

# **Application of CAD Programs for a Gearing** Geometric Model

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Abstract. CAD programs are used in engineering environment on a daily basis, because of their wide use in designing, machining or evaluating process. This fact boosted the development and impact range of CAD software possibilities, but in the field of gears, there may be some shortcomings. These shortcomings are presented in a lack of gear generating tools or addons, and also in a possibilities these tools and addons provide. For uncommon geometries of gears, user may need to use other methods. Some of them are explained in this article. The advantages of explained methods are that they can be used for almost any gear wheel, or the modification of existing gear is quicker.

Keywords: 3D model · Gears · CAD

#### 1 Introduction

A gear wheel is a rotating element of a machine that is capable of transmitting power and torque [1, 2]. The spur gear is the simplest type of gear that has radial gearing. The efficiency of gears is generally almost 98%. Modeling of gears using CAD programs helps to draw complex tooth curves, but also facilitates gear manufacturing, as the CAD model is a necessary input for programming CNC machines [3-5].

Modern high-performance gearboxes require gears with a specially designed tooth geometry to ensure that the design requirements of such wheels are met. Among other factors, the main requirements include the sufficient strength of the side bearings and the heel of the teeth and, last but not least, the minimization of noise emissions during operation [6, 7]. In order to be able to adjust the strength of the tooth, it is necessary to change the angle of engagement or, if conditions allow, to increase the profile of the tooth [8].

Modeling of curves and surfaces for graphical interpretation using computer-aided methods of geometric design is widespread in many industries, especially for functional analysis, control of production conditions and for a possible increase in aesthetics. In preliminary design, geometric modeling is usually performed in an interactive manner This means that designers start with a sketch and then modify it until aesthetic and functional requirements are met. Generally, modeling techniques and procedures such as polynomial and spline functions are used to model volumetric 3D bodies where the positions of multiple points on curves or surfaces are precisely known [9, 10].

The aim of this study is to present a couple of ways, in which the gearing geometry can be created. One of the explained methods is to virtually simulate manufacturing process. Second method is creation of gearing model by parametrically defined curves, which ones can also be made by graphical method, but this way is much more time consuming. Last method is the most universal of them all, but excels in creation of geometrically harder gearing e.g., face gears.

## 2 Materials and Methods

### 2.1 Mixed CAD Modeling

One of the methods for gear generation is simulation of the whole or just a part of the machining process. Such CAD model generation consists of solid modeling of semifinished product, which will become gear wheel, and the tool, which is used in the machining process. These two models have to be aligned into a correct position. In the next step the simulation is proceeded by given movement steps for tool, or in some cases for both tool and workpiece as well. The gearing is made as subtraction of one volume against another, i.e., tool volume is subtracted from workpiece volume wherever they intersect. Disadvantage of this method (Fig. 1) is in steps-precision combination. The more precise model should be the smaller steps should tool carry out, but smaller steps mean longer generation time [11].

Mixed CAD modeling is also a simulation of the same machining process, but instead of solid tool, there is used only a profile of the tool (Fig. 2). This profile carries out the same step movement like in subtraction method described above, but the main difference is that this profile only generates points. Generated points make a point cloud, which has to be edited, preferably by algorithm, in a way that excessive points are removed (Fig. 3a). At the final stage, the generated and edited point cloud is used for surface generation (NURBS modeling) Fig. 3b. By adding volume inside generated surfaces, the CAD model is generated. By obtaining point cloud, the gear generation is much faster and can also be more precise than subtraction of volumes [11].

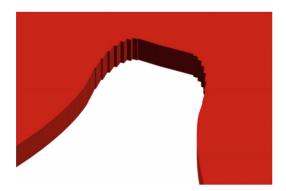


Fig. 1. Tooth gap made by subtraction [9].

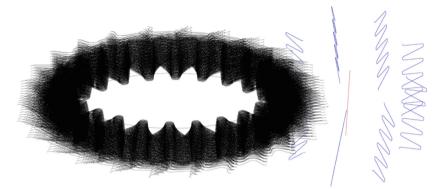


Fig. 2. Principle of mixed CAD modeling method [11].

