Analysis of a Four-Bar Linkage Mechanism in Its Classical and Compliant Form - A Comparison

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Abstract. A four-bar linkage mechanism to accomplish different tasks. Those mechanisms cis used in a vast number of application in different fields accomplish different tasks. Those mechanisms can have some theoretical and practical limitations, depending on their applications. From a theoretical standpoint, four-bar linkages mechanisms are modelled with rigid bodies with rotary joints, and, from a practical standpoint, those joints require additional maintenance during operation and lubrication. On the other hand, compliant mechanisms do not have joints in a classical sense but the relative movement between linkages is accomplished through the deformation. Certain segments of the mechanism are thinned to achieve relatively localized large deformation, which will allow movement between stiffer segments (linkages) of the mechanism. Compliant mechanisms have several advantages and disadvantages compared to classical mechanisms. Those mechanisms are essentially one part that doesn't require any lubrication and there is also no backslash between movable parts. Neverless, due to the mechanism design principles, the strength of the overall structure and movement range can be very limited. Besides basic parametres, in the design process of complaint mechanisms, the dimensions type of its "joints" has sn interesting part of overall performance. The stiffness of theese joints and their size will cause the relative movement of the rotary axis between linkages. The variable position of the rotary axis between adjacent linkages gives one more degree of freedom in the design process. This phenomenon opens up possibilities for fine-tuning, especially in cases where a classical mechanism is used as a design template for compliant mechanisms. An example of this process is a classical mechanism that produces approximately straight-line motion where the transition to a compliant mechanism can further improve straightness of its trajectory.

Keywords: Straight-line mechanism · Compliant mechanism · Cad modelling

1 Introduction

During several hundred years of engineering practice, in many cases, engineers formed a distinct viewpoint on material and component flexibility. In most applications, naturally occurring flexibility of materials is a property that is best avoided if at all possible or

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reduced to be negligible. With this approach, systems are analysed and designed as rigid, especially since the calculations of rigid systems are less complicated [1, 2]. On the other hand, there is the approach where the natural flexibility of materials is utilized to realize system movement. The mechanisms where The mechanism movement is achieved through the deformation of their structure are called compliant mechanisms. Compliant mechanisms are not a new technology. Namely, the examples where material flexibility is used to generate movement can be found throughout human history. Perhaps one of the first tools that can be classified as the compliant mechanism is the bow and arrow, where elastic energy of bow deflection is used to propel the arrow into the air [3, 4].

Based on the way in which motion, force and energy are transmitted, mechanisms can be classified into two categories: rigid body mechanisms and compliant (deformable) mechanisms [4]. Both of theese categories have some advantages and disadvantages. Compliant mechanisms do not require assembly - in most cases, the complete mechanism is one part, the gap between parts does not exist so backlash can not occur during movement, and, since there is no sliding between elements, there is no need for lubrication. On the other hand, a compliant mechanism can not have constant motion as input (for example, an electric motor that will provide continuous rotation, as input) - increasing distance from the mechanism's equilibrium position requires an increase in active force values and, if deflection from equilibrium becomes significant, the mechanism structure can exceed the ultimate strength of the material [3, 4].

However, while compliant mechanisms had been used throughout human history in their simple shapes and forms they did not require complicated calculations, and complete design was based on the designer's talent and skills. With the development of computers and the increase in their computational power, in the last 40 years, the analysis and synthesis of complex shapes and topologies have become relatively easy and fast [4]. With progress in design and manufacturing technology complaint mechanisms have found applications ranging from everyday products to sophisticated technologies. Everz day we use some type of a compliant mechanism without even realizing it - for example, a simple shampoo bottle lid utilizes compliant hinges for repeatable opening and closing of a bottle. This example best illustrates the benefits of the application of part reduction when compliant mechanisms are used. One part simply Scompletes the whole action of opening and closing the bottle without any joint in the classical sense [3, 4]. Much more sophisticated applications can be found in different MEMS (Micro-Electromechanical Systems) devices, where the whole mechanism is no bigger than 100 μm. On these scales, manipulation of separate parts and their assembly can be very challenging, so there is a clear advantage of using compliant mechanisms, as one complete part, in those devices. Without MEMS technology modern smartphones packed with a large number of sensors (such as accelerometers, magnetic or gravity sensors etc.), would not exist [3–6].

