



# Contribution to the Determination of the Coefficient $K_{H\beta-C}$ According to the Standard ISO 6336

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**Abstract.** The standard ISO 6336 was released in 2006 and is a key standard for gearing loading capacity calculation. The first part of this standard is a basic one for a determination of very important factors ( $K_A$ ,  $K_V$ ,  $K_{H\alpha}$ ,  $K_{H\beta}$ ,  $K_{F\alpha}$ ,  $K_{F\beta}$ ). There are some new annex. Annex C is used to analyse crowning conditions in this Part 1 [1]. The aim is a simplified determination of the very important coefficient  $K_{H\beta}$  by the simplified method C for crowned teeth. In Annex C, the main part is a graph in which the factor  $K_{H\beta-C}$  can be read as a function of  $C^*_\beta$  and  $F^*_{\beta x}$ . This paper deals with the determination of the coefficient  $K_{H\beta-C}$  in this Annex C. It is quite confusing and there are also some typos.

**Keywords:** Coefficient  $K_{H\beta}$  · ISO 6336 Part 1 · Annex C · Typos · Errors

## 1 Introduction

The standard ISO 6336 – *Calculation of load capacity of spur and helical gears*, was released in 2006 and is a key standard for gearing checking and loading capacity assessment [2, 3]. It also addresses its service life. In any task related to the design and optimization of the drive of mechanical systems, its application is absolutely necessary. *Basic principles, introduction and general influence factors* which is the first part of this standard. This part was amended in 2008 with *Technical corrigendum 1*, and it is still valid. This is a basic standard and instruction for a determination of very important factors ( $K_A$ ,  $K_V$ ,  $K_{H\alpha}$ ,  $K_{H\beta}$ ,  $K_{F\alpha}$ ,  $K_{F\beta}$ ). This last edition of this part of the standard is bigger than previous one (109 + 8 against 96 + 4 pgs.). It is interesting that the simplified calculation method C is still using for calculations of some factors in this part. Moreover there are some new annex. Annex C is used to analyze crowning conditions. The aim is a simplified determination of the very important coefficient  $K_{H\beta}$  by the simplified method C for teeth modified longitudinally. Determination of the coefficient  $K_{H\beta}$  at the strength calculation of gearing is always very complicated. In all computational standards, it is one of the most complicated calculations. Usually, the actual calculation procedure takes up several dozen pages of the appropriate standard. Obviously also for this reason, in addition to the detailed calculation method B, the simplified method C is still available for the calculation of the coefficient  $K_{H\beta}$  in ISO 6336. In doing so, method C in this

standard has abandoned in a number of other calculations in this standard (comparing the 1996 edition to 2006). This is doubly true for modified gears. Probably for these reasons, the latest edition of ISO 6336, Part 1 (2006), also includes Annex C. This Annex deals with the calculation of the coefficient  $K_{H\beta-C}$  for modified teeth. In Annex C, the main part is a graph in which the factor  $K_{H\beta-C}$  can be read as a function of  $C_{\beta}^*$  and  $F_{\beta x}^*$ . The aim of the study is to facilitate the calculation of the important coefficient  $K_{H\beta}$ .

### 1.1 Crowning Modification

## 2 Typos and Errors

Some errors in the standard need to be clarified before the main part of the article is implemented:

Page 82, eq. (C.1) – replace ...  $C_{\beta}^* = \frac{c_{\beta} \cdot c_{\gamma\beta}}{F_m/b}$  ... with...  $C_{\beta}^* = \frac{C_{\beta} \cdot c_{\gamma\beta}}{F_m/b}$

where  $\rightarrow C_{\beta}^*$  – non dimensional crowning height [-].

$C_{\beta}$  – crowning height [ $\mu\text{m}$ ]

$c_{\gamma\beta}$  – mean value of mesh stiffness per unit face width [ $\text{N}/(\text{mm} \cdot \mu\text{m})$ ]

$F_m/b$  – mean transv. spec. loading [ $\text{N}/\text{mm}$ ]

Page 82, title of Clause C.2.2 – replace ... mesh misalignment ... with ... initial equivalent misalignment ...

Page 84, eq. (C.5) – replace ...  $K_{H\beta} = 1 + \frac{C_{\beta}^*}{3} + \frac{(F_{\beta x}^*)^2}{16 C_{\beta}^*}$  ... with ...  $K_{H\beta} = 1 + \frac{C_{\beta}^*}{3} + \frac{(F_{\beta x}^*)^2}{16 C_{\beta}^*}$

$F_{\beta x}^*$  – non dimensional initial equivalent misalignment [-]

$$F_{\beta x}^* = (1,33 B_3 f_{sh} + f_{ma}) \frac{c_{\gamma\beta}}{F_m/b} \tag{1}$$

