Design of an Experiment Verifying the Effect of the Cardan Shaft on the Drivetrain Vibration

Jiri Struz(✉), Lukas Hruzik, and Jiri Havlik

Faculty of Mechanical Engineering, Department of Machine Parts and Mechanism, VSB – Technical University of Ostrava, 17. Listopadu 15/2172, CZ 708 33 Ostrava, Czech Republic

jiri.struz@vsb.cz

Abstract. At present, the unevenness of the cardan shaft, more precisely the Hook joint, can be described mathematically. This formulation of the non-uniformity is based on the knowledge of the angular velocity of the drive and driven shaft. However, there is no real understanding of how the non-uniformity affects the behavior of the drivetrain, in particular the vibration of the drivetrain. This paper describes an experiment that may shed light on this problem. Another contribution of this research could be to investigate how the length of the cardan shaft affects drivetrain vibration.

Keywords: Length of cardan shaft · vibrations · drivetrain · torque · angular velocity · frequency

1 Introduction

This article deals with the analysis of the drivetrain vibration and is based on the results of previous measurements. [1] The results of previous measurements show that the main source of vibration is the cardan shaft, because the dominant vibration is manifested on the second rotational frequency of the cardan shaft. In this case, the cardan shaft is part of the drivetrain in which the torque is transmitted from the main transmission to the second transmission. The main problem is that the natural frequency of the cardan shaft is higher than the operating frequency. Another problem is that the deflection (bend) angle is not of great value, since the non-uniformity is approximately 0.2% and appears to be insignificant. This raises the question, "How does the non-uniformity affect the motion in reality?" However, other parts of the drivetrain must be considered. For example, there is a planetary gearbox in the main gearbox whose position is ahead of the cardan shaft and the central gear has the same rotational frequency as the cardan shaft. Therefore, there is an assumption that the planetary gearbox is shown in the frequency spectrum instead of the cardan shaft.

The first part of this article describes the load on the cardan shaft more specifically the load on the Hook Joint. In this part, a mathematical formulation is derived that describes the evolution of the dependence of the driven shaft torque on the angle of rotation of the drive shaft. These results will be compared with the results from the rigid

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dynamic analysis. At the end of this investigation, the undamped natural frequency of oscillation will be evaluated. The second part of this paper focuses on the description of the experiment itself. The main purpose of this experiment is to determine the dependence of the shaft speed on the drivetrain oscillation. Another purpose is to eliminate the influence of other parts of the drivetrain on the frequency spectrum.

2 Materials and Methods

2.1 Analytical Procedure

The uneven running causes a change in torque within one revolution of the car shaft. The change in torque causes a change in the forces acting on the joint bearings. This change in force can cause unwanted vibration. To determine the change in torque, it is necessary to know the kinematic relationships. The mathematical relationship between the angle of rotation of the driveshaft and the angle of rotation of the intermediate shaft is described by Eq. (1). [2–5]

$$\alpha_2 = arctan[tan(\alpha_1) \cdot cos(\beta_1)] \tag{1}$$

If we perform the time derivative of the mathematical relation (1), we obtain relations defining the dependence of the angular velocity of the driven shaft on the angular velocity of the drive shaft. [6] Also, the angular velocity of the driven shaft depends on the deflection angle and the angle of rotation of the drive shaft. For the calculation, the angular velocity of the drive shaft is assumed to be constant.

$$\dot{\alpha_2} = \frac{d\alpha_2}{dt} = \omega_2 = \frac{\omega_1 \cdot cos(\beta_1)}{sin^2(\alpha_1) \cdot cos^2(\beta_1) + cos^2(\alpha_1)} \tag{2}$$

If we know the angular velocity and torque applied to the shaft, we can calculate the power. We have assumed a lossless power transfer, see Eq. (3). [7]

$$T_1 \cdot \omega_1 = T_2 \cdot \omega_2 \tag{3}$$

The mathematical description (4) for the torque acting on the intermediate shaft is derived from formula (3).

$$T_2 = \frac{T_1 \cdot \left[sin^2(\alpha_1) \cdot cos^2(\beta_1) + cos^2(\alpha_1)\right]}{cos(\beta_1)} \tag{4}$$

Figure 1 shows the arrangement of the cardan shaft in this application. Equation (4) describes the evolution in time, precisely the dependence of the torque of the driven shaft on the rotation angle of the driveshaft. In this application, Eq. (4) describes the torque applied to a part of the cardan shaft, called the intermediate part, as a function of the angle of rotation of the drive shaft.