Positioning parts with high precision (in micrometre/nanometre scales) is necessary for many areas of the modern industry, especially in semiconductor manufacturing processes. The manipulation and positioning of chips during the manufacturing processes must be controlled in several micrometres or even nanometres, and nence the only logical solution for these precision ranges are compliant mechanisms. In some special cases, macro devices also require such kind of precision, where it is necessary to control the device position in a couple of microns over a wide range of shapes of different kinds of mirrors. Maybe the best example of these applications are several optical control systems on the James Webb space telescope. The construction and extreme conditions in which the telescope operates have created some unique problems that need to be solved. The primary mirror consists of 18 separate hexagonal mirrors and each of them needs to have nanometre position precision for each of 6 degrees of freedom. This is accomplished by high reduction motors that rotate eccentric shafts which drive a compliant mechanism connected to a mirror construction. Using this system, the shape of mirrors is controlled to provide the best geometrical shape for an object that will be recorded with a precision smaller than the wavelength of light [7]. The Webb telescope was designed to capture the farthest galaxies. With this in mind, even if Webb is focused on a small patch of the sky, it captures a large number of objects. To capture an even smaller patch of sky or analyse spectra from one object, a mechanical slit mask was designed using a compliant mechanism to improve accuracy. In a 137×137 mm^2 field of view, this mechanism can form 24 optical slits, with variable sizes (from 50 μm up to 137 mm) with position accuracy $+ -5$ μm and with accuracy $+ -8$ μm [8].

In the past 30 to 40 years, several approaches for designing compliant mechanisms were developed. Those approaches can be divided into 3 groups: the Kinematics-based approach, the building blocks approach, and topology optimization [1, 9]. In the kinematics approach, knowledge of the rigid bodies' mechanisms is applied to compliant ones. This approach has two dominant methods, the FACT (Freedom and Constraints Topologies), and rigid body replacement (RBR) methods. The FACT deliveres only a set of topologies that requires additional analysis. The RBR is a method where flexible segments are replaced with equivalent rigid links, pin joints, and torsional springs [1, 3, 9, 11–14].

The idea behind the building blocks approach is that the main mechanism is composed of several smaller mechanisms which represent the building blocks of the mechanism [15, 16].

The topology optimization can be seen as an approach where part of the material continuum is optimized in such a way that material distribution satisfies given constraints [17, 18].

2 The Idea

In the past several centuries, a lot of effort was invested into mechanism development by a large number of people. The mechanisms are used to solve a vast number of problems. Some mechanisms were used to make everyday life easier, while others controlled huge machines and systems [19]. Modern humans in everyday life rely on electronic devices for various things. For example, modern smartphones are packed with sensors that can measure the magnetic field, humidity, acceleration, used as calculators, etc. [20] Before the electronic age, all of those values were measured with a mechanical sensor and mechanism that transformed one type of energy and motion into another more useful form. The complexity and precision necessary to manufacture such devices limited their use by a large number of people, but modern mass production manufacturing technology has menaged to overcome those limitations. To produce a cheap, yet relatively reliable device, besides advance in parts production, the design process must adapt too.

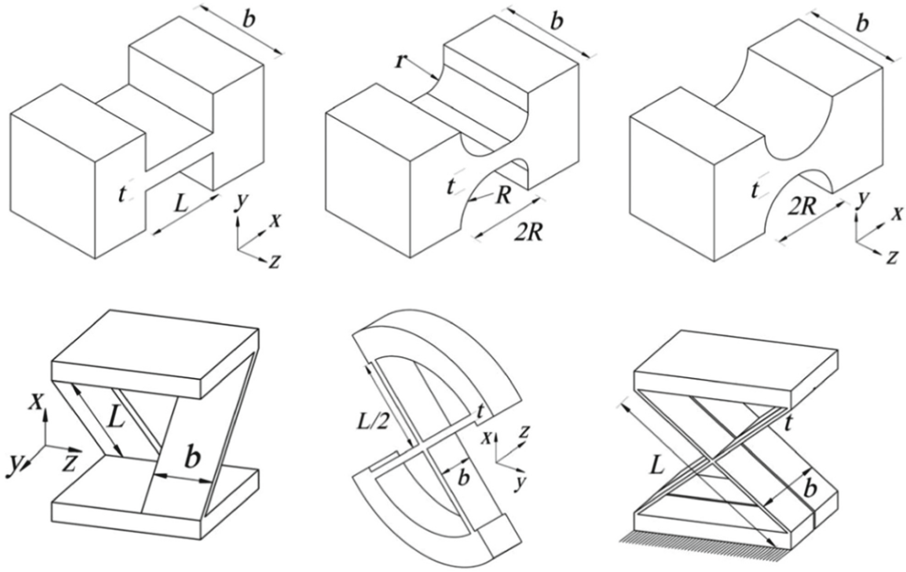

Fig. 1. Some examples of compliant joints [21]

In the past two centuries, a tremendous amount of knowledge was collected by academics in every field of human activity. Various examples of solutions to many problems can be found in such knowledge collections. In the field of mechanis, one such example are the m books by Artobolevsky, where examples and solutions for many practical and theoretical problems, that solved by the scientists before him, can be found.