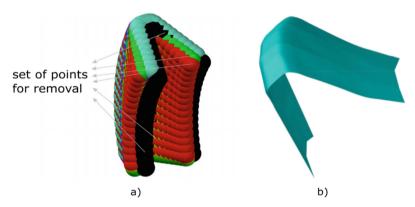


Fig. 3. Tooth generated by a) points, b) surfaces made through points [11]

#### 2.2 Parametric Modeling

This method is applicable to spur gears, helical gears and also gears with a special tooth shape. The principle of this method is to calculate all of the values of the gear wheel first, and then the involute curve and the transition curve in the CAD program are created using the parametric curve function, or other means of modeling. The gearing is created by removing the profile that forms the tooth gap, but there are also methods, which generate teeth as a solids. Subsequent copying of this profile around the circumference of the gear wheel at equal intervals results in total gearing.

#### 2.3 Modeling of Spur, Helical and Bevel Gears

The fundamental basis for good meshing conditions is involute shape gear tooth. In parametric modeling, the involute is driven by these Eqs. (1), (2) and by given parameters (Fig. 4).

$$x = r_b \cdot (\cos(t) + t \cdot \sin(t)) \tag{1}$$

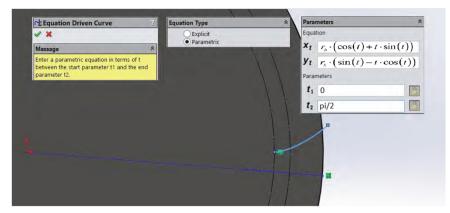


Fig. 4. Parametrically defined involute.

$$y = r_b \cdot (\sin(t) - t \cdot \cos(t)) \tag{2}$$

Profile of the gearing is formed not only by the involute curve but also by transition curve. The shape, which this curve holds, is given by the machining process and have a significant effect on the bending stress. Two kinds of geometries of transition curves are: trochoidal fillet and circular fillet. Trochoidal curve geometry is defined indirectly by the rack geometry used in machining process, while the circular fillet geometry is defined directly [10].

#### **Trochoidal Fillet**

When the gear is made with rack tool in a hobbing process, the tip of this tool creates a cut to the tooth geometry in a shape, which is represented by trochoidal curve (Fig. 5). The equations representing this curve in polar coordinate system are:

$$\theta_t = tan^{-1} \left( \frac{\sqrt{r_t^2 - (r_b - h_k)^2}}{(r_b - h_k)} \right) - \frac{\sqrt{r_t^2 - (r_b - h_k)^2}}{r_b}$$
(3)

$$\psi_t = tan^{-1} \left( \frac{r_b(r_b - h_k) - r_t^2}{r_b \sqrt{r_t^2 - (r_b - h_k)^2}} \right)$$
(4)

For the cartesian coordinates of the trochoidal curve:

$$\theta_t'' = \theta' - \delta \pm \theta_t$$

where:  $\theta'$  - original angle at the center of the tooth,

 $\theta_t$  – vectorial angle of the trochoid,

 $\theta_t''$  – vectorial angle of the trochoid with Y angle center of the tooth.

$$x_t = r_t \sin \theta_t^{\prime\prime} \tag{5}$$

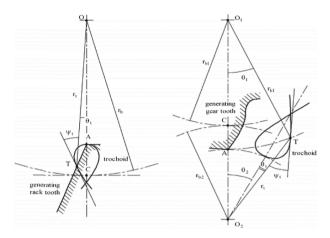


Fig. 5. Trochoidal fillet geometry [13].

$$y_t = r_t cos \theta_t'' \tag{6}$$

where:  $r_t$  – any radius of the trochoid. [13, 14]

#### **Circular Fillet**

The sketch of gear tooth is illustrated in Fig. 6, where the principle of circular fillet is demonstrated graphically. Point O is the center of the gear and for the coordinate system. Axis y is chosen to be the axis of symmetry for the sketched tooth. Point B lays on a base circle which radii is  $r_s$ . Point of tangency of the circular fillet with the root circle  $r_f$  is A. Point D is situated a line  $\varepsilon_2 = OA$  and is the center point of the circular fillet. Line  $\varepsilon_3$  is tangent to the root circle at point A intersecting with line  $\varepsilon_1$  generating point C. The fillet curve is tangent to the line  $\varepsilon_1$  at point E. In equation  $r_s > r_f$  signify that the circular fillet can be made without exceptions on all spur gears [13, 14].