The mathematical description of the torque on the driven part can be evaluated in a similar way. The final formulation of this mathematical relationship is in Eq. (5).

$$T_3 = \frac{T_2 \cdot \left[cos^2(\beta_2) \cdot cos^2(\alpha_2) + sin^2(\alpha_2)\right]}{\cos(\beta_2)} \tag{5}$$

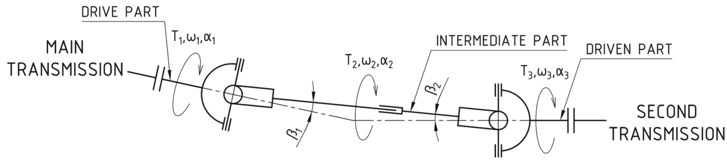

Fig. 1. Arrangement of cardan shaft.

Assuming constant angular velocity and torque on the drive shaft at the same angles between the shafts, the torque applied to the intermediate shaft is variable, but the torque applied to the driven shaft must be constant and the same as the torque on the drive shaft. [2] Fig. 1 shows that the angle between the shafts, or the angle between the driven shaft and the intermediate shaft, and the angle between the intermediate shaft and the driven shaft are not the same. This causes different torque and angular velocity on the drive, intermediate and driven shafts. The time evolution of the torque is shown in Fig. 2.

Due to the unequal angles between the shafts, the torque on the driven shaft is variable and is not the same as the torque on the driven shaft, which can be seen in Fig. 2. This causes time-varying loads on shaft and gearbox mountings and other parts. These parts can create vibrations that spread through the structure. Figure 2 shows that the torque amplitude on the intermediate shaft is very small, approximately 15 Nm. In the case of the driven shaft, the amplitude is about 7 Nm. If we compare the magnitude of the amplitude with the magnitude of the torque on the driven shaft, the amplitudes are very small, but the torque and forces acting on the cross bearings are quite large. It is not clear whether this phenomenon can cause high vibration of the drive train. This is the main reason why conducting this experiment is necessary. Another reason is to exclude the influence of other parts of the driveline.

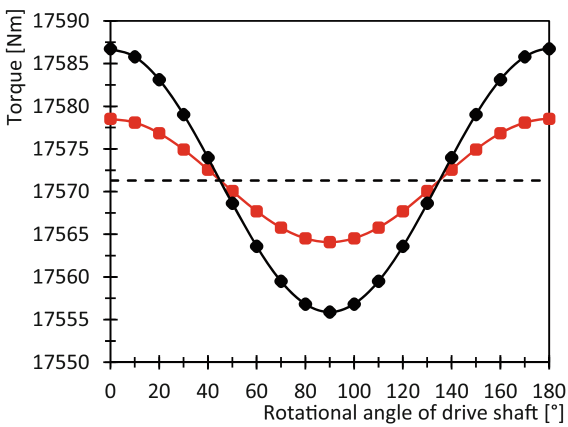

Fig. 2. The theoretical functions describe the dependence of the torque on the rotation angle of the drive shaft (the red line describes the torque acting on the driven shaft, the black line describes the torque acting on the intermediate shaft).

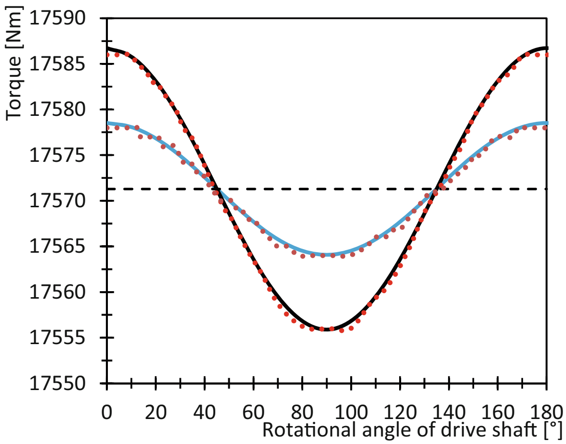

Fig. 3. Comparison of analytical results and rigid body dynamics results (red dotted line describes the torque acting on the intermediate shaft and orange dotted line describes the torque acting on the driven shaft, solid lines describe analytical results).