The main idea of this paper is to analyse the possibility of improving classical rigid body mechanisms through their simple conversion into compliant ones, with the focus on the simple conversion. In his book, L. Howell analyses the rigid body replacement method for compliant mechanism synthesis. Under the joint section, several examples of compliant joints have been explained. Each of those joint types has its advantages and disadvantages. From the manufacturing standpoint, probably the easiest and the simplest joint type is the living hinge (See Fig. 1). A living hinge is formed simply by thinning the material on the joint location. Other joint types may have better kinematics characteristics in special applications but the simplicity of living hinges represents a great advantage [4]. With modern CNC machines (Cutting laser, waterjet, CNC mills, 3D printers), simple 2D geometry of living hinges is relatively easy to create and implement into classical planar mechanisms.

The mechanisms that can produce linear or nearly linear motion, are called “straight-line mechanisms” in the literature. Mechanisms where linear motion is achieved only by rotary joints are a spetial case, these are also the most interesting ones. Mechanisms that were designed by James Watt are probably the best-known, fnad later on in this paper the four-bar linkage developed by James Watt will be analysed [22]. The example mechanism will be converted into a compliant one, where joints will be replaced by living hinges. In further analysis, it will be identified how the size and dimension of living hinges influence the mechanisms of movement. Methods for linear curve fitting will be used to qualitatively and quantitatively measure the impact of the compliant joint on the straightness movement. In addition, the impact length and thickness of living hinges will be analysed on bar movement after the pivot. In this case, the goal will be to examine whether at least possible at least possible to qualitatively describe the hinge size impact by calculating the position of poles of planar displacement.

3 Problem Formulation

Watt's four-bar linkage that will be analysed is shown in Fig. 1. Points A and D are fixed and linkages AB and CD rotate around them respectively. Points C and B are connected to point E with a triangular plate BCE. Point E follows a straight line QQ when AB and CD rotate. To obtain straight-line movement linkages of mechanism must satisfy certain relations:

$$AD = BE = 0.68 \cdot AB; \tag{1}$$

$$DC = 0.51 \cdot AB; \tag{2}$$

$$CB = 0.49 \cdot AB; \tag{3}$$

$$CE = 1.1 \cdot AB. \tag{4}$$

To model, the mechanism in SolidWorks of the length of AB = 100 mm has been chosen, according to Eqs. (1), (2), (3), and (4) AD = BE = 68 mm, DC = 51 mm, CB = 49 mm, and CE = 110 mm respectively (see Fig. 2). A small difference in triangular plate design can be noticed if Fig. 2 is compared to Fig. 1. In the case where point E is the sharp point, this design choice has been made to make simulation easier within the SolidWorks environment.

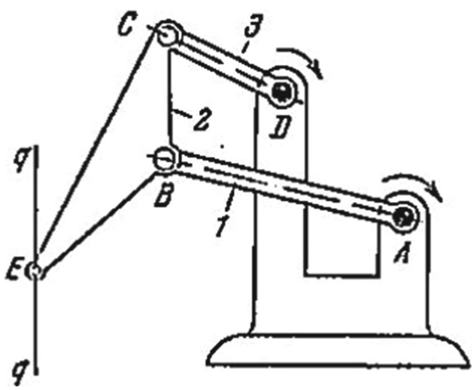

Fig. 2. Four-bar Watt's mechanism [22]

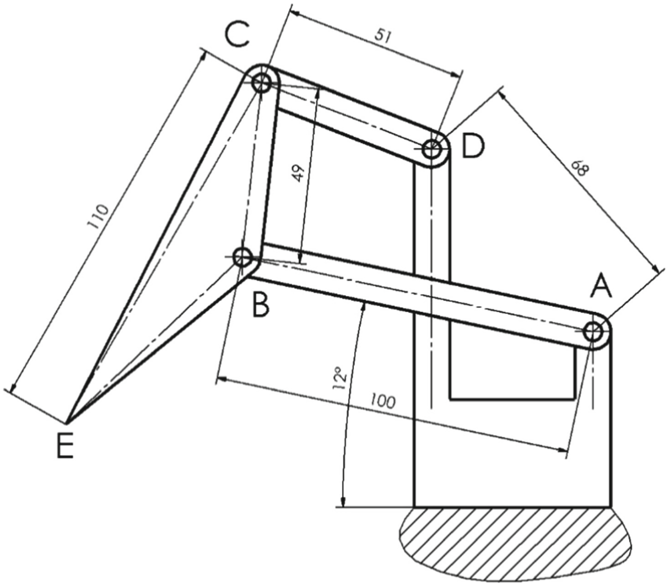

Fig. 3. Classical Mechanism design – SolidWorks model.