To make a circular fillet, the only thing needed is exact coordinates of point D. This point is center point of the arc, which is tangent to root circle in a point A. the equations for cartesian system coordinates are:

$$x_D = (r_f + AD)sin(\zeta + \Omega_s) \tag{7}$$

$$y_D = (r_f + AD)cos(\zeta + \Omega_s)$$
(8)

The length of AD is determined by the user but the whole bracket must have higher value than  $r_f$ , otherwise there would be no fillet, and the maximum value of the bracket is when point E = B:

$$(r_f + AD) = r_s tan\zeta \tag{9}$$

The  $\zeta$  is defined as:

$$\zeta = \cos^{-1} \left( \frac{2S}{1 + S^2} \right) \tag{10}$$

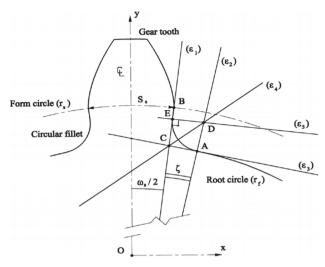


Fig. 6. Circular fillet geometry [13]

where:

And S is non-dimensional parameter given by the equation:

$$S = \frac{r_s}{r_f} > 1 \tag{11}$$

The  $\Omega_s$  is:

$$\Omega_s = \frac{s_s}{2r_b} \tag{12}$$

where:  $s_s$  – width of the tooth.

#### 2.4 Modeling of Face Gears

The face gear is a widely researched new transmission mechanism. This mechanism found its use in special transmission type group e.g., in aviation industry. Face gears are characterized by special teeth flanks, which consists of work surface and transition surface as shown in Fig. 7. Therefore, modeling of tooth contour cannot be modeled directly [15].

Given by the exemplary coordinate system in Fig. 8, the two surfaces can be obtained by Eqs. (13, 14). Figure 8 contains of four coordinate systems, which are used for equations for surface generation. Tool and face gear systems are  $s_s = (o_s, x_s, y_s, z_s)$  and  $s_2 = (o_2, x_2, y_2, z_2)$ , where  $z_s$  and  $z_2$  align with the face gear and tool axes. The angle  $\gamma$  stands for the drive axis angle. Next coordinate systems are  $s_{s0} = (o_{s0}, x_{s0}, y_{s0}, z_{s0})$ and  $s_{20} = (o_{20}, x_{20}, y_{20}, z_{20})$ . The angle  $\varphi_s$  stands for the tool rotation angle, while  $\varphi_2$ is the rotational angle for face gear [15].

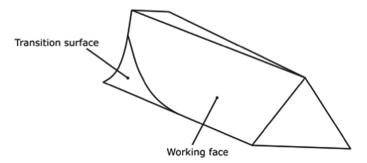


Fig. 7. Schematic representation of face gear tooth.

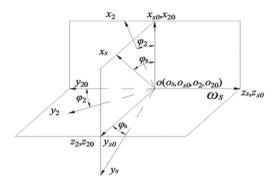


Fig. 8. Coordinate system for face gear generation [15].

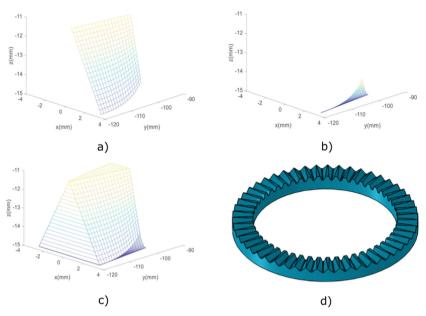
$$\vec{r}_{2}(\theta_{s},\varphi_{s}) = \begin{bmatrix} r_{bs} \Big( cos\varphi_{2}(sin\varphi_{\theta} \pm \theta_{s} \cdot cos\varphi_{\theta}) - \frac{sin\varphi_{2}}{q_{2s} \cdot cos\varphi_{\theta}} \Big) \\ -r_{bs} \Big( sin\varphi_{2}(sin\varphi_{\theta} \pm \theta_{s} \cdot cos\varphi_{\theta}) + \frac{cos\varphi_{2}}{q_{2s} \cdot cos\varphi_{\theta}} \Big) \\ -r_{bs}(cos\varphi_{\theta}) \pm \theta_{s} \cdot sin\varphi_{\theta} \end{bmatrix}$$
(13)

where:  $r_{bs}$  - the base circle radius of the tool,

 $\varphi_2$  - face gear rotation angle  $\varphi_{\theta} - \varphi_{\theta} = \varphi_s \pm (\theta_{s0} + \theta_s).$   $\theta_s$  - angle parameter of the point on the involute curve of the tool.  $\theta_{s0} - \theta_{s0} = \frac{\pi}{2N_s} - inv\alpha_s.$   $N_s$  - number of teeth of the tool  $q_{2s}$  - inverse gear ratio