2.2 Rigid Body Dynamics

To verify the mathematical relationships described above, a rigid dynamics analysis was performed. The rigid dynamics conditions are the same as in the analytical description in Chap. 1 (Fig. 3).

3 Results and Discussing

The calculations in Chap. 2 describe the main reason why this experiment is necessary. Another reason is to exclude the influence of other parts of the drivetrain. In this application, the drivetrain is made up of different parts such as the main gearbox, the second gearbox, the differential and the wheel reducer. For this reason, the test rig will be considerably simpler. In essence, it will consist of a drive electric motor or an internal combustion diesel engine, a cardan shaft and a brake, as an open circuit test rig concept has been chosen. The measurement principle is shown in Fig. 4. The movement of the driven part changes the length of the cardan shaft and thus the angle between the shafts.

Changing the length of the shaft will cause the natural frequency to change. To get a better idea of how the natural frequency will change, a calculation is necessary. For initial considerations, the analytical method is chosen.

Since the evaluation of the results by the analytical method is very complicated for the drivetrain, it is necessary to introduce some simplifications here. The simplified torsion system is called the reduced torsion system. Basically, it is a reduction in mass and length. The simplified conditions can be defined according to [8] as follows:

- The mass is constant and independent of time;
- The length is constant and independent of time;

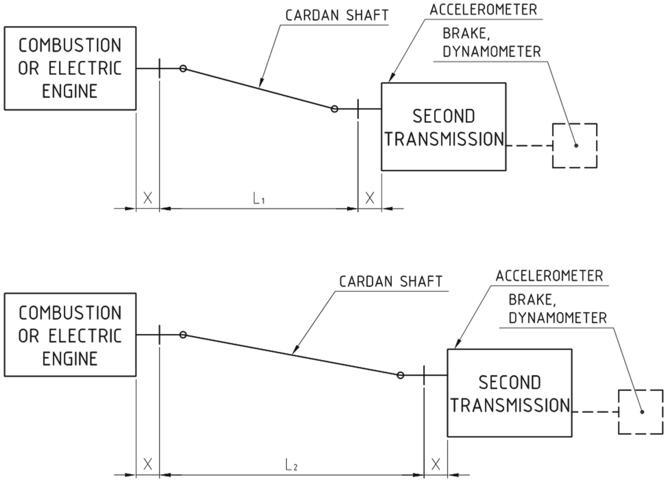

Fig. 4. Experimental scheme.

- The system is homogeneous.

It was also necessary to simplify the model of the cardan shaft. This meant that all the curves and chamfers were neglected. The result of the reduction was a shaft with a constant diameter of $d_{red} = 97.1$ mm with a reduction length of $L_{red} = 172.4$ mm. Knowing the length and diameter of the reduction as well as the moment of inertia, the stiffness of the reduced cardan shaft was evaluated according to Eq. (6):

$$c = \frac{G \cdot \pi \cdot d_{red}^4}{32 \cdot L_{red}} \tag{6}$$

Mass moment of inertia for the reduced cardan shaft:

$$I_{red} = \frac{1}{2} \cdot m_{red} \cdot \left(\frac{d_{red}}{2}\right)^2 \tag{7}$$

Undamped natural frequency of oscillation:

$$\Omega_o = \sqrt{\frac{c}{I_{red}}} \tag{8}$$

Subsequently, the dependence of the length of the cardan shaft on its natural frequency was evaluated. The following characteristic was evaluated for the given conditions:

From the Fig. 5, it is seen that natural frequency is very high. It can be assumed that real natural (eigen) frequency will be lower, because the resulting stiffness will be reduced by pliancy of the cardan shaft cross, shaft mounting and others. This fact is confirmed by the modal analysis of the given cardan shaft, from which these facts can be observed (Figs. 6 and 7).