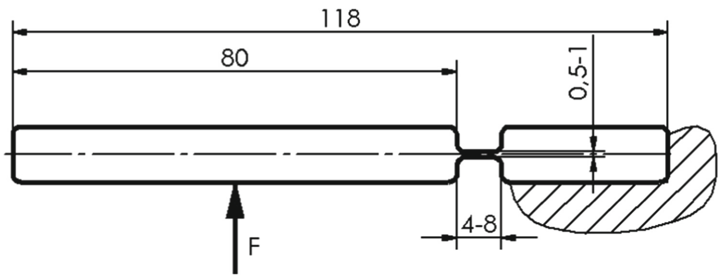

Fig. 4. Single hinge mechanism design – SolidWorks model.

The design of the compliant mechanism is shown in the next image (see Fig. 3). Several modifications have againbeen chosen to make the simulation process much less complicated. All pin joints have been replaced with living hinges. The placement of the living hinges is defined so that the points ABCD are in their centre (see Fig. 3). To investigate the impact of living hinge thickness and length on bar movement, one smaller model with one pivot is created. One bar is fixed (see Fig. 4) and the other bar is pushed with force F. Length and thickness of the hinge are varied.

From classical mechanics, it is well known that planar motion can be divided into translators and rotary parts. For each moment of planar motion, there is a point around which the body rotates. The name of this point is the "pole of planar displacement" (POPD). The position of the POPD determines the movement that the body performs. If that point is stationary then the body only rotates, and if the POPD is in infinity then the movement of the body is completely translatory. In the third case, the body can be

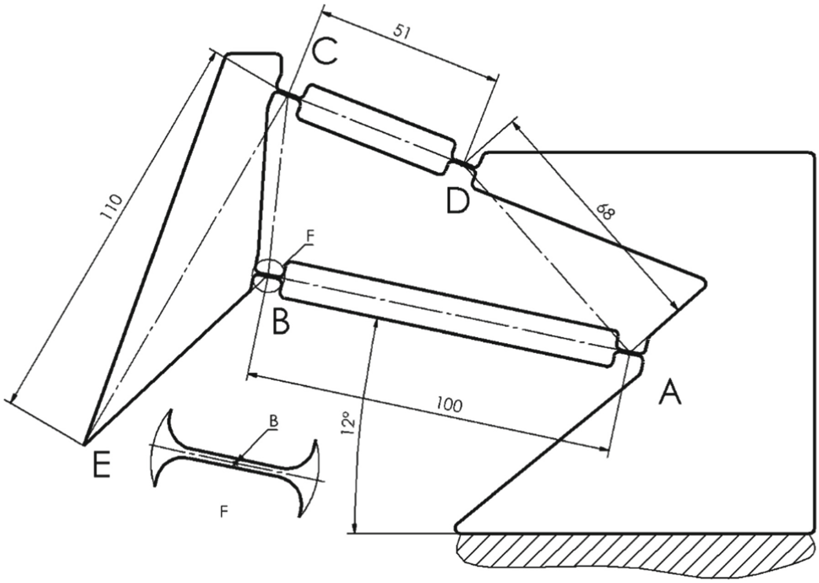

Fig. 5. Compliant mechanism design – SolidWorks model.

moved from one position to another with a combination of both, then The POPD also moves within the plane [23, 24]. The goal of this analysis is simple - if two bodies move in the same way, the POPD trajectory will be the same.

To calculate the POPD position when the body moves from one position to the next it is necessary to know the coordinates of two points on the body in every position.

When the body moves to the next position and takes a new orientation both points can be connected with lines, those lines simply represent the direction of the velocity vector when the body moves between two positions. From mechanics, the velocity vector of a point on the body is perpendicular to the line that connects the POPD and that point. In discrete cases, the POPD can be found as the intersection of two lines, which originates from middle points between two positions (see Fig. 5)

$$x_{am} = 0.5 \cdot (x_{a1} + x_{a2}) \tag{5}$$

$$y_{am} = 0.5 \cdot (y_{a1} + y_{a2}) \tag{6}$$

$$x_{bm} = 0.5 \cdot (x_{b1} + x_{b2}) \tag{7}$$

$$y_{bm} = 0.5 \cdot (y_{b1} + y_{b2}) \tag{8}$$

To find an angle of the direction vector to the defined coordinate system, the following equation needs to be calculated (Fig. 6):

$$\alpha = \arctan\left(\frac{y_{a2} - y_{a1}}{x_{a2} - x_{a1}}\right) \tag{9}$$

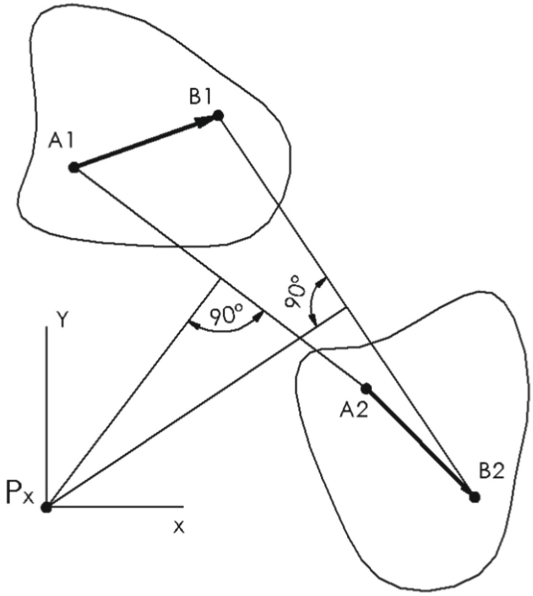

Fig. 6. Planar movement of the body [24].