$$\begin{cases} x_2^*(\varphi_s) = r_{bs} \left[ \cos \varphi_2 \left( \sin \varphi_{\theta}^* \pm \varphi_s^* \cos \varphi_{\theta}^* \right) - \frac{\sin \varphi_2}{q_{2s} \cos \varphi_{\theta}^*} \right] \\ y_2^*(\varphi_s) = -r_{bs} \left[ \sin \varphi_2 \left( \sin \varphi_{\theta}^* \pm \varphi_s^* \cos \varphi_{\theta}^* \right) + \frac{\cos \varphi_2}{q_{2s} \cos \varphi_{\theta}^*} \right] \\ z_2^*(\varphi_s) = -r_{bs} (\cos \varphi_{\theta}^* \pm \theta_s^* \sin \varphi_{\theta}^*) \end{cases}$$
(14)

where:  $\theta_s^* - \theta_s^* = \frac{\sqrt{r_{as}^2 - r_{bs}^2}}{r_{bs}}$ 



**Fig. 9.** Generated a) work surface, b) transition surface, c) surface model of a tooth, d) face gear [15].

 $\varphi_{\theta}^* - \varphi_{\theta}^* = \varphi_s \pm (\theta_{s0} + \theta_s^*).$ 

Due of the shape of the face gear tooth surface profile, a discrete point method can be utilized to produce the tooth surface. In this case it is almost impossible to calculate points positions in a reasonable time without an additional computational software such as Matlab. Chosen supplementary software has to be able to output computed data in a spreadsheet format accepted by modeling CAD program. This method can be used for any gears or in the face gear – pinion combination, it can be used to generate pinion as well [13].

Calculated discreet point cloud of tooth surface was used in Matlab for simulation of the surfaces. Supplementary program does not have to be Matlab tough, a simulation can be done, but is not required. The Matlab simulation is shown in Fig. 9 a, b, c, where user can check for any major geometry defects. The simulation as well as point generation was carried out in two steps. First step was generation of work face, second step contained of generation of transition surface and then these surfaces were connected, forming a tooth. The calculated point coordinates are edited to generate point cloud in CAD software accepted files. The surface of the tooth is generated in a CAD program. Whole gear wheel is created by array of individual teeth and discs (Fig. 9 d) [15].

### 3 Results and Discussion

Mixed CAD modeling is direct upgrade from so called subtraction modeling, where final product is obtained by subtracting one volume of another. Advantages are in much faster generating time, the tool used in process is made of just a profile curve of such tool and

therefore the software needs to only generate points, rather than subtracted volumes. Generation of points eases up computing power needed, so user can define the whole process to take smaller i.e., more precise steps in point cloud generation. All of this means that the process is faster and more accurate.

Parametric modeling is form of subtraction modeling, where key parts are defined on one point (gear tooth or gear gap) and then the volume of such feature is made. This volume is then subtracted or merged and patterned around to form a gear. Great advantage of this method is in how accurate and fast the gearing can be modelled. Although the weakness of this method is that not every CAD program dispose of the parametrically driven curve function, but there is possibility to define such curve with a graphical method, which almost nullifies the speed of the method. Another disadvantage is that this method is harder to use for geometrically difficult gearing.

Modeling of face gears was defined by discretization method. Face gear was used as an exemplary demonstration of the principles. This method can be used for any gear wheel, but two step process of obtaining point clouds and then transformation of this clouds to 3D solids in CAD software must be sticked to. Generation of the gearing is similar to mixed CAD method, although discreditation method uses additional software for point clod generation. This can mean that the point clod can be denser, which improves accuracy of modeled gearing and can be easier for CAD software in a meaning that point cloud is imported not computed in CAD environment.

### 4 Conclusions

CAD programs have become an integral tool for designers after their appearance on the market. Thanks to their advantages, they have been very rapidly expanded and developed by developers to such degree that they are now applicable in every aspect of engineering practice. One of these aspects is the design and construction of gears.

When designing gears, it is necessary to be able to create a 3D model, for better visualization and later for control, by the finite element method or others, as quickly and accurately as possible. An unwritten rule is that the more accurate the model, the longer it takes to create or generate it. However, with the continuing development of CAD programs and increasingly powerful computing technology, this difference is disappearing.

CAD programs can provide designers with both the aforementioned visualization and the ability to inspect a given gear, as well as the ability to modify an existing gear. Last but not least, a CAD model is needed for programming some of the production machines.

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