parameters (working centre distance a_w, working transverse pressure angle α_{tw}, specific sliding at pinion's root ϑ_{A1}). For this reason, it is necessary to choose

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a constant sum of shift coefficients x_Σ. The division of shift coefficients is optimised to reach the same values of specific sliding at the wheel and pinion's tip and root. See Eqs. (1) and (2).

$$x_{1v} = x_\Sigma - x_{2v} \tag{1}$$

$$\begin{aligned}
&\frac{1}{\sqrt{\left(\frac{a_w}{m_n} + h_{a1}^* - \frac{z_2}{2\cdot\cos\beta} - x_{2v}\right)^2 \cdot \frac{4\cdot(tg^2\alpha_n + \cos^2\beta)}{z_1^2} - 1}} \\
&- \frac{1}{\sqrt{\left(\frac{a_w}{m_n} + h_{a1}^* - \frac{z_2}{2\cdot\cos\beta} - x_{2v}\right)^2 \cdot \frac{4\cdot(tg^2\alpha_n + \cos^2\beta)}{z_1^2} - 1}} \\
&- \frac{\frac{z_2}{z_1}}{\sqrt{\left(\frac{a_w}{m_n} + h_{a2}^* - \frac{z_1}{2\cdot\cos\beta} + x_{2v} - x_\Sigma\right)^2 \cdot \frac{4\cdot(tg^2\alpha_n + \cos^2\beta)}{z_2^2} - 1}} \\
&+ \frac{\frac{z_2}{z_1} - 1}{\sqrt{\left(\frac{a_w}{m_n}\right)^2 \cdot \frac{4\cdot(tg^2\alpha_n + \cos^2\beta)}{(z_1+z_2)^2} - 1}} = 0
\end{aligned} \tag{2}$$

2.2 Constant Sum of Profile Shift Coefficients X_Σ

Firstly, it is necessary to set a constant value of shift coefficient sum; in this case, its value equals $x_\Sigma = 0{,}7$. Table 1 shows the basic parameters of the gearset. The derivation of the working centre distance a_w is based on relations (3), (4), and (5). The pressure angle α_n in the frontal plane is given by relation (6). Finally, relation (7) calculates the working centre distance a_w concerning the profile angle α_n.

$$a_w = a \cdot \frac{\cos\alpha_t}{\cos\alpha_{tw}} \tag{3}$$

Table 1. Basic parameters of the gearset for pressure angle calculation

Number of pinion teeth	z_1	20	[-]
Number of wheel teeth	z_2	40	[-]
Normal module	m_n	1	[-]
Helix angle	β	10	[°]
Pinion addendum factor	$h_{\mathrm{a}1}^*$	1,5	[-]
Wheel addendum factor	$h_{\mathrm{a}2}^*$	1,5	[-]
Pinion tip clearance factor	c_1^*	0,25	[-]
Wheel tip clearance factor	c_2^*	0,25	[-]

$$a = \frac{(z_1 + z_2) \cdot m_n}{2 \cdot \cos\beta} \tag{4}$$

$$a_w = a \cdot \frac{\cos\alpha_t}{\cos\alpha_{tw}} \tag{5}$$

$$\alpha_t = \tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta}\right) \tag{6}$$

$$a_w = \frac{(z_1 + z_2) \cdot m_n}{2 \cdot \cos\beta} \cdot \frac{\cos\left(\tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta}\right)\right)}{\cos\alpha_{tw}} \tag{7}$$

It is also necessary to find the dependence of the pressure angle α_n on the working pressure angle α_{tw}. The relation (8) is used for this purpose. The working pressure angle α_{tw} is calculated from the involute (9).

$$\text{inv}\,\alpha_{tw} = \frac{2 \cdot x_\Sigma}{(z_1 + z_2)} \cdot \tan\alpha_n + \text{inv}\,\alpha_t \tag{8}$$

$$\tan\alpha_{tw} - \alpha_{tw} - \text{inv}\,\alpha_{tw} = 0 \tag{9}$$

The last relation (10) shows the specific sliding at the pinion root ϑ_{A1} concerning the pressure angle, where the working centre distance a_w and d_{a2} must be specified parametrically.