$$\beta = \arctan\left(\frac{y_{b2} - y_{b1}}{x_{b2} - x_{b1}}\right) \tag{10}$$

Finally from these equation, we can calculate the coordinates of the POPD:

$$x_p = \frac{(y_{bm} - y_{am})\sin\alpha\sin\beta + x_{bm}\sin\alpha\cos\beta + x_{am}\cos\alpha\sin\beta}{\sin(\alpha - \beta)} \tag{11}$$

$$y_p = \frac{(x_{am} - x_{bm})\cos\alpha\cos\beta + y_{am}\sin\alpha\cos\beta + y_{bm}\cos\alpha\sin\beta}{\sin(\alpha - \beta)} \tag{12}$$

Watt's mechanism will produce an almost straight line, but not a straight line - in this case, it would be great to find one number to define the goodness (straightness) of the mechanism trajectory. The data obtained from simulations can be observed as a dataset through which a straight line could be fitted. For [26] measurement of the goodness of a fitted line, the Sum of Squares Due to Error (SSE) will be calculated. Smaller SSE is better fit. [25, 26]. For a given data set, a straight line can be fitted to satisfy the equation (see Fig. 5):

$$y = k \cdot x + n \tag{13}$$

where k and n are unknown. In the case of only two variables, where one is independent, a simple least square method can be used [26]:

$$k = \frac{\sum(x_i - \overline{x})(y_i - \overline{y})}{\sum(x_i - \overline{x})^2} \tag{14}$$

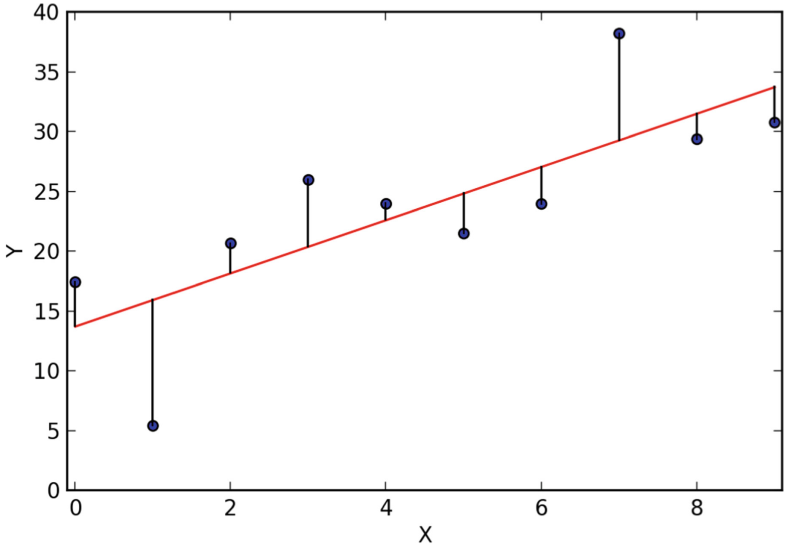

Fig. 7. Linear Fitting example [26]

$$n = \overline{y} - k \cdot \overline{x} \tag{15}$$

where $\overline{y}$ and $\overline{x}$ imply values. Now it is possible to calculate SSE:

$$SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \tag{16}$$

where y_i is a data point and $\hat{y}_i$ is a calculated value using (13), (14), and (15).

4 Simulations

Complete modelling of 3D models, simulations, and calculations was performed in SolidWorks 2021 and MatLab R2020a. For the simulation of the compliant mechanism, it was necessary to use non-linear simulation because the mechanism is subject to large deformations. According to the SolidWorks manual nonlinear static stimulation was performed as a sequence of stress-strain calculations using finite element analysis (FEA). The desired load value of a model was divided into a specified number of steps, and, for each step, force was increased, and new stresses and deformation calculated. Finally, a complete stress-strain state was determined as a combination of stresses and strains from all the previous steps.

Classical mechanisms have a different module for simulating (motion analysis), where parts are represented by rigid bodies. Here, the SolidWorks calculates only kinematics or dynamic properties of the system without deformation.

In both cases, only the movement of several points is necessary for further calculations. Those points are point E and at least one more point for the POPD calculation.