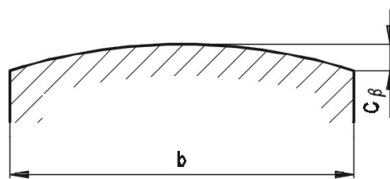


Fig. 1 Crowning for straight gears

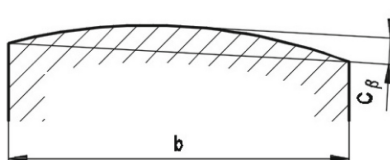
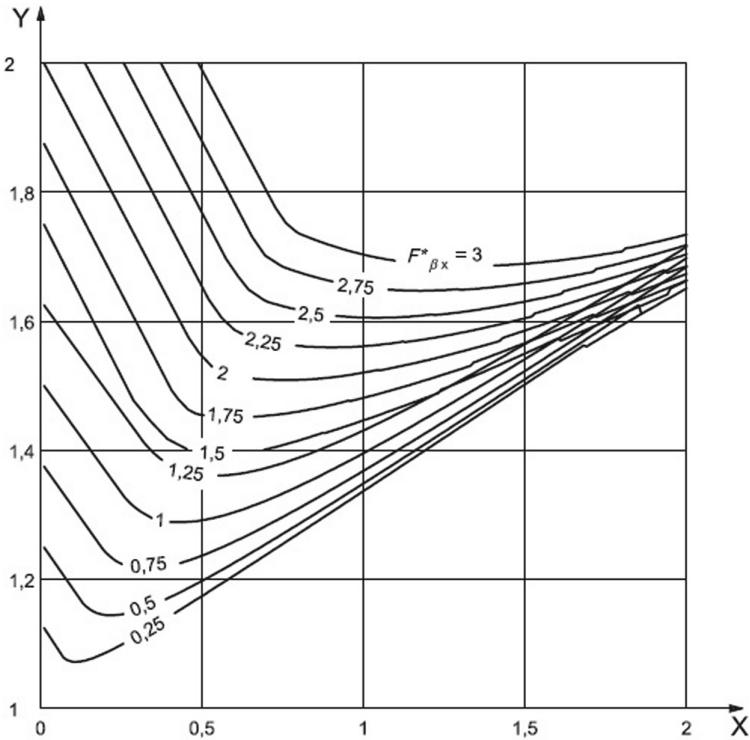


Fig. 2 Crowning for helical gears



**Key**

X crowning height,  $C_{\beta}^*$

Y face load factor,  $K_{H\beta}$

**Figure C.1** — Face load factors  $K_{H\beta}$  for crowned gears

**Fig. 3** Figure C.1 taken from the standard

where  $\rightarrow f_{sh}$  – component of equiv. misalignment due to deformations of pinion and wheel shafts [ $\mu\text{m}$ ]  $f_{ma}$  – mesh misalignment due to manufacturing deviations [ $\mu\text{m}$ ]  $B_3$  – 0,1 if a helix modification is carefully calc. otherwise is equal to 1,0.

### 3 Results and Discussing

The most important part of Annex C is the Fig. 1. This graph is described by a series of equations that are sometimes misleading. Therefore, the entire graph will be divided into several areas and equations will be assigned to these, which will simplify the search for the  $K_{H\beta}$  factor. There are areas in the graph where the inputs for are overlapped. For this reason, it is necessary to proceed very carefully in the calculation. An appropriate procedure follows. The graph is divided into 4 priority areas (A,B,C,D). Important is the curve  $C_{\beta}^* = 0,25 \cdot F_{\beta x}^*$  which separates the area A from the areas C and D. This curve and all areas are shown in Fig. 2.

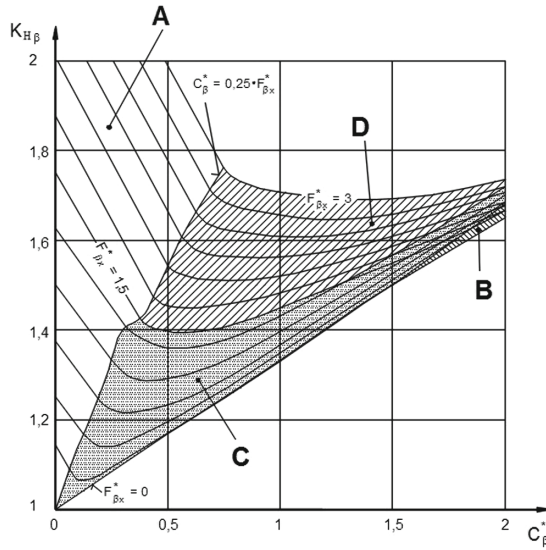


Fig. 4 Figure C.1 with four areas

### 3.1 Area A

Simple relationships apply to gears without crowning ( $C_{\beta} = 0$  [ $\mu\text{m}$ ]). These are intersections of the curves  $F_{\beta x}^*$  with the vertical y-axis of the graph.

$$\text{if } F_{\beta x}^* < 2 \text{ then } K_{H\beta} = 1 + \frac{F_{\beta x}^*}{2} \text{ if } C_{\beta}^* \geq 2 \text{ then } K_{H\beta} = \sqrt{2 F_{\beta x}^*}$$

The linear parts of the curves  $F_{\beta x}^*$  start from the vertical axis y. They ended at the limit, which is the curve  $C_{\beta}^* = 0,25 \cdot F_{\beta x}^*$ . The values of  $K_{H\beta}$  for the end points of the lines lying on the curve  $C_{\beta}^* = 0,25 \cdot F_{\beta x}^*$  can be calculated using equations from adjacent regions C or D. The resulting value of the factor  $K_{H\beta}$  can be solved by using interpolation between two lines (Figs. 3, 4 and 5).