$$\vartheta_{A1} = 1 - \frac{z_1}{z_2} \cdot \frac{\sqrt{d_{a2}^2 - d_{b2}^2}}{2 \cdot a_w \cdot \sin\alpha_{tw} - \sqrt{d_{a2}^2 - d_{b2}^2}} \tag{10}$$

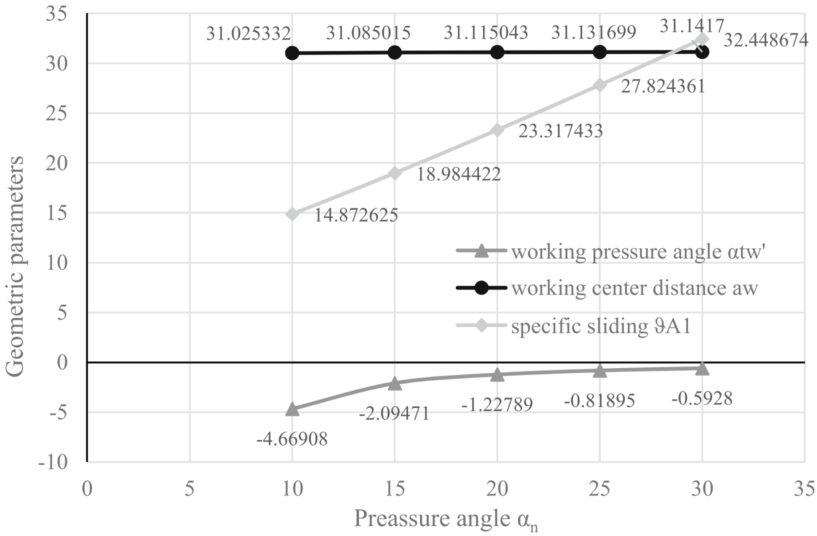

Fig. 1. Geometrical parameters dependencies on pressure angle α_n.

All dependencies are shown in Fig. 1.

Figure 1 shows that as the pressure angle α_n increases, both the working pressure angle α_{tw} and working centre distance a_w increase; conversely the value of specific sliding ϑ_{A1} decrease.

3 Results and Discussing

Another parameter that affects the specific sliding ϑ_{A1} is the helix angle β. To find dependence of the helix angle β on other geometrical parameters (a_w, α_{tw}, ϑ_{A1}, x_Σ,) the problem is divided into two subproblems as follows.

3.1 Constant Sum of Profile Shift Coefficients X_Σ

The above is also based on the same equations as mentioned in Sect. 2.1. The only difference is that the pressure angle α_n is constant, according to Table 2, and the helix angle β changes. Again, the shift coefficient is necessary to compensate for the specific sliding at the gears tip and root. The sum of shift coefficients value equals the same value as shown in the previous chapter $x_\Sigma = 0{,}7$.

All dependencies are shown in Fig. 2.

Figure 2 shows that as the helix angle β increases, both the working pressure angle α_{tw} and working centre distance a_w increase, conversely the value of specific sliding ϑ_{A1} decrease.

3.2 Constant Working Trans. Pressure Angle α_{tw}

In this problem, the working pressure angle α_{tw} is constant. The dependence of the helix angle β on the following parameters (a_w, x_Σ, ϑ_{A1}) must be found. Dependence of the helix angle β on the working centre distance a_w is based on Eq. (7), the mutual dependence between helix angle β and the sum of shift coefficients x_Σ is calculated in Eq. (11) [1].

$$x_\Sigma = \left(\tan\alpha_{tw} - \alpha_{tw} - \frac{\tan\alpha_n}{\cos\beta} + \tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta} \right) \right) \cdot \frac{z_1 + z_2}{2 \cdot \tan\alpha_n} \tag{11}$$