The compliant version of the mechanism is highly dependent on live hinge dimension. The design study optimization is also performed to find hinge configurations that have the biggest deflection for a given force. All simulations of the compliant mechanism were performed with the prescribed force of 5N because the prescribed angular movement would try to satisfy rigid body conditions.

In the design study, optimization was done on each of the A, B, C, and D hinges. For each hinge, two thicknesses (0.5 mm and 1 mm) and two lengths (4 mm and 8 mm) were optimized. This setup provided 256 different cases that needed to be checked with nonlinear FEA. Also, the compliant mechanism dictated that the whole 256 cases should be repeated in other directions. In Fig. 2 and Fig. 3 equilibrium state is defined by the angle of the AB lever and its value is 12 degrees.

The material used in the FEA simulation is ABS plastics with a yield strength of 30 MPa, and Elastic modulus of 2000 MPA, and a Passion's ratio of 0.3. To further simplify, and speed up the simulation process, a planar mesh is used with the following parameters. Mesh type - 2D Planar Mesh and standard mesh, maximum element size up to 2 mm, and minimal element size up to 0.1 mm mesh quality was high. All other simulation parameters were left as default values except for steps size. In the design study, the step size was automatic to speed up the simulation but, for the final model, at least 50 points were calculated in both directions.

In the motion study, the step size was the default value, 20 frames/s, and the mechanism was driven by a rotary motor with a constant speed of 2 RMP, applied to joint A.

From these simulations *.csv files were obtained and they were further used in Matlab scripts to calculate the coordinates of the POPD and SSE of the fitted lines.

5 Results and Discussion

Since in the the FEA the linear materials used were only defined by Elastic modulus,in all analyses, the models were pushed over their yield strength in order to easily analyse their kinematics. With linear material, stresses and deformation will linearly grow and their values do not have an impact on mechanism kinematics and movement.

The results from the FEA performed on a model from Fig. 3 were used to calculate the POPD, the values for thickness and length of the live hinge are the same as those used in the design study, and hence 4 configurations in total were calculated. As a result of the performed simulation and calculation, a diagram was created (see Figs. 7 and 8).

From this diagram, ity can be seen that the hinge with the lowest stiffness has the biggest movement of the POPD, while the thickest and shortest have the smallest range of the POPD movement, which was to be expected. All trajectories of the POPD start relatively smoothy, but, as the deformation increases, they become rough. This result can be a consequence of an increased load, which deforms even the thick lever elements, or numerical inaccuracies in the simulation process. Further investigation is necessary. One

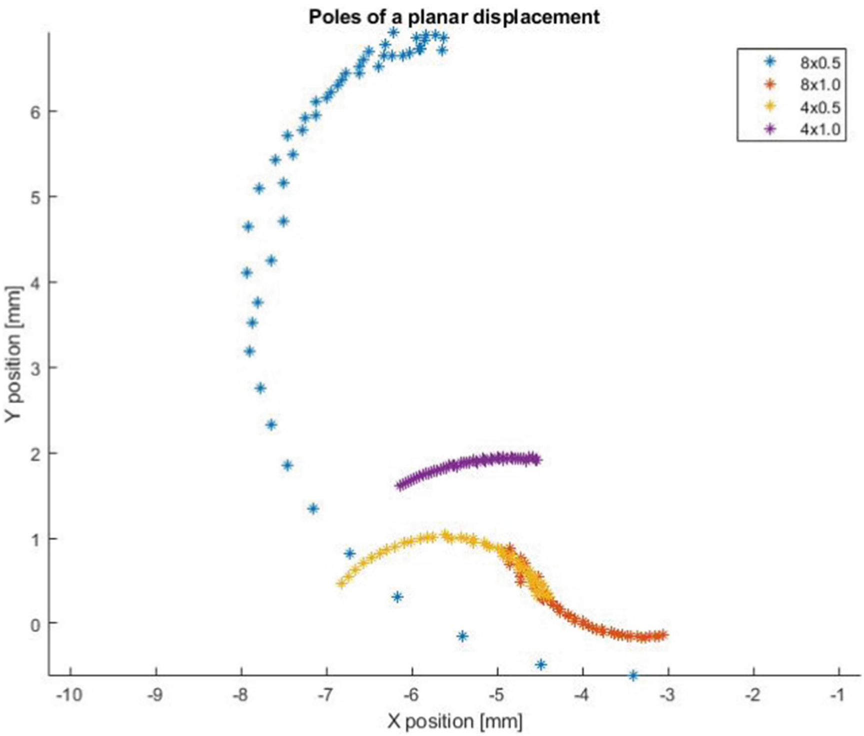

Fig. 8. Poles of Planar displacement for single hinge

more interesting result is trajectory similar bulge orientations for the same length, the 8 mm hinge has a down left orientation and the 4 mm hinge has an upward orientation. A more flexible hinge starts its rotation closer to the central axis (Blue dots, $Y = 0$), and the POPD for stiffer hinges is moved further in the direction of bending. This is simply a qualitative analysis obtained from Fig. 7.