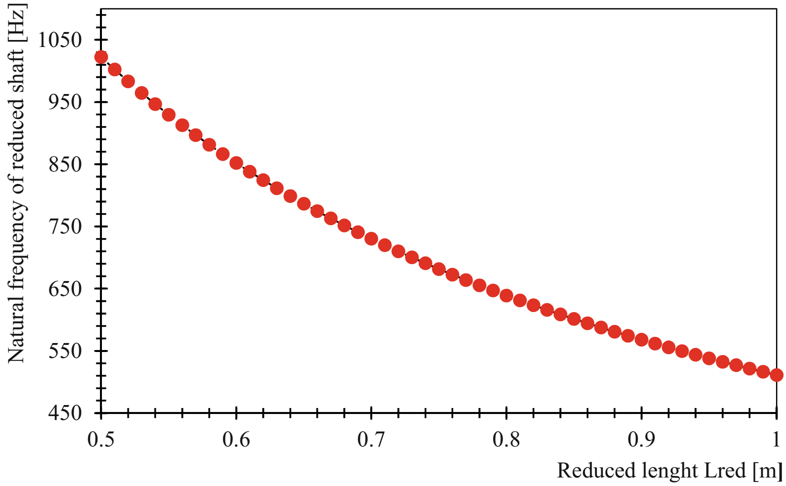

Fig. 5. Dependence of the natural frequency on reduced length.

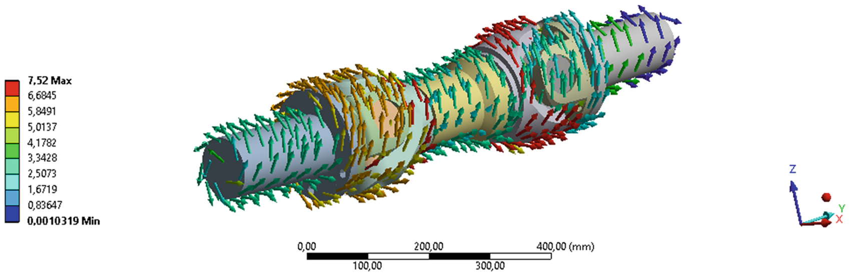

Fig. 6. Direction of deformation at the first eigenfrequency.

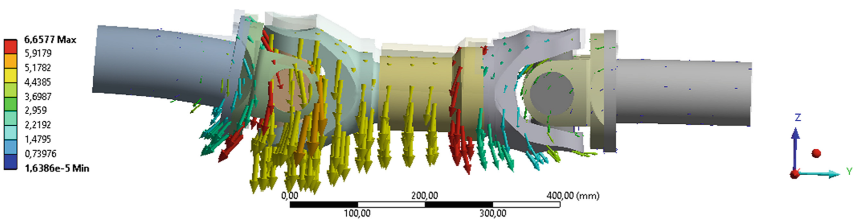

Fig. 7. Direction of deformation at the second eigenfrequency.

Based on the results of the modal analyses performed by the FEM, a similar dependence was established and is shown in Fig. 5 (Fig. 8).

Moreover, the amplitude of the torsional oscillation can be determined. Even though it is more convenient to use the concept of an experimental device with an electric motor to perform the measurements, an internal combustion engine will be considered in the following considerations, precisely because of the application under investigation.