Table 2. Basic parameters of the gearset for helix angle calculation

Number of pinion teeth	z_1	20	[-]
Number of wheel teeth	z_2	40	[-]
Normal module	m_n	1	[-]
Pressure angle	α_n	20	[°]
Pinion addendum factor	h^*_{a1}	1,5	[-]
Wheel addendum factor	h^*_{a2}	1,5	[-]
Pinion tip clearance factor	c^*_1	0,25	[-]
Wheel tip clearance factor	c^*_2	0,25	[-]

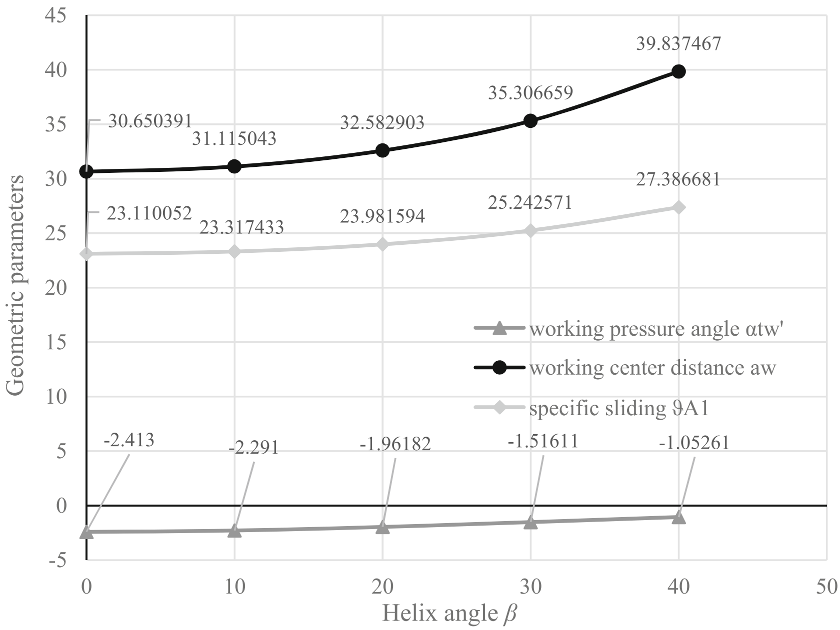

Fig. 2. Geometrical parameters dependencies on helix angle β (constant x_Σ).

The last relation is a mutual dependence of the helix angle β and the value of specific sliding ϑ_{A1}. The shift coefficient required to achieve balanced sliding is calculated using relations (1) and (2). Finally, the numerical value of the specific sliding at the pinion and wheel's root is determined by the following relation (10). In this equation, the parameters a_w and d_{a2} must be expressed as a function of the helix angle β, which gives a system of transcendental equations that must be solved numerically[1]. All three dependencies described above are shown in Fig. 3.

Figure 3 shows that as the helix angle β increases, the working centre distance a_w increases; conversely, the value of specific sliding ϑ_{A1} and the sum of shift coefficients x_Σ decrease.

3.3 Profile Shift Coefficients

Shift coefficients are an essential parameter to optimise the specific sliding at the pinion and wheel's tips and roots. When shift coefficients are changed, the working diameters change, which affects the value of specific sliding. Sum of shift coefficients' calculation is based on a value $x_\Sigma = 0{,}8667$ for a constant working centre distance $a_w = 65$ mm to determine the dependence of shift coefficients and specific sliding values. Ten different scenarios, where ratios of shift coefficients x_1/x_2 vary while their sum remains constant, are established. The last step is to calculate the specific sliding's value at the gear's tips and roots. The following graph shows the shift coefficients' dependencies on specific sliding.