The results of the design study and nonlinear simulation provided a range for motion study. With the same force intensity in the up and down directions, point E of the compliant mechanism moved by different values. Point E moved upward by almost 80 mm and downward only by 30 mm with the same external load. To simplify the analysis, only segments from -30 mm to $+30$ mm will be analysed. Trajectories around the equilibrium match almost perfectly, however, with increasing distance from the equilibrium the trajectories get more separated (see Fig. 9). If we pay attention to the scale of the axes, it can be concluded that the maximum difference between the two trajectories is less than 0.5 mm at a length of 60 mm, which means that the maximum deviation is less than 1%.

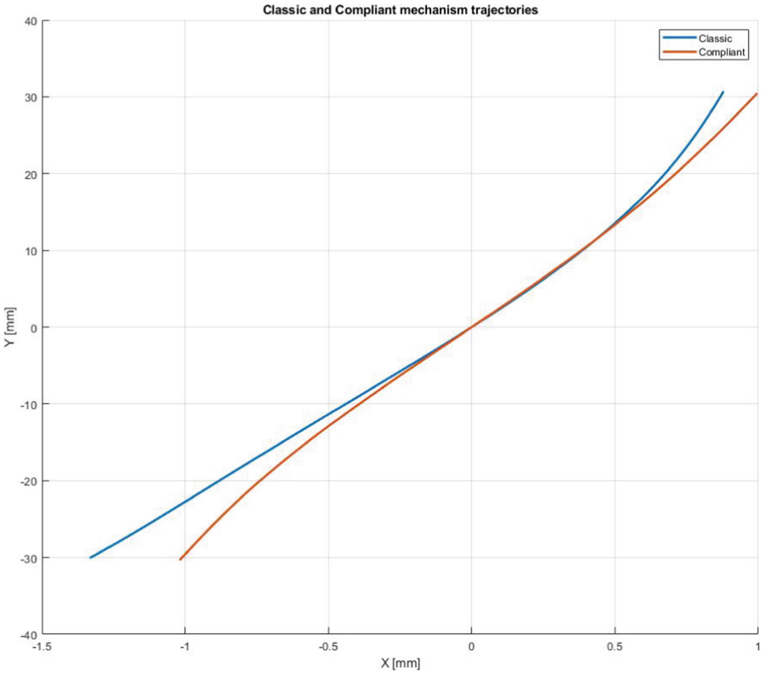

Fig. 9. Classic and Compliant mechanism trajectories

Looking at the POPD trajectory, it can be easily seen that there is a very large jump in one part of the diagram, which implies that there is a moment when the BCE plate moves in a translatory manner (see Fig. 10). This jump is much larger in the classical mechanism than in the compliant mechanism. The difference in the jump may be due to the accuracy of the simulations - still, there is a clear difference in the remainder of the trajectory. Despite all the differences, it is clear that the shape of the trajectory is similar.

The most interesting result is acquired from linear fitting. Equations for the compliant and classical mechanism are $y = 0.0351 \cdot x - 0.009372$ and $y = 0.03849 \cdot x - 0.08112$, with $SSE = 0.0688$ and $SSE = 0.5599$ respectively (see Fig. 11 and Fig. 12). It is important to note that the axes have changed places to avoid high values of the coefficients. Following the previous statement, the compliant mechanism creates a more vertical path for point E. Also, the SSE Value is almost 10 times smaller for the compliant mechanism compared to the classical one, which means that the path of point E is flatter. This is also confirmed with residual values. Compliant mechanism values are about 0.05 mm (see Fig. 12) but the classical oneshave almost double deviations even though there is a segment where the deviation is Higher than 0.2 mm.