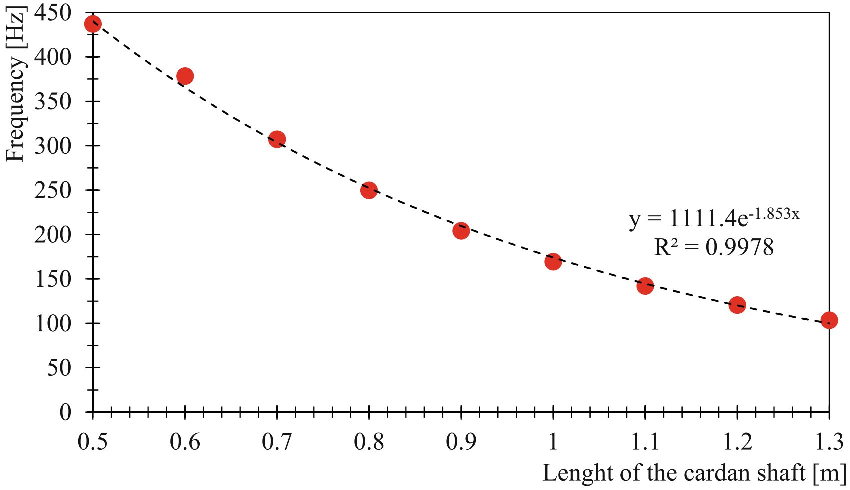

Fig. 8. Cardan shaft length dependence on first natural frequency.

According to [8], the drive torque of diesel engine is defined as follows:

$$T(t) = T_a \cdot sin(\omega_m \cdot t) \tag{9}$$

The amplitude was determined by the Eq. (10):

$$\phi_a = \frac{M_a}{I_{red}} \cdot \frac{1}{\sqrt{\left(\Omega_0^2 - \omega_0^2\right)^2 + (2 \cdot \delta \cdot \omega_0)^2}} \tag{10}$$

In Eq. (10), ω_0 represents excitation frequency. Obtained amplitude characteristics is visible on Fig. 9. In Fig. 9, there is a comparison of results obtained by FEM analysis (blue line) and analytic method (red line). Figure 9 also shows that the FEM results are on the left. This is because the analytical calculation does not take into account the bearing stiffness, hook joint cross stiffness and other parts because their analytical solution is difficult.

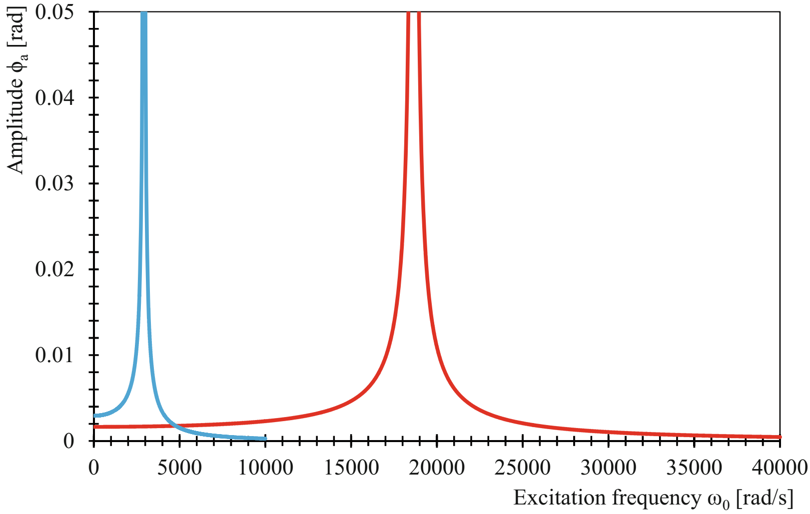

Fig. 9. Amplitude characteristics.

4 Conclusions

This article describes part of the solution to a problem in which the cardan shaft plays a major role. The first part of the article deals with the mathematical description of the load on the cardan shaft, in particular the mathematical description of the torque. In this part, a mathematical relationship is derived that describes the magnitude of the torque on the intermediate and driven shafts. These analytical results have been compared with the results obtained by rigid dynamic analysis. The determination of the magnitude and effect of each component on vibration is the subject of further investigation. The second part of this article describes an experiment that could provide a new perspective on the issue of cardan shafts. Through this experiment we will be able to understand the behaviour of the cardan shaft and in particular have a better understanding of the non-uniformity. So far, we have been able to describe the non-uniformity mathematically, but no one has any idea of what effect the non-uniformity has on the behaviour of the cardan shaft in reality. Furthermore, with this measurement we can get more accurate information about the effect of the length of the cardan shaft on the vibration of the driveline. At the same time, two approaches used to calculate natural frequencies are compared. This shows that the analytical approach can often be burdened with error due to the difficulty of describing the stiffnesses of some parts. In fact, the given structure is considered stiffer than in the case of FEM calculation and reality.

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