### 3.2 Area B

The smallest area at the right side of the graph is bounded by two restrictions  $C_{\beta}^* > 1,5$  and:

$$F_{\beta x}^* < 4 \cdot C_{\beta}^* \cdot \left( 1 - \left( \frac{1,5}{C_{\beta}^*} \right)^{1/3} \right) \quad (2)$$

The thick boundary curve between B and C is given by the Eq. (2). Factor  $K_{H\beta}$  is then easily enumerated:

$$K_{H\beta} = \left( 2,25 C_{\beta}^* \right)^{1/3} \quad (3)$$

The calculations in this area are somewhat unclear. There is a possibility of overlapping two areas for different  $F_{\beta x}^*$  values. This means that for two different  $F_{\beta x}^*$  values, the

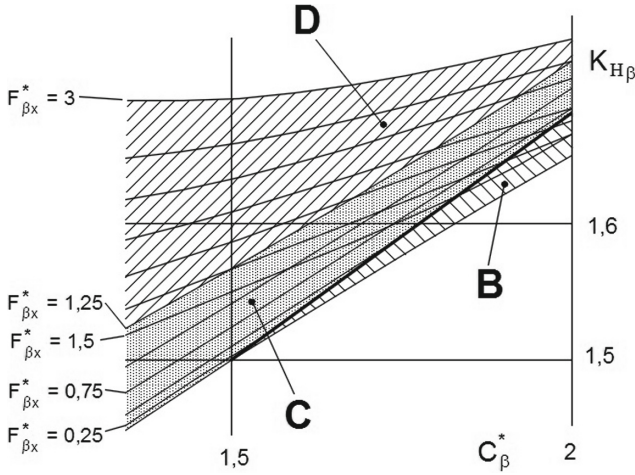


Fig. 5 Detail

same value of the factor  $K_{H\beta}$  is obtained. For example, this thick boundary curve between B and C starts for values  $F_{\beta x}^* = 0$  and  $C_{\beta}^* = 1,5$  and ends for  $F_{\beta x}^* = 0,7315$  and  $C_{\beta}^* = 2$ . This means that for some values  $F_{\beta x}^*$ , it is necessary to calculate by equations valid in area C or D even when passing through area B.

**3.3 Area C**

This area is bounded by two curves -  $C_{\beta}^* > 0,25 \cdot F_{\beta x}^*$  and  $F_{\beta x}^* < 1,5$ , (but there is one small area B where the calculation does not apply for some values  $F_{\beta x}^*$ ). For this area applies:

$$K_{H\beta} = 1 + \frac{C_{\beta}^*}{3} + \frac{(F_{\beta x}^*)^2}{16 C_{\beta}^*} \tag{4}$$

For this area it is now possible to find the curve  $F_{\beta x}^* = 0$ . It is a line starting from the origin of coordinates and going up just below the curve  $F_{\beta x}^* = 0,25$ . It is also true here that for the values of  $F_{\beta x}^* > 1,5$  passing through this regions C it is necessary to calculate according to the equations valid for the region D.

**3.4 Area D**

The last area is bounded by curves  $C_{\beta}^* > 0,25 \cdot F_{\beta x}^*$  and  $F_{\beta x}^* \geq 1,5$  with small exception given by crossing curves for different values of  $F_{\beta x}^*$ . The calculation of  $K_{H\beta}$  is done by simple numerical calculation, which converges relatively quickly. But near the curve  $C_{\beta}^* > 0,25 \cdot F_{\beta x}^*$  it is quite inaccurate (Fig. 4).

Starting value for the loop is  $q = 1$ .

$$k = \sqrt{\frac{q}{C_{\beta}^*}} \tag{5}$$

$$m = \frac{F_{\beta x}^*}{8 \cdot C_{\beta}^*} + \frac{k - 1}{2} \quad (6)$$

$$t = 4 \cdot C_{\beta}^* \cdot m(k - m) \quad (7)$$

$$A = \frac{2 \cdot q \cdot k - C_{\beta}^* \cdot m^3}{3} - \frac{m \cdot t}{2} \quad (8)$$

$$q = q - \frac{1,5 \cdot (A - 1)}{k} \quad (9)$$

End is for  $A = 1$ , and then  $K_{H\beta} = q$ .

## 4 Conclusions

For  $F_{\beta x}^*$  values that are not represented in the graph, the value of factor  $K_{H\beta}$  is found by simple linear approximation. Reading  $K_{H\beta}$  values directly in the graph is quite confusing and inaccurate. In addition, there are areas with some problems in the graph. For these reasons, it is much more convenient and accurate to work with a mathematical graph replacement. However, the errors and uncertainties mentioned in this Annex of the standard as described in this article must be eliminated.

## References

1. ISO 6336 Part 1 :2006. Calculation of load capacity of spur and helical gears – Basic principles, introduction and general influence factors.
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