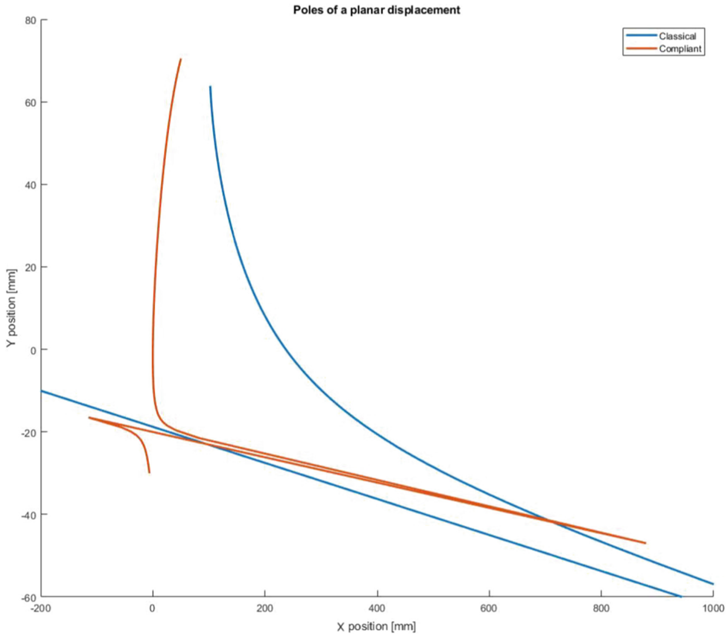

Fig. 10. ThePOPD trajectories

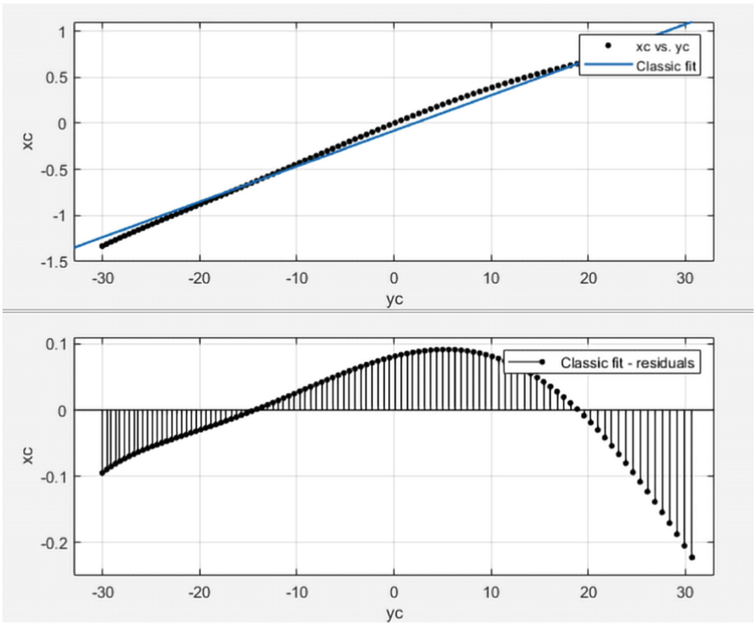

Fig. 11. Classical mechanism

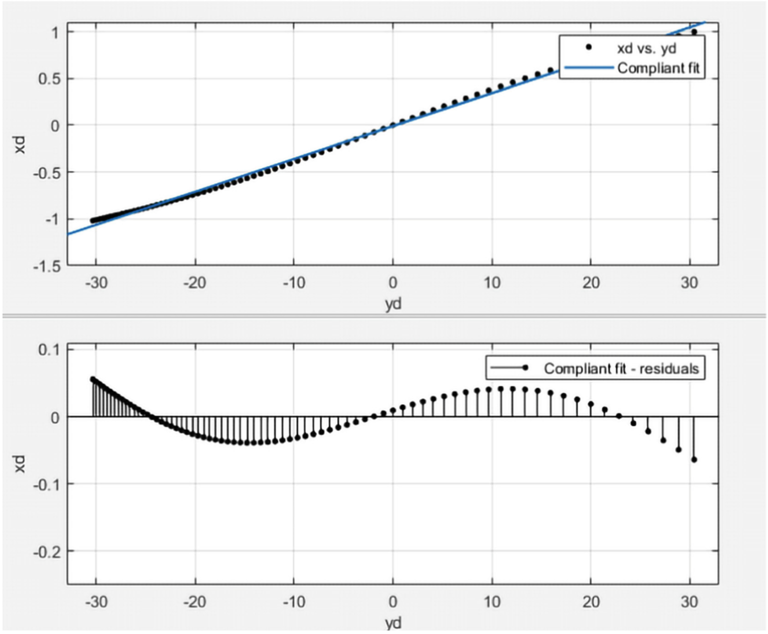

Fig. 12. Compliant mechanism

6 Conclusion

The main idea of this paper was to check the possibility of using topologies of classical mechanisms in the compliant ones. Further, one of the goals was to use the simplest flexible hinged as a replacement for pin joints. These ideas were tested on the example of a four-bar linkage mechanism for rectilinear motion. All analyses were performed with the help of the SolidWorks software and additional calculations were done in MatLab. The result of mechanisms simulations in SolidWorks are the trajectories of point E for both types of mechanisms. An interesting result is that a compliant mechanism has a better performance compared to a classical one without any topological optimization.

These results support the initial idea that it is possible to improve the behaviour of classical mechanismsby converting them into complaint mechanisms. This behaviour is the result of changing the POPD position of the levers with compliant hinge deformation. The size (length and thickness) of a compliant hinge can have a significant impact on the POPD trajectory. Analysing the POPD trajectory can accelerate qualitative evaluation of compliant hinge performance.

The linear fitting has proven to be a relatively simple method to assess the quality of this type of mechanism, since only SSE is sufficient to inferl about mechanism performance.

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