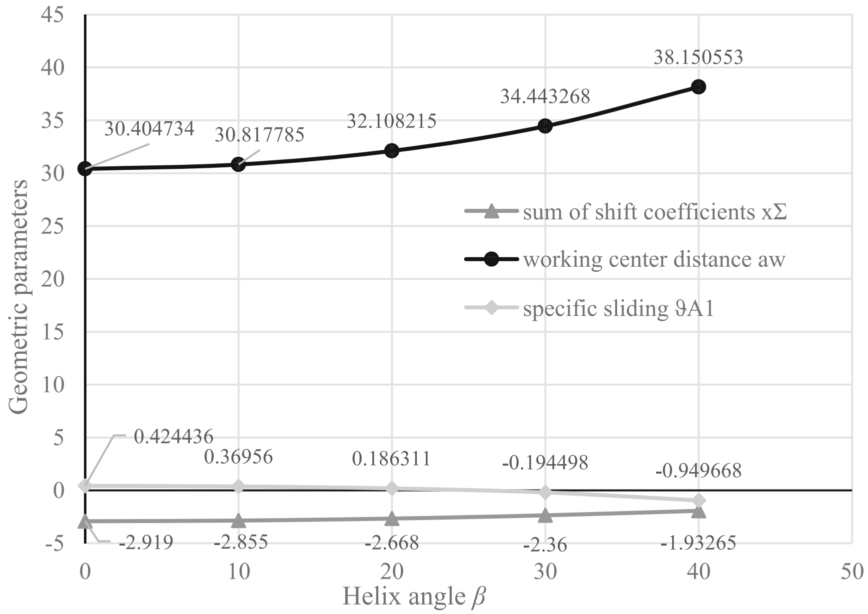

Fig. 3. Geometrical parameters dependencies on helix angle β (constant α_{tw}).

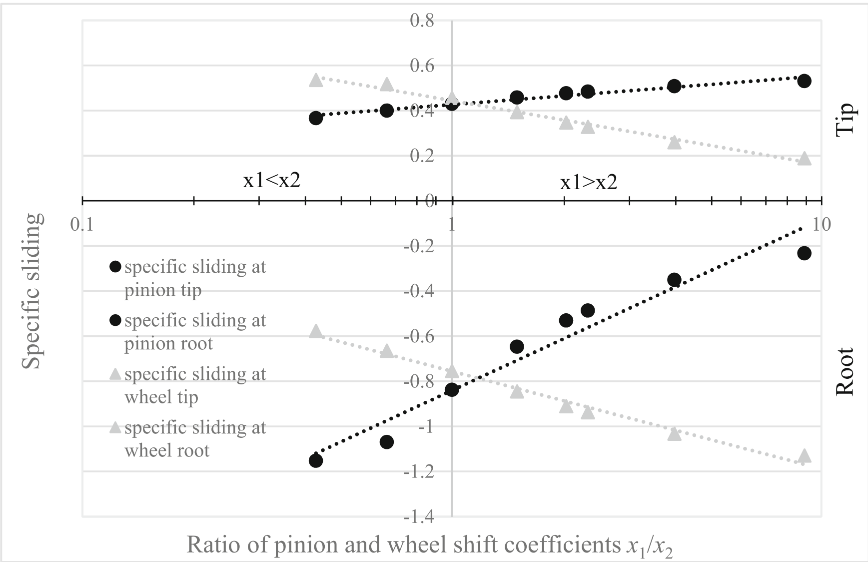

Fig. 4. Shift coefficients dependencies on specific sliding [2].

Figure 4 shows that if the pinion's shift coefficient is lesser than the wheel's shift coefficient $x_1/x_2 < 1$, the absolute value of specific sliding at the pinion's tip is lower, and the value of the specific sliding at the pinion's root is higher than in case of the opposite ratio of shift coefficients. The same applies to the wheel.

4 Conclusions

All the above-presented dependencies support the following findings. Higher values of pressure angle α_n and helix angle β can be used to design gear without a significant risk of increasing the value of the specific sliding; quite the opposite, this value decreases with increasing angles α_n and β. Another important finding is that when the pressure angle α_n is changed, the constant values of a_w and α_{tw} are no more valid for the constant sum of shift coefficients x_Σ. The same applies for changing the helix angle β. Consequently, designer can easily be mistaken. The last finding is based on the dependence of shift coefficients on the specific sliding value. As already mentioned above, the specific sliding value at the pinion's root reaches the top values. For this reason, it is recommended to choose a higher value of the pinion's shift coefficient than the wheel's shift coefficient. The above only applies when it is feasible in the gear design.

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